Introduction to Time - Varying Electrical Networks
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Lecture 54
Determining H_0 for input frequency deviations from f_s

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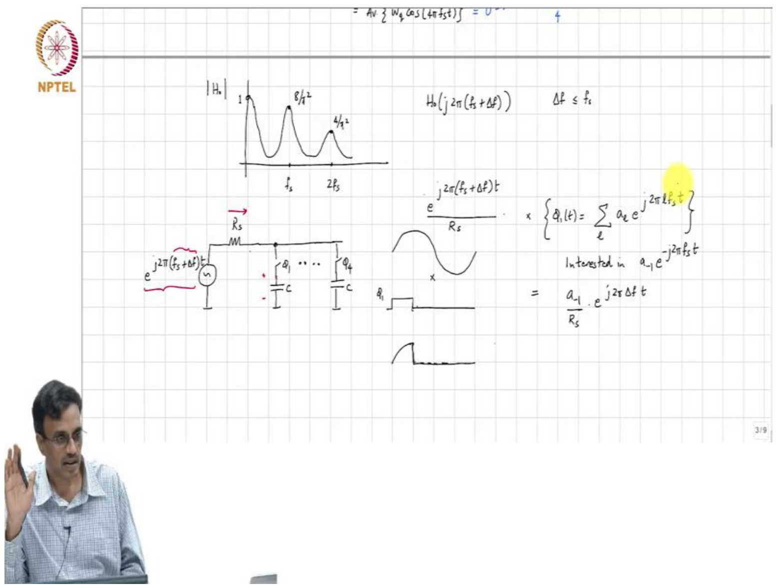

Now, the question is, well H0, I plot mod H0, DC clearly it is 1, at fs it is 8 by pi square at 2 fs it is 4 by pi square and so on. So, the question is what happens in the middle? So, for example, what is H of j 2 pi fs plus delta f where delta f is much, much smaller than fs? So, we change the frequency from fs a little bit, the input frequency, what do you think will happen to the output?

And to do that, well there is I am not going to get into the details now but I am just going to give you a rough idea of the, of how to proceed and intuition about we already have seen that if you, if the input frequency is not exactly fs then each capacitor is going to see different sections of the input waveform over that quarter period. And therefore, the magnitude of the voltage across the capacitor is going to be much smaller than what it would have been if the input is an integral multiple of fs.

So, we should expect the magnitude to be, to fall off, if you deviate from fs. So, basically you should expect behavior something like this, some kind of qualitative behavior that you should expect to be quantitative of course we need to do a little bit of analysis. And to do that we are going to let us say, we excite the input with e to the j 2 pi into fs plus delta f times t.

So, when phi 1 is on, see remember, what comment can you, when delta fs was 0, in other words when the input was e to the j 2 pi fs times t that is both cos or sin, what did we see for the voltage across this capacitor? This was simply DC, it is not changing very much during that period when the switch is turned on. Now, what do you think will happen if instead of the input being fs the frequency changes slightly to fs plus delta f?

Student: DC will vary but…

Professor: Exactly, so what, when the input is exactly at a frequency fs, this voltage would remain constant from cycle to cycle. So, in steady state what you would see here, it would, I mean it would be, in this cycle it would be some waveform and the next cycle would be exactly the same waveform.

And intuitively that is, you can think of it as the following, this input frequency at fs is being switched at fs, switching is basically, I mean in other words this current yeah, we are sampling or multiplying it with a square wave at fs, so what is going into the capacitor is the low frequency component which happens to be DC.

And at the same time the capacitor is trying to discharge through the resistor which is connected only for a fraction of the period. So, when the 2 effects basically equal each other you will have an equilibrium voltage, whatever voltage results is a result of the equilibrium between two opposing effects, one is the mixed down version of the current flowing through the high frequency current flowing through RS, and the other one is the loss of voltage of charge from the capacitor due to this periodic connection with RS.

Now, so basically if I, so if I think of this as e to the j 2 pi fs into t. So, this is vi and we can think of this as being multiplied by phi 1 of t. So, when we do this you are only getting these currents and so on. So, only one period, so this is the, so this is, so this divided, so the voltage divided by RS is, if the capacitor voltage is constant during a clock face, the RF current going in through the resistor is simply e to the j 2 pi fst divided by RS that gets multiplied by this waveform and consequently you can see that this is getting I mean the waveform of the current going through the capacitor is something like this.

