Introduction to Time–Varying Electrical Networks
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Lecture 41
Determining H_{-k} (j2\pi kf_s)

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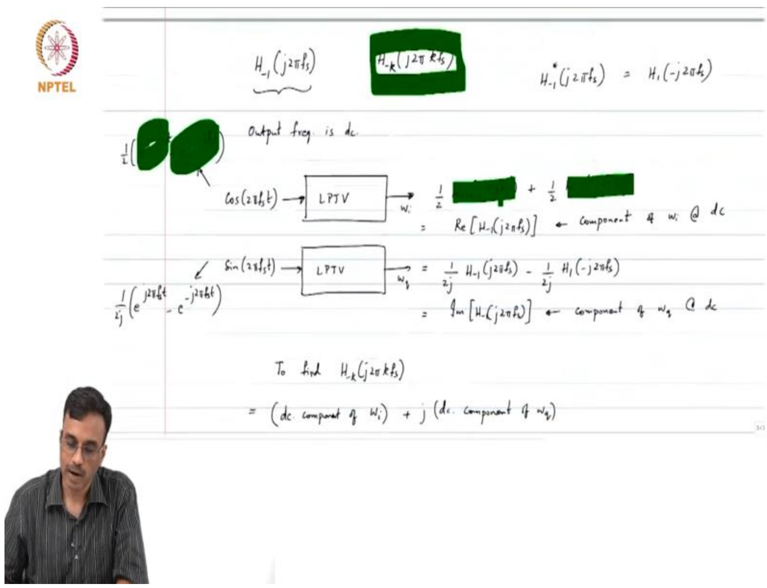

In many cases, it is so turns out that in many practical situations require you to find say, harmonic transfer functions of the form H minus 1 of j2 pi fs; or in general, H minus k of j 2 pi times k fs. And so, what is so special about the situations? It is demodulation; so, the output frequency is always at dc.

So, in these cases it turns out that you do not really need rather trick you can use. We do not really need to find Wi Wq and plot the real part and plot the imaginary part; and then do the fourier series, and then find the relevant fourier components. It turns out that there is a shortcut and that is a following.

So, basically if you put e to the j 2, I am going to do it for the case where k equal to minus 1; and then you can easily generalize that two other parts. So, if you put e to the j 2 pi fs times t, what will you get? You have this LPTV system here. What will you get at the output? Or rather let say we put cos 2 pi fs times t; what will you get at the output? See standard, we are doing this is to do is to exited, with cos 2 pi fs t, sin 2 pi fs t. And then get Wi, Wq and do all the math; so, what is Wi now?

Student: (())(02:56)

Professor: This is nothing, cos 2 pi is nothing but half of e to the j 2 pi fs times t, plus e to the minus j 2 pi fs times t. And this is 1 over 2j e to the j 2 pi fs times t minus e to the j 2 pi fs times t. So, what will you get here? You will get half times H.

Student: H minus 1.

Professor: H minus 1 of j 2 pi fs and.

Student: H of plus 1.

Professor: You will get half H plus 1 of minus j 2 pi fs; because this is getting translated 1 fs below. And minus fs will also calls. Remember multiple input frequencies can translate to the same output frequency, when you have an LPTV system. So, e to the plus j 2 pi fs t will come out as dc, through a gain H minus 1 of j 2 pi fs. And e to the minus j 2 pi fs will will get translated by plus 1 to get up to dc. And what is so special about these two quantities? What comment can we make about H minus 1 of j 2 pi fs and H1 of minus j 2 pi fs? That the complex conjugate of each other.

So, H1 of H1 of minus j 2 pi fs is nothing but H minus 1 of j 2 pi fs times star. So, therefore when you add these two what will you get? You basically get real part of H minus 1 of j 2 pi fs. And what will you get, so this will be the component of Wi of t at at dc. Now, what about what will you get here? You get Wq will. The dc component of Wq will be? It is 1 by 2j times H minus 1 of j 2 pi fs minus 1 over 2j H1 of minus j 2 pi fs. And that should give you the imaginary part of H minus 1 of j 2 pi fs. So, if you are only interested in finding and likewise if you wanted to find h minus k of j 2 pi k fs. What will you do?

