Introduction to Time–Varying Electrical Networks
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Lecture 37
MNP stamp of a Periodically Time Varying Conductance

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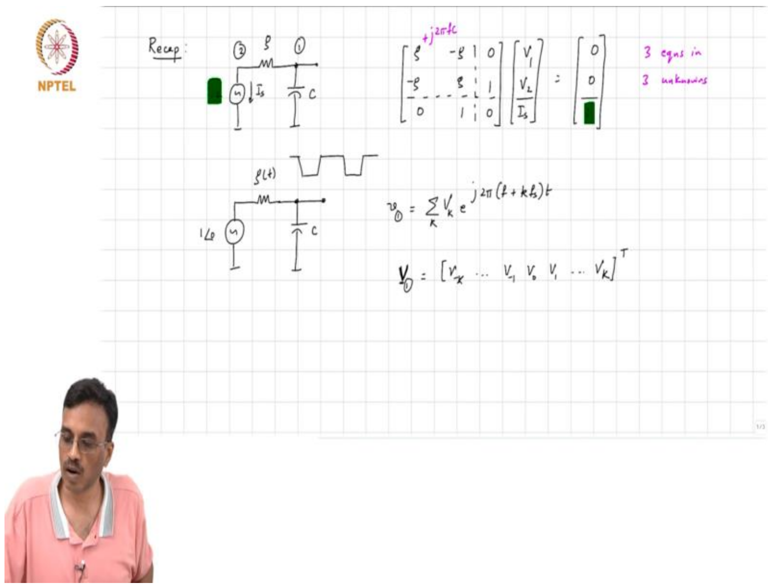

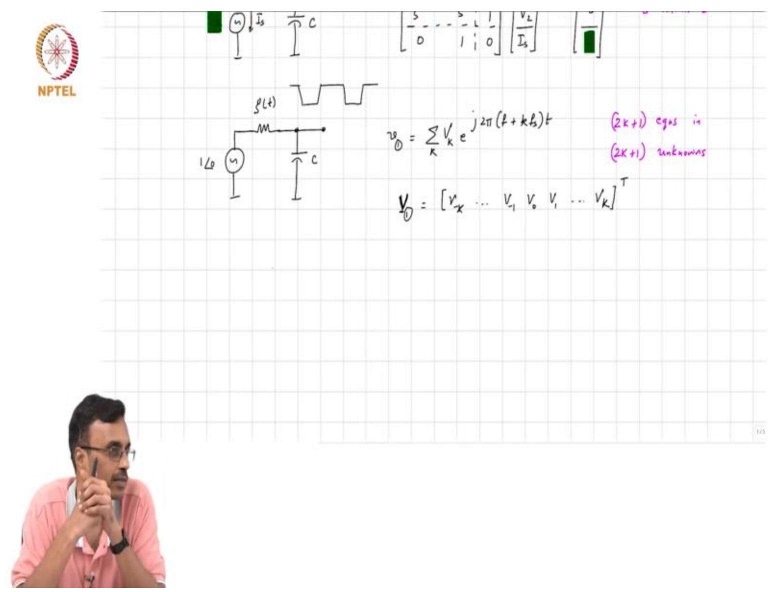

Quick recap what we were doing in the last class; we are learning about linear periodically time varying systems. And we would like to do that in a manner as close as possible, to what we are already used to namely the analysis of time invariant network. And we took our familiar first order example; let say we are interested in trying to solve for the branch currents and branch voltages of a network like this. The first thing we do is set up kcl and kvl and a systematic way of doing this would be to simply write the MNA equations. And if this phasor current phase is denoted by I sub s.

Then as we saw during the last class basically, we have v1, v2, Is equals 0, 0 and 1. And between 1 and 2 there is going to be g, minus g, minus g, g; and you will add a plus j 2 pi fc. And this becomes 0 1 and this becomes 0, 1; or the other way wrong this right. And the phasors, now you must understand that this 1 here stands for the one angle 0; that you are applying here. And straight forward solution of this set of equations is gives you the phasors for all the node voltages as well as the branch current. So, this is you we have seen this already in great detail.

Now, the question is what happens when you have a periodically time varying network; so, this is g of t, which is doing some some periodic variation. And if you look at this, we one thing we know is that every node voltage and every branch current is now consists of, is of the form sigma some Vk e to the j 2 pi, f plus k fs times t. And if we kind of assume that before sufficiently large K perhaps this Vk becomes so small, that you can neglect it. And therefore, you denote this branch voltage, you express this as the phasor corresponding to that branch voltage is something like this.

Where, you start off with V0, V minus 1, V plus 1 blah blah blah; V minus K capital K, and V plus capital K. And this is basically a column vector and since writing this stuff here, is seems awkward and much easier to write the stuff this way. It is minus Vk blah blah blah, V0 sorry not minus Vk; this is V minus k; V minus 1, V0, V1. This is capital K all the way up to V capital K and this transpose. And this this this underscore is just there to kind of remind ourselves that this is a this is a vector.