And what do you call this on average is the capacitor is simply averaging this and therefore there is some voltage established. So, we are interested in the low frequency component of this waveform multiplied by the width of the phi 1 pulse. So, now by the same token if the input was e to the j 2 pi fs plus delta f times t, the current is the voltage divided by RS.

And we are interested in figuring out what the low frequency component of this multiplied by this phi 1 of ts, phi 1 of t is basically 1 for ts by 4 and 0 otherwise, so as this can be expressed as a Fourier series. So, a sub l e to the j 2 pi lfs times t. So, what will be the component of this current at delta f, which component, I mean what do we need to, which component to the Fourier series?

We are interested in the low frequency component of the product but the input is at e to the j 2 pi fs plus delta f we are interested in the delta f component. So we have to worry about which series coefficient here? We need to worry about in a minus 1 e to the minus j 2 pi fs times t. Yeah, when this gets, this will, I mean so the RF current will get multiplied by the minus fs term in the Fourier series and will yield the result is a minus 1 by RS e to the j 2 pi delta f times t.

Student: (()) (10:19) we are sampling that input signal and whatever fs plus delta f will be there, it will be (()) (10:26) DC current.

Professor: Correct. If the input was exactly fs then it will (())(10:30) component.

Student: Delta f will be DC plus fs.

Professor: Exactly, exactly, exactly.

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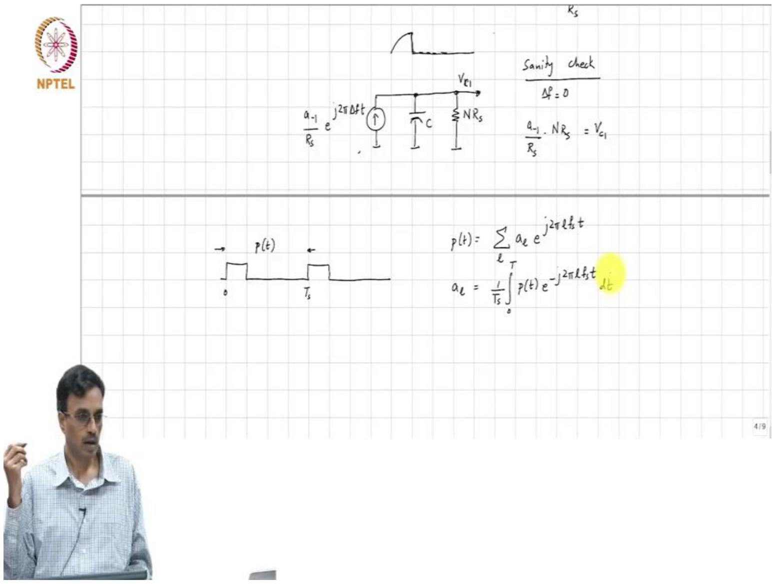

So, you can think of this therefore as the equivalent low frequency current that is flowing in into the capacitor. So, this is a minus 1 by RS e to the j 2 pi delta f times t and this is the capacitor but how is the, but the capacitor is also losing charge. How is it losing charge? When the rest, when the switch is turned on, the voltage across the capacitor is tending to go down because it is going to get discharge through RS.

So, what is the, but this RS is only there for 1 by Nth of the period. So, this is equivalent to having a resistor which is N times larger all the time. Because if per cycle, here it is only on for a small period and yeah, so if you want to represent it as a fixed resistor as far as the low frequency current is concerned you can think of it as having a resistor which is N times larger but is on for the entire duration.

So, this will therefore give us the amplitude, I mean so if you take this current and pass it through this impedance, you will be able to get the low frequency component of the voltage across the capacitor. So, sanity check, what is the sanity check? Delta f is 0. So, what do you think will be the voltage here?

So, a minus 1 by RS times NRS is the voltage across the let me call that VC1. And what is a minus 1? a minus 1 is the minus 1th Fourier series I mean coefficient of this phi 1 pulse and that is, to do that well we use standard results from Fourier transform stuff. Remember if you have a periodic waveform, let us call this P of t, that is the waveform within one period. And what is P of t? P of t is simply sum over l a sub l e to the j 2 pi lfs times t and what is a l, how do you find a l? From P of t it is nothing but integral 1 over Ts P of t e to the minus j 2 pi l fs times t dt.

