Introduction to Time - Varying Electrical Networks
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Lecture 46
Time-domain intuition of the N-path principle

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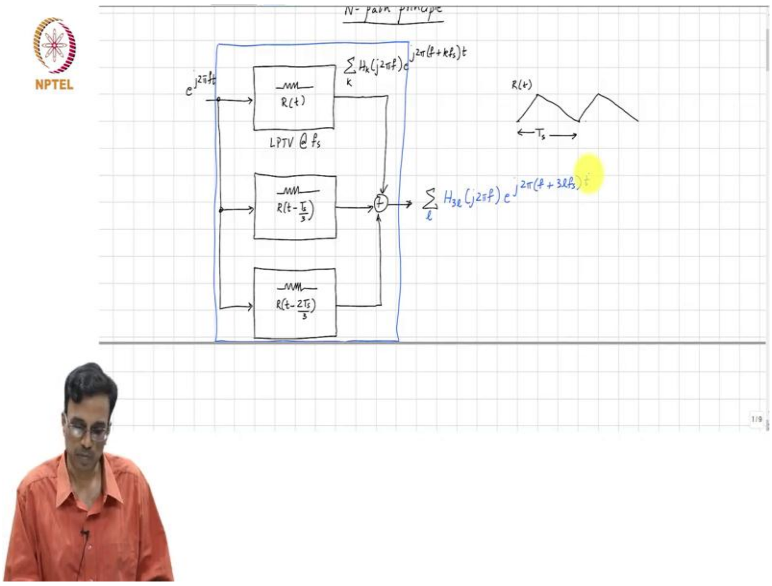

Quick recap of what we did in the last class. So, we were talking about the N path principle. And the basic idea as we saw the last time around was that we take an LPTV system. It is time varying at frequency fs. And for arguments sake, let us say and we are going to just show one resistor which is varying with time, but in general, they can have multiple conductances inside which are all varying with time with the same frequency of course.

And let us assume that this R of t is varying in some fashion say like this. And this of course is Ts, so this is the R of t and if we excite the system somehow with an input e to the j 2 pi ft as we saw yesterday, the output will simply be something of the form sigma over k sum over k H sub k of j 2 pi f times e to the j 2 pi f plus k fs times t.

Now, what we said was that if we took n such systems, as today, we looked at the example where n was 3 and here the time variation was shifted by 3, R into t minus 2 Ts by 3 and this basically as we saw would result in an LPTV system if we put all of this inside a box and on the face of it, this also is an LPTV system and since every component inside is varying at the rate fs, this is also an LPTV system which is varying at fs but given the special nature of 3 building blocks inside as we saw yesterday this can be written as H sub l, I mean sum over all l of 3 sub l H sub 3 l of j 2 pi f e to the j 2 pi f plus 3 l fs times t.

And we saw why this comes about, yesterday we saw the whole this thing in the frequency domain. Today, I will give you the intuition behind this in the time domain as well as show you an example of the use of this n path principle.

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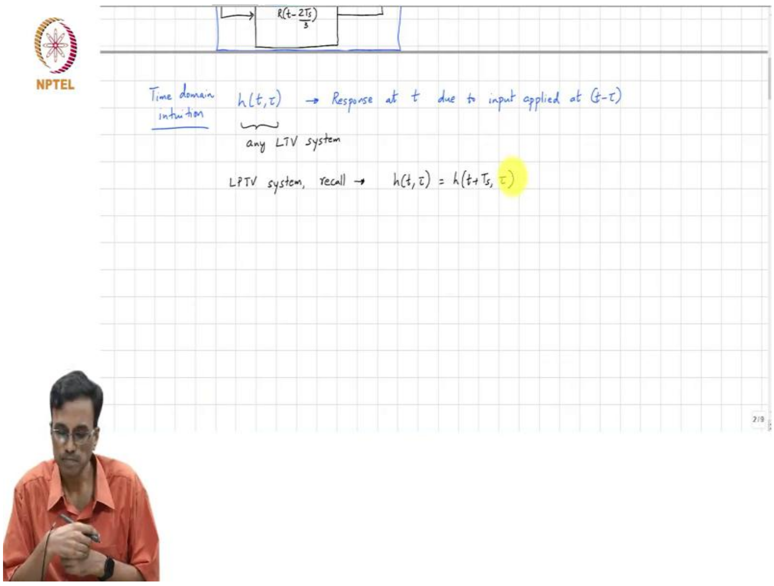

So, remember, when we talked about LPTV system, we said that in the time domain remember h of t comma tau represents in an LPTV system the response at time t due to input apply that at tau before the point of observation we namely at t minus tau and what happens periodically time varying system this is true for any linear time varying system. So, for any LPTV system recall that h of t comma tau is the same as h of t plus T s comma tau. So, in other words, if you move the time of observation and the time at which the impulse response, the impulse is applied by the same Ts, then you will get the same response.