You will get the same; the only thing that will change is that this H minus k of j 2 pi k fs; and H plus k of minus j 2 pi k fs, and you will get exactly the same. So, what is the moral of the story therefore? If you want to find H minus 1 H minus k of j 2 pi k fs; what should you do? It is nothing but it is the dc component of Wi, plus j times dc component of Wq.

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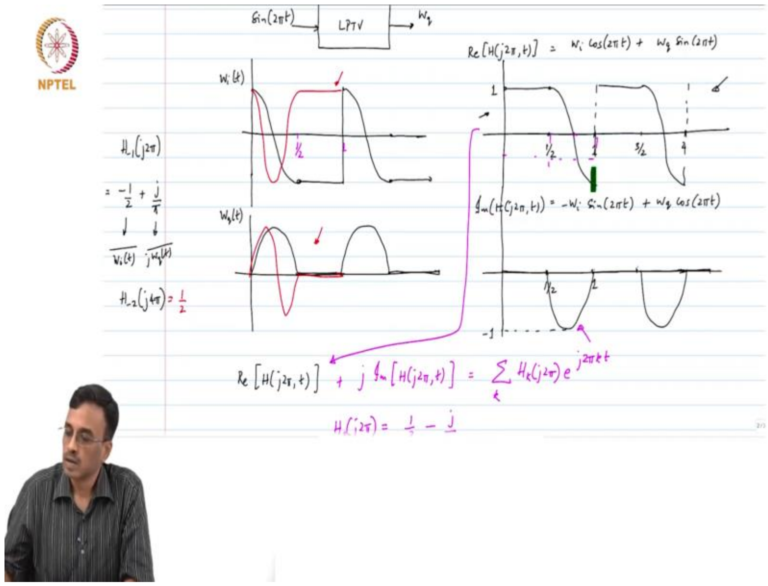

So, let us go back to our system here, this is Wi and Wq. So, what is H minus 1 of j 2 pi? What we need to do? What do we need to do?

Student: To find the average value of Wi of t.

Professor: What is the average value of Wi of t?

Student: Minus half.

Professor: Minus half and so this is nothing but the average of Wi of t; and what is the average value of Wq of t? 1 by. So, and what should you expect to see for H minus 2 of j 4 pi. You would put in, what do what frequency are you putting in?

Student: 2 Hertz.

Professor: You basically would then get Wi would be, you would have two cycles here; you will have one whole cycle and half clock period. And it will be it would stay like that, and the sin would be you would have one whole cycle like that, and then stay like that. So, what would H minus 2 of j 4 pi be? What is the average of this waveform guys?

Student: 1 by 2.

Professor: 1 by 2; What about this guy? So, H minus 2 of j 4 pi would be.

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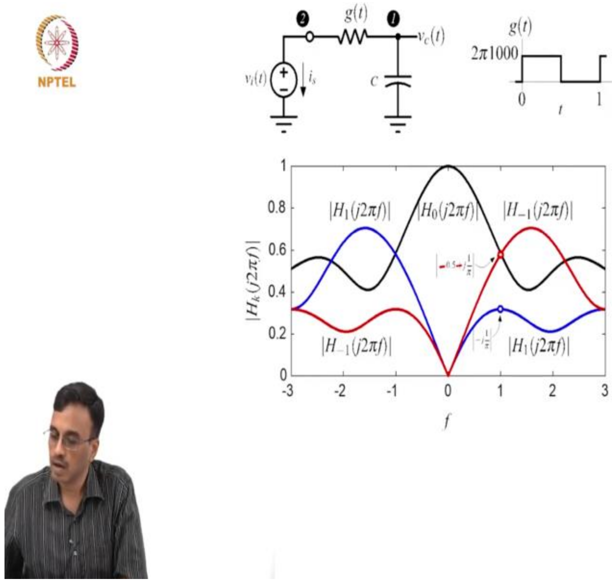