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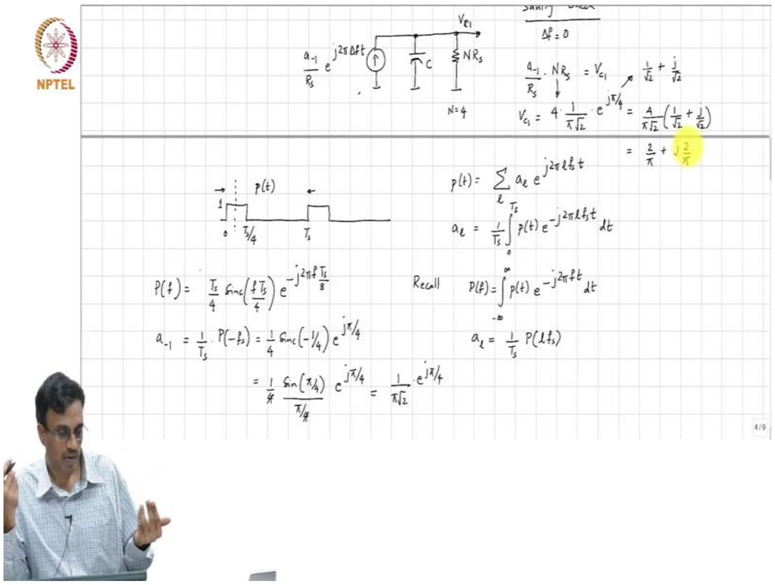

