Introduction to Time-Varying Electrical Networks
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Lecture 42
Impedance and Admittance in LTI and LPTV Networks

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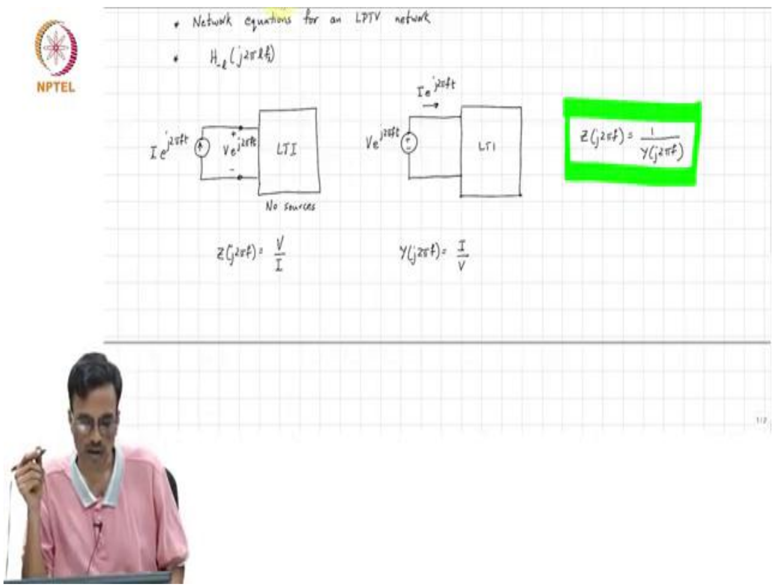

So far, in this course, we have seen how one can write the network equations for an LPTV network and this basically involved creating a conductance matrix, where each entry is now a matrix in itself and that contains the Fourier coefficients of the time varying element. Now, we also saw how one can determine special cases where one can determine H sub minus l of j 2 pi l times f or l times f s, and this was a special case where the output frequency is DC and we have said that we can find this by exciting the LPTV system with cosine, exciting with the sine and then just simply looking at the average parts of the of the waveform.

Now, today, let us continue on with some other properties of LPTV networks. I would especially like to highlight the fact that things that we are very familiar with when we talk about time invariant networks will not necessarily be true, when you have a network with periodically time varying components and one example is the following: we know that if we had a time invariant network, this was LTI then and let us say the network had no sources inside then if you excite the network with a current e to the j 2 pi f t, the voltage developed is say V, the phasor V.

Then we know that the impedance Z of j 2 pi f is simply nothing but V by I technically speaking, this must be if I is the phasor, then what you are exciting the network with is I times e to the j 2 pi f t, the voltage that is developed is V e to the j 2 pi f t, where V is the voltage phasor and the impedance is simply nothing but Z of j 2 pi f.

Now, if on the other hand we took the same LTI network and excited it with a voltage phasor V meaning that the waveform is V e to the j 2 pi f t then it will turn out that the current will be I e to the j 2 pi f t and the ratio of the current developed to the voltage developed is nothing but I by V and for an (LPTV) LTI system Z of j 2 pi f is simply 1 over Y of j 2 pi f. This is of course very well-known from our earlier classes.

In other words, it does not matter in an LTI network if you want to find the admittance, it does not matter whether you apply a voltage and measure the current or you apply a current and measure the voltage and the impedance is simply what do you call the reciprocal of the admittance. Now, well, it turns out that, that is definitely not the case, not necessarily the case, when you deal with periodically time varying networks and we will spend a little bit of time trying to understand why that is so.