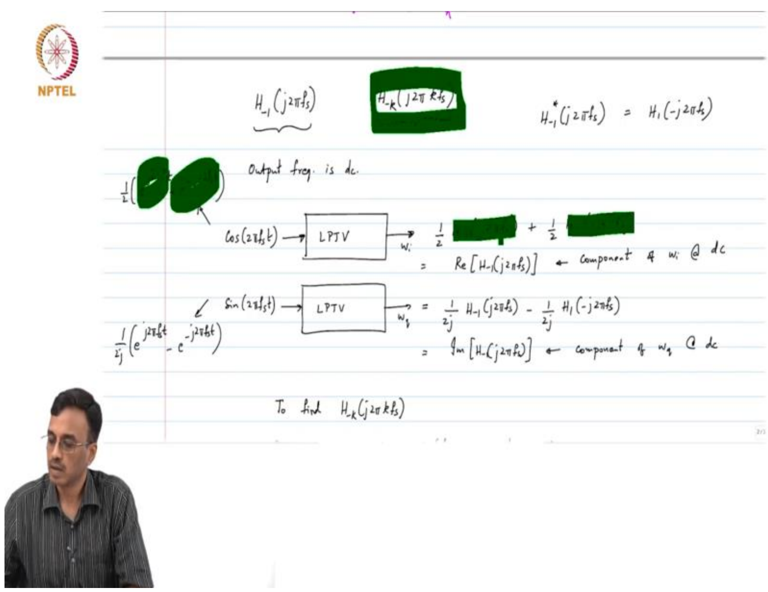

So, let see if that makes sense. So, H minus 1 of j 2 pi f, it is not visible here; so, we calculate minus half plus j by pi. And certainly, I think it is a checks out and what is this? This is. So, H1 of j 2 pi, you do the math actually and then calculate it. But what interested in to say was that if in many practical situations, especially I think you are able to see what the application of this is.

And this is down conversion to see how much of this stuff will villous to dc; and simple trick is to you took. For this special case, you do not really need to find Wi times cos and plus W; you do not find the real and imaginary parts of H, by doing Wi cos plus Wq sin. And the other one and then expand that in Fourier series and all that. You can simply look at the average value of

Wi and Wq, and that will give you the H minus k of j 2 pi k times fs. The question is will I be able to do this, if I want say, H1 of j2 pi fs; will the same trick work? Which is.

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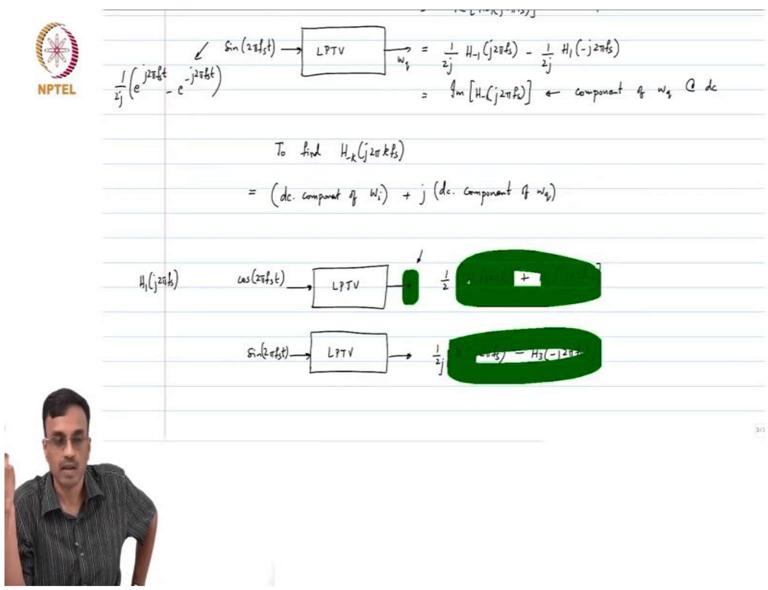