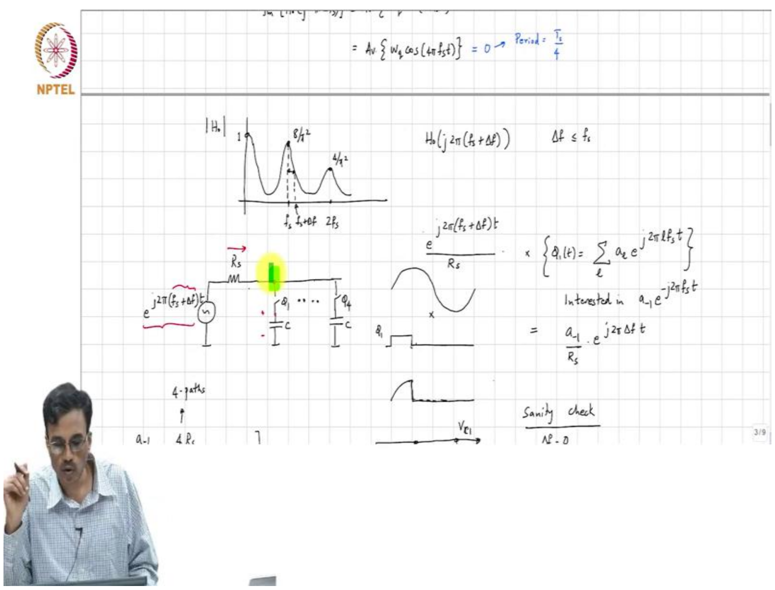

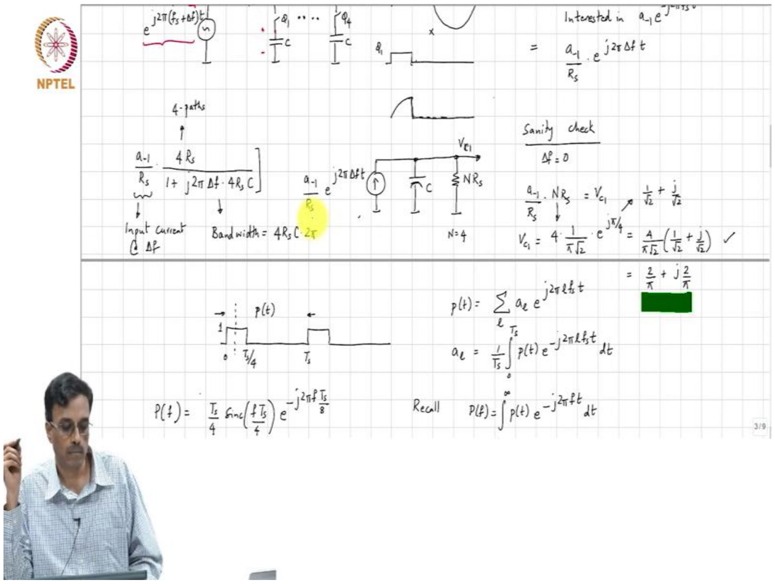

And recall however, that integral 0 to infinity or minus infinity to infinity P of t e to the minus j 2 pi ft dt is nothing but, this is nothing but the Fourier transform of P of t. So, to get a sub l all that you need to do is do 1 over Ts P of, evaluate the Fourier transform of that one period of the waveform at l times fs.

So, in our case P of t is basically 1 for a duration Ts by 4 and therefore P of f is nothing but it is a rectangular pulse, the Fourier transform of a rectangular pulse happens to be TS by 4 sync f TS by 4 that would be the Fourier transform of a pulse centered at the origin but this is now moved a little bit to the right, so this will be e to the minus j 2 pi f TS by 8 because if the origin was here then it would be just the sink, now it is moved to the right by the pulse is moved to the right by TS by 8 and this is what you get.

Now, therefore, a minus 1 therefore is 1 over TS times P of minus fs which is why 1/4th sink of minus 1/4th times e to the j 2 pi fs at times TS is simply 1, so 2 pi by 8 that becomes pi by 4 which therefore must be 1/4th sink is an even function so that is basically sin pi by 4 divided by sin, divided by pi by 4 and times e to the j pi by 4.

So, in our case this N is 4, so VC1 therefore is a minus 1 which happens to be or it is N which is 4 times that is this is simplify this, so this and this goes away and therefore you have 1 by pi root 2 times, yeah e to the j pi by 2, I hope there is no error, seems okay. So, VC1 therefore will be 4 1 by pi root 2 times e to the j pi by 4 which is nothing but what is e to the j pi by 4, 1 by root 2 plus j by root.

So, this is nothing but 4 by pi root 2 times 1 by root 2 plus j by root 2 which is nothing but 2 by pi plus j 2 by pi. And this is consistent with when we put cos in that, the first capacitor would have 2 by pi, when we put sin the first capacitor would have, when we put cos or sin, what would be, if we put cos 2 pi fs times t what would be the voltage here? What would be the voltage across this capacitor if this was a, well if that was fs?

It would average the first quarter of the clock period of the input period that would be 2 by pi. Similarly, when you put sin also you would average this and that is indeed therefore is consistent with what we derived earlier. So, all that I want to show here now is that as delta f, so sanity check this is fine, so as delta f increases what comment can you make about VC1?

It basically will be a minus 1 by RS times the impedance of this RC network which is NRS by 1 plus j 2 pi delta f times NRS times C, that makes sense. So, this is simply an input current, so this is the magnitude of the input current at delta f and this is nothing but a current flowing into an RC network.

And so therefore, the voltage across the amplitude of the voltage developed across the capacitor at delta f will be given by this expression which is a minus 1 times in our case n is 4 as because the 4 paths that we have. And so the what is this telling us, it is I mean this is telling us stuff that we know already, namely if delta f is 0 then you get the maximum voltage, as delta f deviates from 0 what comment can you make about the magnitude of this? It decrease, and within quotes “what is the bandwidth that you can, I mean for what delta f will the amplitude reduce by a factor of root 2”?

Student: (()) (23:03)

Professor: Yeah exactly. So, basically this is n is 4, so the bandwidth is basically 4RS times I mean 2 pi of course by 4s. So, what this is looking like? So, H0 of j 2 pi f basically at fs has a gain of 8 by pi square and as you deviate this is the delta f, this is fs plus delta f. What happens? As you deviate from fs the magnitude of the voltage across the capacitors keeps falling which then means that the voltage at this node also keeps falling because the voltage at this node is simply we are taking the voltages across each one of those capacitors and putting them all together.

So, and the bandwidth basically happens to be 4 RS times C times 2 pi. And intuitively this makes sense because around that center frequency it looks like a low pass filter with the C of course is the capacitance and the reason why we have 4 RS is that that RS is only being connected to the capacitor for only 1/4th with them.