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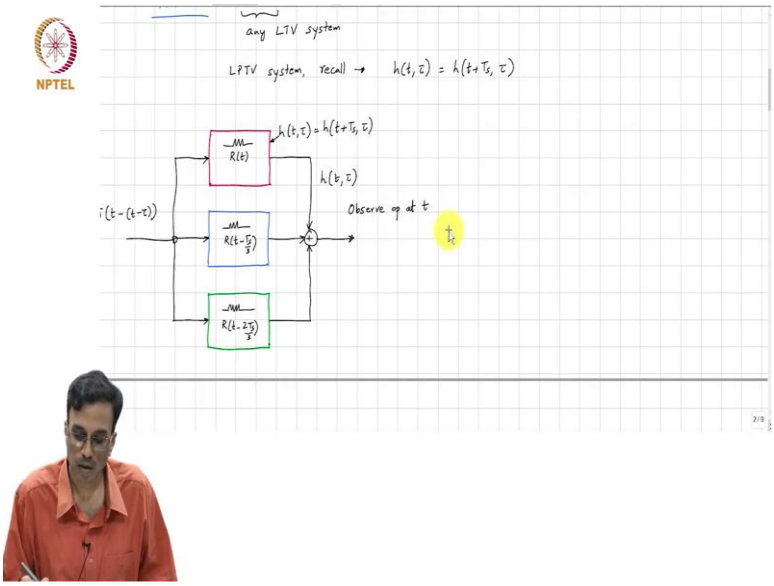

Now, let us take this composite system, I am going to just for visualization show them in different colours red, blue and green. So, this is some resistor R t, this is R of t minus Ts by 3, this is R of t minus 2 Ts by 3. So, if we looked at any just one of these systems in isolation, then clearly you basically as you can see for each of these systems in isolation the impulse response will be periodic with (respect) with time Ts because each one, because this system is periodic at Ts is LPTV with fs.

Now, so if I apply an impulse at delta of t minus tau I am sorry, I apply an impulse at t minus tau so that basically means the input is delta of t minus t minus tau and I observed the output at t then what do I get here I will get H of t comma tau. Now, if I take a snapshot of this system exactly at Ts by 3 later. So, what happens?

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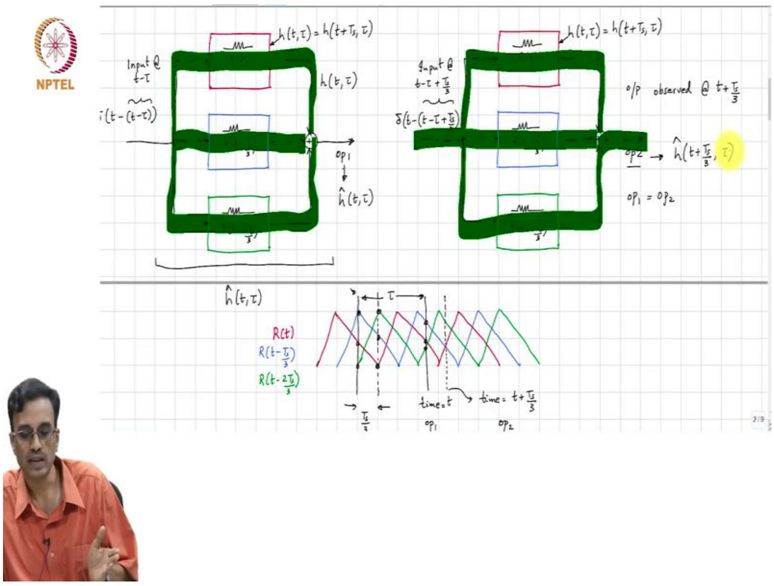

Let me copy paste. So, I will now if I observe the output at time t plus Ts by 3 and what would happen with regard to the input and if I apply the input at t minus tau plus Ts by 3 so here the input is applied at t minus tau, here the input is applied at t minus tau plus Ts by 3. In other words, I have moved the input by Ts by 3 I have moved the output, the time of observation also to, yeah output observed at t plus Ts by 3. So, let us call this op 1 and we call this op 2.

So, if you move t, I mean remember R of t was like this and what happens with R of t minus Ts by 3, well that does something like this and R of t minus 2 Ts by 3. Now, at time t let us say this was some time at some observation time is equal to t for some particular observation time that is the time I choose to look at. And what do we, how does the system look like? There is one of these boxes operating with a resistor of that value.

One of these boxes operating with a resistor of that value and the third box is operating at with that resistance value. Now, let us see what happens if you go and as a result you have some output at time t, which is op 1. So, now, what comment can we make when we take the time of observation to be t plus Ts by 3, this is time is t plus Ts by 3 and the output we are observing is op 2 that corresponds to this guy here.

So, it is the same system which has been excited at a time, with an impulse at time t plus at Ts by 3. So, for example, in the first system you would have excited let us call this, let us say we exited the system here this was tau. And so, the corresponding excitation in the second system we will have to come here that is Ts by 3, it is delayed by Ts by 3, the time of observation is also delayed by Ts by 3.

So, think about this way. So, in the fifth picture to the left we excited the system here, where there was one, the blue box had this value of resistance, the red box had this value of resistance and the green box had this value of resistance. Now, at t if we excited the system after Ts by 3 well there is the green box is has that value of resistance, the blue box has this value of resistance, I think my diagram drawing is not really as good as it should be but and the red box has this value of resistance.