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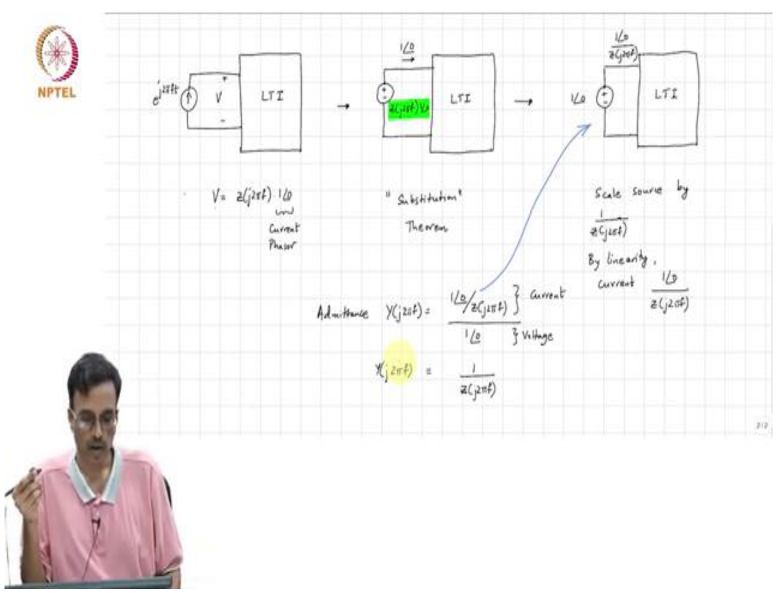

Before we go there, let us basically, prove this for an LTI network and then we use the same way of proving, we use the same method to go and analyze an LPTV network and see where that leads us. So in other words, we would like to understand why in a linear time invariant network, the impedance and the admittance are reciprocals of each other. So let us say, we excite this with e to the j 2 pi f t of current and this voltage V is developed. This is LTI. Now, what I am going to do is replace the, what I am going to do is the following.

I am going to, we know that V is nothing but Z of j 2 pi f times one angle 0, because this is the phasor corresponding to the current excitation. Now, this is a network and this voltage here is this phasor V. Now, the network would not know if I replaced the current source with the voltage source with, which is Z of j 2 pi f times one angle 0, the current that would flow there, the current phasor that would flow there would still be one angle 0 because as far as the network is concerned, it would not know the difference, whether it is being driven by a voltage or if it is being driven by a current. So this is often what is called the substitution theorem.

And now, the next thing I am going to do is basically use linearity. If you know that, this is a linear network after all, so if I scale the strength of the source by a number, then the response will also change by the same factor and therefore, what do I do next is linearity. So, scale the source by 1 by Z of j 2 pi f and therefore, this would be one angle 0 divided by Z of j 2 pi f. So this, so by linearity, current must be. So, but by definition, the admittance Y j 2 pi f is simply the ratio of the current to, that is which is basically the current that flows into the network, when it is excited by a voltage.

And that is basically as we see in this picture and as you can see, this is simply nothing but 1 over 0, j 2 pi f. So the admittance is simply the reciprocal of the impedance, which is something that we knew already, there is nothing new here.

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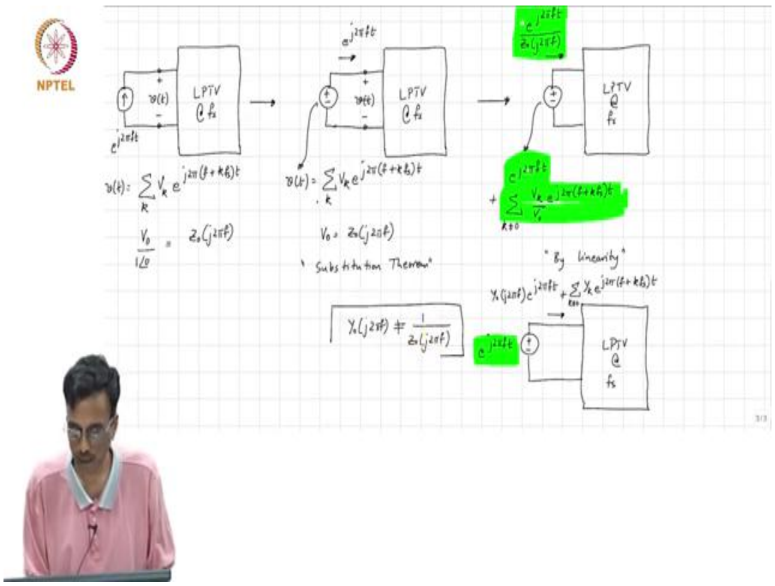