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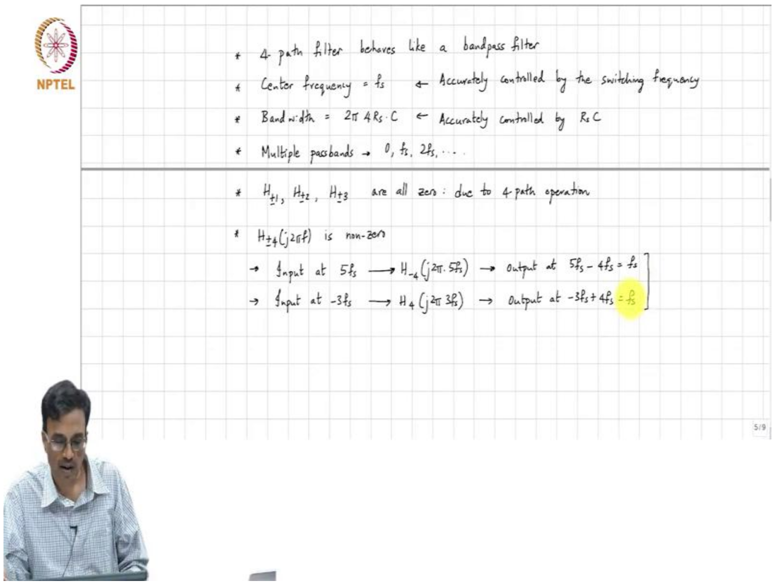

So, to summarize therefore, the N path filter or the 4 path filter behaves like a bandpass filter, center frequency is fs, bandwidth is 2 pi times 4 RS times C, there are, and so therefore, center frequency is accurately controlled by the switching frequency and the bandwidth is accurately controlled by the RS C product. So, this now allows us to have a wide programmability. Unfortunately, there are also multiple passbands. So, where are these passband? At 0, at fs, 2 fs and so on. And there is also translation of frequencies. For example, H sub minus 4 of 5 fs.

Remember if you put 5 fs also you will, I mean the capacitor voltages will basically have some nonzero averages. And when you go and compute the real part of H of j 2 pi f, an imaginary part of H j 2 pi f, you will get a periodic waveform at with a period of TS by 4. So, basically if you put an input at 5 fs and H minus 4 is not going to be 0. Remember, only what is 0? H1, let me write that down before we go here. H plus minus 1, H plus minus 2, H plus minus 3, are all 0. And why are they all 0? Because this is due to 4 path operation.

But H minus 4 is nonzero, is nonzero means input at 5 fs, yeah, will get down converted through H minus 4 of j 2 pi into 5 fs and yield output at 5 fs minus 4 fs which is fs. Similarly, an input at minus 3 fs will go through H plus 4 of j 2 pi times 3 fs and yield an output at minus 3 fs plus 4 fs which is fs. So, not only are there multiple passbands, there is also potential for out of band frequencies folding frequency translation. But we knew this already. This is an LPTV network, this is exactly what we signed up for. The hope is that it is, I mean so if you want to prevent this what do you suggest we should do?

So, in other words we want to filter something around fs and if we want to have a narrowband filter around whose center frequency is at fs and so we build this thing. But unfortunately, and around fs it obviously does a good job because it seems to be rejecting stuff from, it only lets frequencies close to fs pass through, as you keep going farther and farther away from fs you will start to see problems. I mean it is attenuating signals farther away from fs.

However, if the input frequency is very far away. So, if it is say at 3 fs, minus 3 fs or if it is at 5 fs basically the LPTV nature of the system seems to be causing frequency translation from these frequencies to 2 fs. So, we are at this point, will be confused as to whether the output at fs is genuinely our filtered input which is at fs or it is some extraneous tone which is at either 5 fs or minus 3 fs or whatever which is getting translate. So, this is however an easier problem to fix. And how will you do this?