And likewise, every current will also have will now be characterized by these every branch current; which in the time invariant case was only characterized by one number one complex number, is now characterized by 2K plus 1 complex numbers. And there is no point in complaining about it and saying I mean what is this seems terribly complicated. But that is what life is, the network is varying with time and it seems reasonable that. If you have something which is varying with time to capture its behavior; you need within quotes you need to work a lot hard.

So, fortunately for us kcl and kvl must be satisfied not merely at a frequency f; it must be satisfied at every frequency of the form f plus k times fs. And that therefore means that while early in the linear time invariant case, you had 3 equations in 3 unknowns. Now, so you I am going to write this down here; so, 3 equations in 3 unknowns. And all of a sudden now what do you what comment can we make? So, how many equations do we have and we need. So, if kcl kvl had to be satisfied at one frequency; we needed 3 equations and we had three unknowns, per frequency evidently.

So, now you have 2K plus 1 frequency, so you basically have 2K plus 1 equations, in 2K plus 1 unknowns correct. So, already as you can see even a simple first order RC circuit; whereas as soon as you make the resistor time varying, the complexity of the calculation becomes kind of insane. But fortunately once you understand the principles, you go to a computer and then invert those matrices. So, now how do you write Kcl and kvl? We can write this in the same it turns out, as we will see going forward today; that we can use the same formulation that we the MNA formulation that we used for the time invariant case.

If you know to do that then this becomes very straightforward to write the set of equations. The solution of course you go you punch it into MATLAB or your favorite tool; and then get the answers. And just like you see a complicated network and you know that if it is a linear network; I can actually do the math and get the answer. But you go to a computer anyway for if the order is say 2 or 3 or 4 or something; you do not start doing stuff with pen and paper. You go to a computer, it just so turns out that when you have a time varying network. Even for a first order system, you need to go to a computer; that is how like this.

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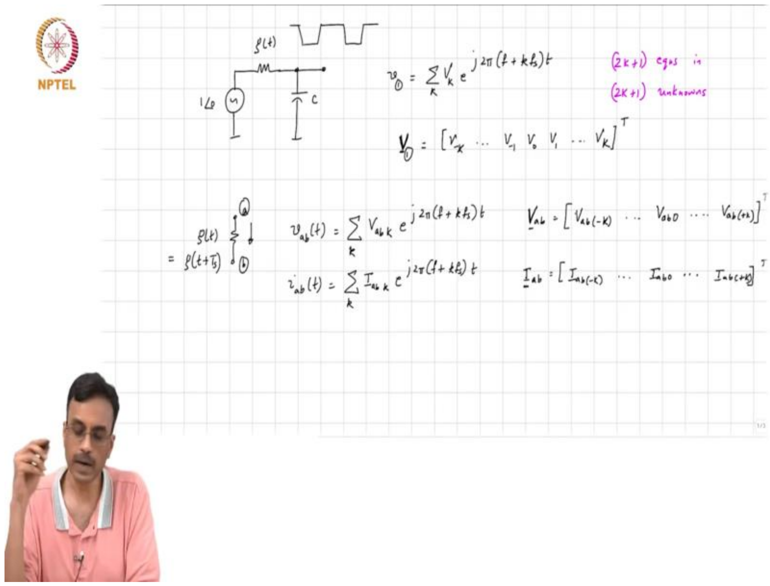

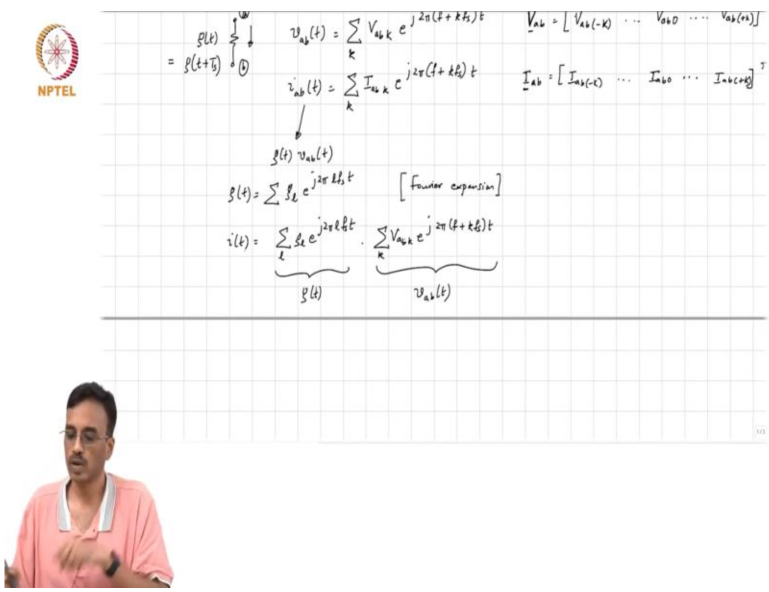

So, as far as we are concerned we are more interested in figuring out how to write the equations in a systematic way. And we will be very happy, if we are able to link it to what we did with the time invariant case, correct. So, you do not want to learn a new set of rules now great; so, let us go element by element. So, therefore remember in the time invariant case, we wrote the MNA stamps for each element and built up that g matrix, by adding the corresponding MNA stamps. So, we have to do the same thing here too; except now that we have to deal with the fact that if you have element.

