Introduction to Time - Varying Electrical Networks
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Lecture 53
Computing H_0(j2pi 2 f_s) for a 4-path filter

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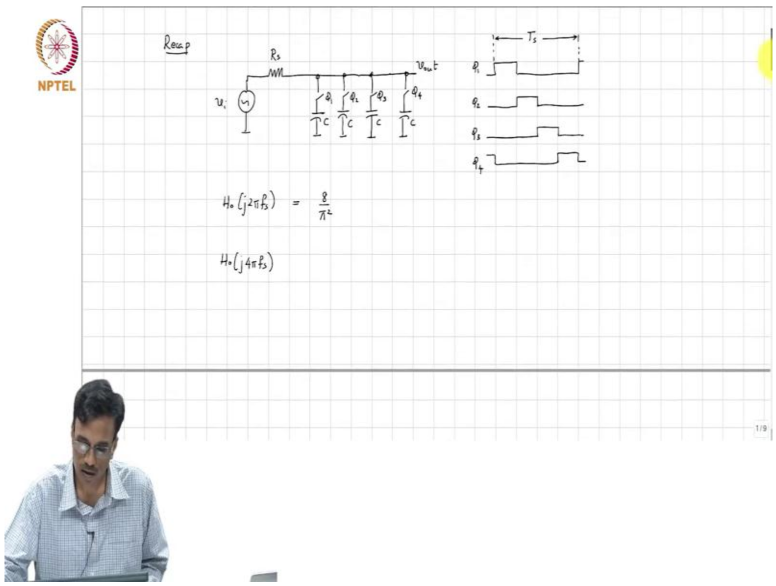

A quick recap of what we were doing yesterday, we were looking at what is called N-path filter example of which, so you have four identical capacitors and these are the clock waveforms and this is Ts, this is a 25 percent each one of these clocks this is phi 1, phi 2, phi 3 and phi 4, each one of these waveforms is a 25 percent intuitive cycle clock.

And we were yesterday we spent a lot of time and figure out what H 0 of j 2 pi fs was and we found that that is equal to 8 by pi square. Today let us continue and try and figure out what is H 0 of j 4 pi fs which is the zeroth order harmonic transfer function at when the input frequency is 2 fs when it is twice the input the switching frequency.

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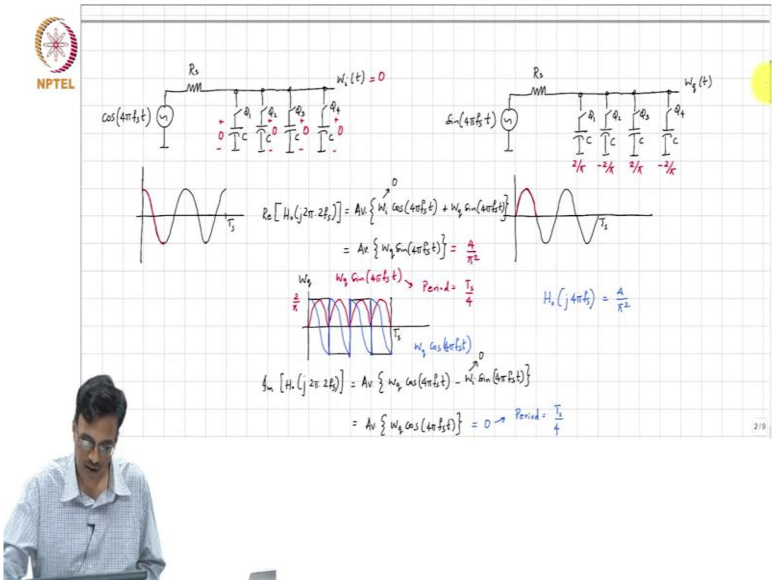

So, to do that let me copy this over, as usual we put in cos 2 pi; cos 4 pi fs times t and sin 4 pi f s times t this is Wi of t and this is Wq of t. And what are the voltages on this capacitor? Well the, if you have an input which is at twice the switching frequency then in one quarter of the clock period you basically have one half of the input cycle, then, so this is Ts, so this is cos omega t.

So, during the first capacitor is going to average out this part of the waveform and therefore the average here is going to be 0, second capacitor is going to average out that part of the waveform and so therefore that is going to be 0, and similarly the third capacitor and the fourth capacitor all will have 0 voltage across them, so Wi of t is 0.

Remember Wi of t is simply the voltage across the first capacitor during phi 1, the second capacitor during phi 2 and so on and therefore Wi of t is 0. What comment can we make about Wq of t? Well, that is Ts, so that is a sin 4 pi fst, the first capacitor is going to average this part of the waveform and that is therefore going to be 2 by pi, the second one is going to do, is going to have a voltage which is, so this I am going to mark the voltages down, so this voltage will be 2 by pi, this will be minus 2 by pi, this will be plus 2 by pi and this will be minus 2 by pi.

And remember H the real part of H0 of j 2 pi times 2 fs times t, I am sorry is simply the average value of Wi cos 4 pi fs times t, W plus Wq sin 4 pi fs times t, which therefore Wi being 0 is nothing but the average value of Wq sin 4 pi fs times. So, Wq is going to be a waveform like this, so it is 2 by pi minus 2 by pi so this is Ts.

And so Wq times sin 4 pi fs times t is going to be, is going to be this, this and therefore what comment can we make about the average? So, this is 2 by pi actually I made a small error, 4 multiply by sin, this should be, so what should, what would be the average value of that waveform? 4 by pi square and one thing that is apparent is that this waveform is periodic with respect, what is the period of this Wq sin 4 pi fs?

The period is, period is Ts by 4 which makes sense because this is a, we knew already that it is an LPTV system and with 4 paths and therefore even though the switching frequency is fs it behaves like an LPTV system with switching frequency 4 fs. Now, what comment can we make about the imaginary part of, this is simply the average value of Wq cos 4 pi fs times t minus Wi sin 4 pi fs times t, this we know to be 0, so this is nothing but the average value of Wq cos 4 pi fs times t.

And what, how will that look like? So, we know Wq is like this, if we plot Wq times cos 4 pi fs times t. Well, this is going to be a something like that, the second half is going to be something like that, and therefore what comment can we make about the average, of the blue waveform is 0. Again, we notice that the period is Ts by 4. So, H sub 0 of j 4 pi fs is nothing but 4 over pi square.

And so it turns out, I mean then similarly you can go and do it for all integral multiples of fs and it will turn out that there will be peaks in the transfer function at multiples of, integer multiples of fs. So, at fs you have a peak, at 2fs you have a peak, at 3fs you have a peak and so on. And what comment can you make about the peak at, the magnitude of the gain at 4fs versus, at 2fs versus fs?

At fs the gain was 8 by pi square, at 2fs it is 4 by pi square and likewise it turns out that at higher and higher frequencies, intuitively that makes sense because as frequencies get higher and higher what will happen is that you will be averaging out the sinusoid over what, I mean over Ts by, over period Ts by 4, but there you will have larger and larger number of integral number of cycles, so only the fractional cycle is what you will, which is what will remain. And therefore, its value will keep decreasing, and.