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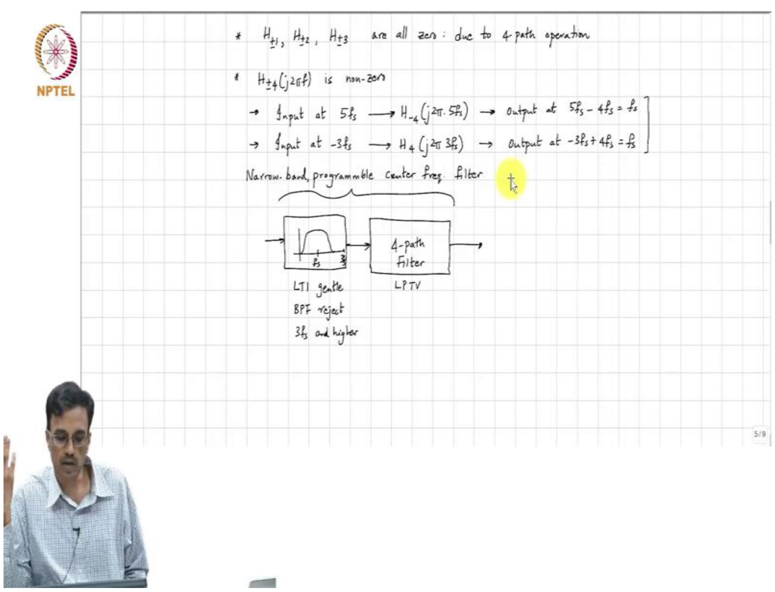

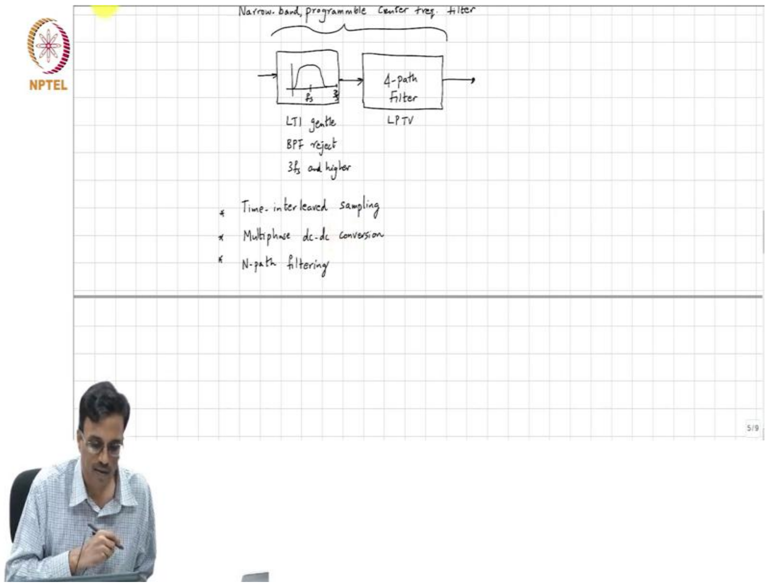

Well, you say here is my 4 path filter. So, this is a LPTV of course. And which is the nearest input frequency that can appear at the output? 3 fs. So, basically if you look at the distance to fs it is 3 fs. So, what you could do for instance is to put a gentle bandpass filter which rejects 3 fs. So, for instance so let us say you have a gentle bandpass filter, so this is fs, so LTI gentle bandpass filter, and so this is 3 fs and therefore at 5 fs it will be even smaller. So, to reject 3 fs and. So, this is easy to do because it is very gentle. And it only has to reject something which is very, very far away from fs.

The biggest challenge with bandpass filtering is that if you have 2 tones which are closely spaced it is very difficult to reject one and pass the other. This the N path filter based bandpass filter basically is good at filtering close tones which are very close to each other, but it has this problem because of the inherent time varying nature of the network that it causes stuff to fold from multiple frequencies to output. So, this is a one way of solving the problem.

So, this is equivalently a very narrowband programmable. And the programming of the center frequency is very accurate because the center frequency depends on the switching frequency, if we change the switching frequency the center frequency or the bandpass filter will also change. So, with this I will stop our discussion on N path techniques.

So, we have seen three applications of N path principles. The first is time interleaved sampling. The second we saw was multi-phase dc-dc conversion. Third is N path filtering. So, in the next class we will see how one can make, how I mean, when we talked about time invariant networks after analyzing, figuring out how to write the matrices and so on and analyze the network, we talked about reciprocity and inter reciprocity, we will do the same thing for periodically time varying networks in the next class.