Now, let us try and see where if we tried working the same way with an LPTV network, let us see where things lead us. So, let us assume now that we are working with an LPTV network, it is varying at f s. We will excite the network with a current e to the j 2 pi f t, like we did in the time invariant case. Now, the voltage developed here is because of time variance, not merely consist of stuff at frequency f, it will also consist of…

Student: (())(11:48)

Professor: It consists of all tones of the form sigma l, I do not remember who you kept using l or k. Let us use k, k V sub k, e to the j 2 pi f plus k f s times t. So that is the voltage V of t developed there and the current of course, is e to the j 2 pi f t. Now, like we did in the time invariant case, we go into, by the way, V of 0 divided by one angle 0 is the ratio of the strength of the voltage at f to the current at f and that is simply, that is by definition is Z 0 of j 2 pi f.

Now, let us see what happens when we now replace the current source with a voltage source by the substitution theorem, what do we need to do? We need to replace, if you want the current here to remain e to the j 2 pi f t, what you need is you simply cannot put just V 0 e to the j 2 pi f t, you need to add all the components.

So, basically, what you need to do therefore is to add this voltage source has to be V of t and that remember is simply, has to be not merely V 0 but you must have all the other components that are present in V of t. Now, the next thing is that we will scale. So remember that V 0 is Z naught of j 2 pi f. Now, what we are going to do, this is the substitution theorem.

Next, we are going to resort to using linearity. So, this is LPTV at f s again. Now, I am going to scale this by Z naught. So, we will call, this is now, e to the j 2 pi f t plus sum over all k, k not equal to 0, V k over V 0, e to the j 2 pi f plus k f s times t and the current that flows through here will therefore be e to the j 2 pi f t divided by Z naught or j 2 pi f. This is by linearity.

So, the key point to notice is that if you wanted to measure admittance, what you would have to do would be, or what you tend to do would be to excite this with a voltage one angle 0, which is equivalent to saying that, I will put in a voltage of e to the j 2 pi f t and the current that would flow here, you would, that would be Y 0 of j 2 pi f plus a whole bunch of other terms, which I will actually write down here. Y 0 of j 2 pi f to the j 2 pi f t plus sum of k not equal to 0, Y sub k e to the j 2 pi f plus k f s times t.

So, what this is saying is that, clearly, if you simply excite the voltage, the network with a voltage e to the j 2 pi f t, the current Y 0 of j 2 pi f times e to the j 2 pi f t which is basically the admittance seen at the frequency f, is not 1 over Z naught because for the current to be 1 over Z naught times e to the j 2 pi f t it is not merely enough to add or to excite the network with e to the j 2 pi f t, it needs all these extra components.

Because since the network is LPTV these components are all getting either down converted or up converted by appropriate amounts and then they all add up and that is how the current becomes simply 1 over 0. So it is therefore, Y naught of j 2 pi f is not necessarily equal to 1 over Z naught, j 2 pi.