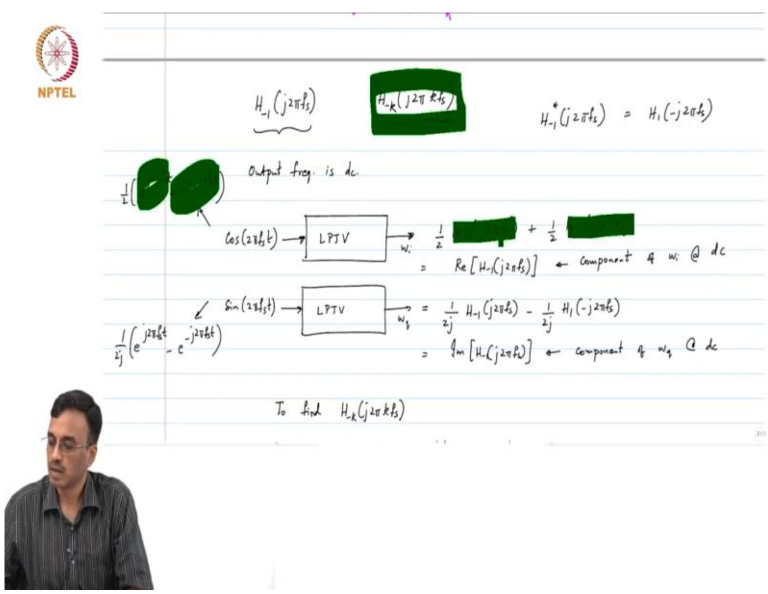

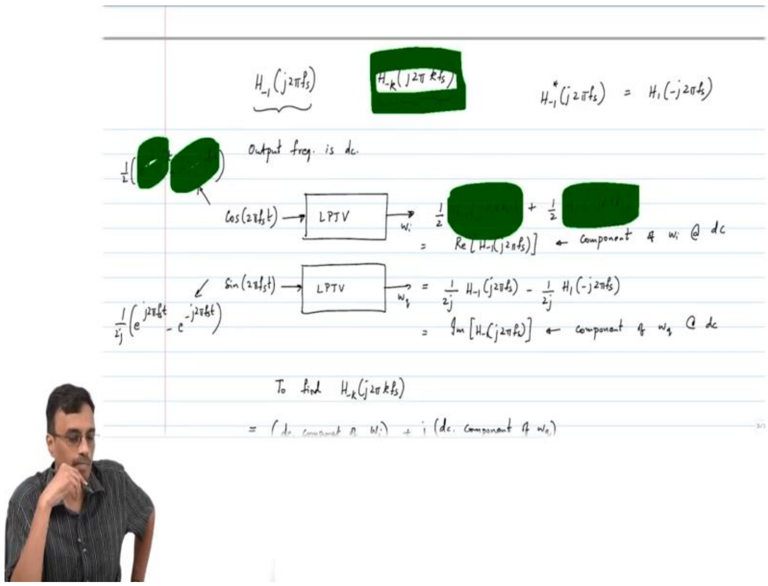

So, I claim that I want to find H1 of j 2 pi fs; so, I will look at the output here. I will look at the component at, what is the output frequency now?

Student: (())(13:56)

Professor: Can I look at 2 fs? If I know the component at 2 fs for both. By look at the component at 2 fs for at the output of both these guys; do you think I will be able to get H sub 1?

Student: (())(14:38)

Professor: Like what? What I will get here at? What will be the components here at 2 fs?

Student: (())(14:50)

Professor: So, basically you will get from so you will get half times H; I will get H1 of j 2 pi fs that will translate it to 2 fs. Then but this also, so H 3 of minus j 2 pi fs that will give me the amplitude at at 2 fs. And similarly, here you will get H1 of j 2 pi fs and minus H3 of minus j 2 pi fs. If the output happens to be dc, then these two are related.

These two to get plus 1 and then H plus 1 and H minus 1; now, you do not that does not happen. So, you have to expand what you get here as, as a fourier series you will get the second harmonic component. And then that will be this and then similarly likewise you get this; and you have to solve those simultaneous equations to get H1 and H3.

In the, it is only in this special case when the output frequency is dc; that what you get here. These two are related because of mirror symmetry in the harmonic transfer functions; and therefore, you do not really need to do any more work. You simply look at the average value of this thing and you get. Is that clear?