So, whatever output you would have got in with the blue box when you applied the impulse at t minus tau, you would get that same output with the green box in when you apply it at t minus tau plus T by 2. So, whatever in other words, whatever output you are getting here, when you applied the input at t minus tau, you will now be getting this, at what time now, in the in the green box.

So, you will be getting this through that box. Whatever output you are getting through the red box in the first case, you will now get through the blue box. And whatever you were getting through the green box before, you will get through the red box. And since all these three systems are identical, it does not matter. You know, so in either case, basically the bottom line is that yeah, if you add the three up the final output is going to be the same. So output 1 at, so H, the output is going to be H of, let us call this, so output 1 is going to be the same as output 2.

So if you think of the impulse response of this composite system as H hat of t comma tau this after all a LPTV system in its own and so therefore the output of the system at t is simple H hat of t comma tau and you will see that just because of whatever we have seen the output of the system when looked at the time Ts by 3 later is the same, I mean if the impulse was delayed by Ts by 3 and the output was observed Ts by 3 later you would get the same output in both cases.

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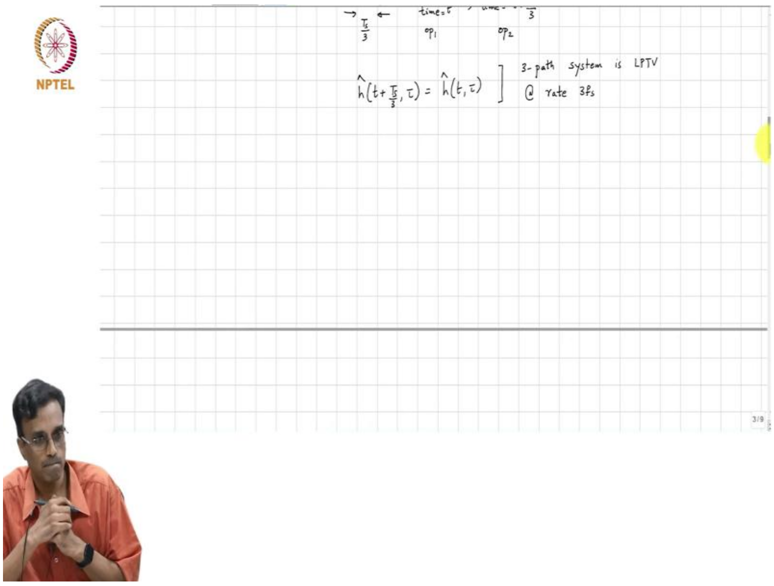

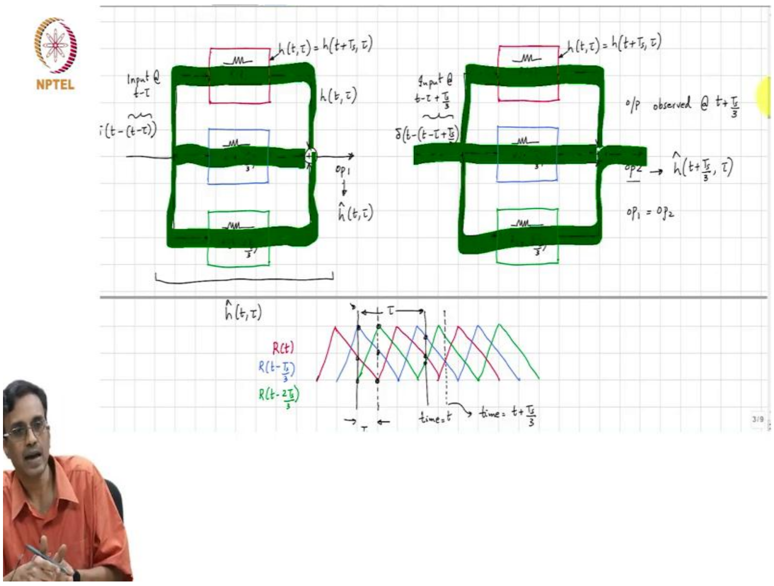

So H hat of t plus Ts by 3 comma tau is the same as H hat of t comma tau and what is this therefore saying, yeah so this is saying that this composite system, the 3-path system is LPTV at the rate 3 fs even though.

Student: Each system is varying.

Professor: Each system is only varying with rate at fs, in other words, another way of thinking about this is that if you take a photograph of the system at certain time the network looks exactly the same at a time Ts by 3 later and that happens because the three sub systems are exactly identical but its only their timing which is changed if you start, if you take a snapshot or photograph of this system Ts by 3 later what the red would have done would be done by as we saw in this case, yeah the green box will basically behave like the red box and the blue box will behave like the green box and so on.

So this is the time domain intuition behind n-path operation, so one way of quickly recognizing whether the LPTV system is periodic at n-times the frequency is to simply look at how the network is at a time Ts over n later and you know apart from simply moving the components around if the network actually looks the same then you know that it is LPTV at Ts over n.