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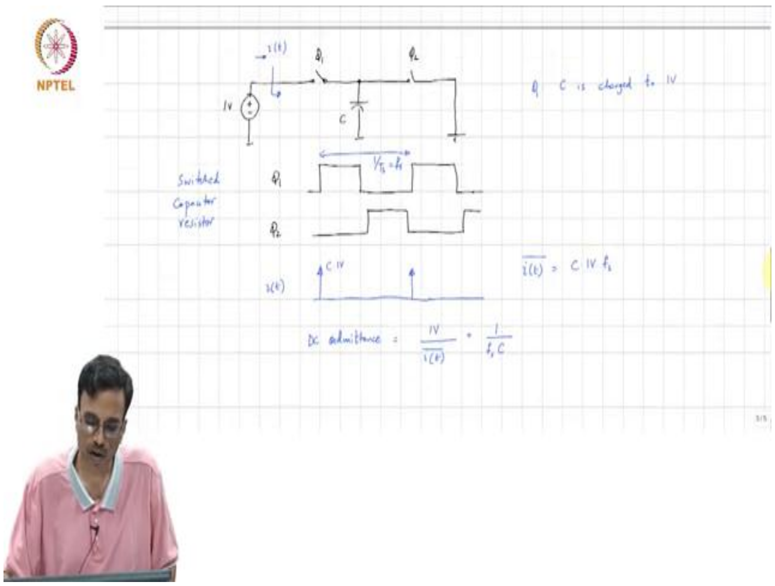

So, let us take a simple example to prove the point. Here is a voltage source let us say it is a DC voltage and we have a capacitor C and when this clock waveform phi 1 is high, so phi 1 looks like this, phi 2 looks like this and the let us take a look at the impedance looking at DC. So, as you can see, so if you look at during phi 1 the capacitor C is charged to 1 volt during phi 2 the capacitor is completely discharged. So, if you look at this current I of t, it basically assuming the switches are ideal, the current I of t basically is an impulse at every rising edge of phi 1.

And the charge contained in this impulse is C times 1 volt and if the frequency of operation of these switches is f s then the average current you see is just simply C times 1 volt times f s. So, the DC impedance or DC admittance actually is 1 volt divided by the average value of the current that is flowing and that is simply 1 over f s times C. So, for those of you who have seen this before, this is nothing but a switched capacitor resistor. Now, the next thing that we would like to do is say in other words, the low frequency admittance is 1 over fs times C.

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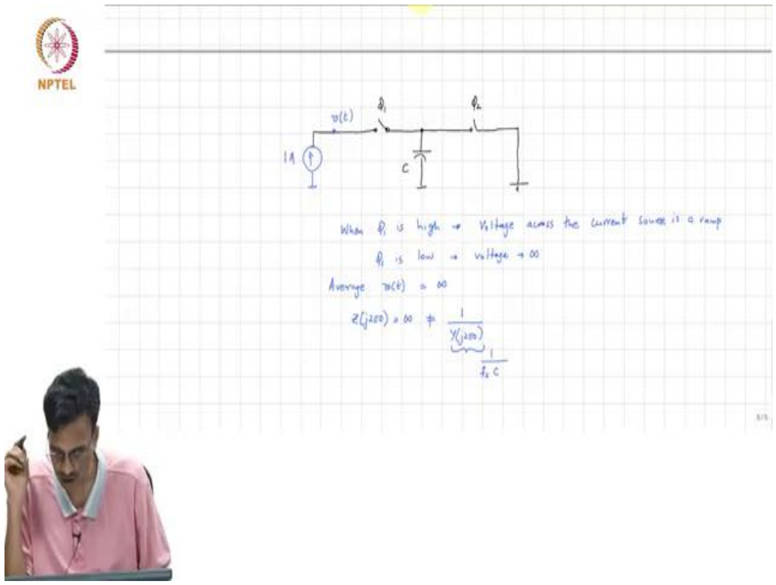

Now, let us see the same network is driven by a current source and let us now measure the impedance and see what that amounts to. So, you can see that clearly when phi 1 is high well, the voltage across the capacitor becomes infinite or rather it keeps going up as a ramp, voltage across the current source is a ramp, well, when phi 1 is low, well the voltage is infinite.

So, the average voltage, therefore, look at this voltage V of t is simply infinite. So, Z therefore of j 2 pi 0 that is the DC value of the impedance is infinity, which is definitely not the same as 1 over Y of j 2 pi 0. Remember this, we calculated to be 1 over f s times C. So, familiar things like impedance and admittance are inverses of each other et cetera do not necessarily work with LPTV circuits.