Say for example this is g of t, which is varying periodically with some at some rate g of t plus Ts. Then you need to be able to relate the voltage phasor with the, or you need to relate the current phasor with the voltage phasor. So, the voltage the branch voltage will be of of some form like this; so let us call this node a and this is node b, and the branch voltage.

So, that Vab or Vab of t will be of the form some Vab K, sum over K e to the j 2 pi f plus k times fs times t. Likewise, the current i ab of t will be some Iab K e to the j 2 pi f plus k fs times t. So, the Vab phasor is simply nothing but, we as usual we will truncate this beyond a certain large number K; which can be made arbitrarily large.

So, you have Vab of minus K, Vab 0, Vab plus K whole transpose. This is a column vector and likewise Iab the phasor vector that we are now interested in is simply Iab minus K blah Iab 0 Iab 0 Iab plus K. Now, the question is how is Vab, how is Iab, how is the vector Iab related to the vector Vab? How is how in other words how is Iab of t related to Vab of t? Iab of t is simply nothing but g of t times Vab of t. And g of t and g of t is nothing, but it is periodic, and therefore can be expressed by the Fourier series which is g sub k. Or, let me use another subscript g sub l, e to the j 2 pi l times fs times t; is simply the fourier expansion.

And again we can we can truncate this expansion beyond the certain large number; that we can set later. At this point we know that all that we need to understand is well. As far as engineering practice is concerned, presumably all the infinite coefficients are not important; we probably stop somewhere. So, therefore i of t therefore must be g l e to the j 2 pi l fs times t, times Vab k this is sum over l; this is sum over K e to the j 2 pi f plus k fs times t. Will make sense people? This is nothing but g of t and this is nothing but Vab of t. And all that this is saying is the current is g of t times the voltage.

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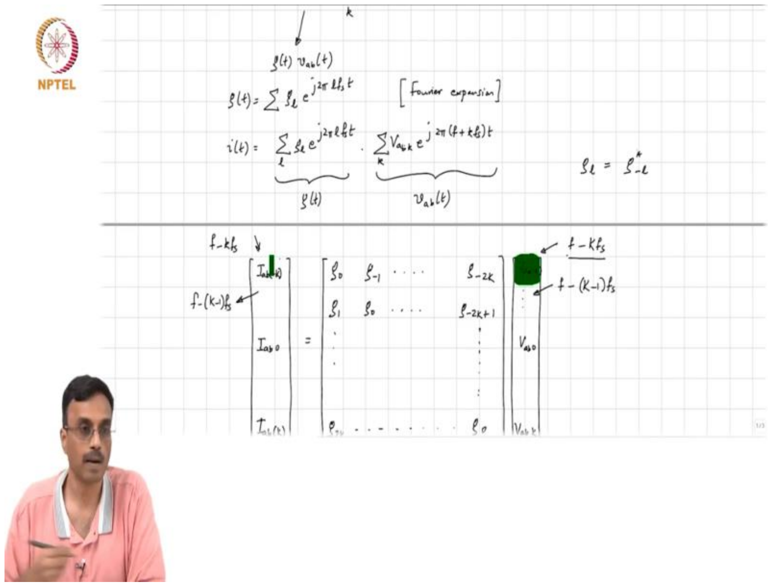

Now, the question is what is the, so we are trying to relate the current phasor which is basically Iab of the minus capital K. All the way let say we try to find the strength of the minus Kth capital Kth harmonic in the current phasor i. Clearly, it must be related to the strengths of the voltage phasors, so Vab minus k V0. This is Iab 0, Iab plus k; this is Vab plus k. So, what comment can you make about Iab of minus K; that is anyway please remind me what this actually signifies. What is that number indicate?

Student: (())(16:53)

Professor: It represents the strength, the complex amplitude of the current at a frequency f minus capital K times upper case K times fs. And likewise, for everyone of those terms, and then specifically Iab 0 quantifies the complex amplitude of the current at f. So, what comment can you make about Iab of minus K in terms of Vab of minus k? What is Vab of minus K signify? It signifies the complex amplitude of the voltage at frequency f minus k fs. With what how will that reflect as the current at the same frequency? This is at a frequency f minus k fs.

This is also at a frequency f minus k fs; so, whatever what should be this entry here? I mean which two terms this term is Vab of plus k e to the minus k, e to the j f minus k fs. If this has to be the same frequency, the only term that has; so, this frequency the g term that. Remember that when you multiply two of these terms, what you get on the left hand side is simply the sum of those frequencies. So, if the right-hand side if I or v has got a frequency of f plus k fs; and the left-hand side has the same frequency. The only term that can cause it is the g0 term, is that clear? Yes/no?

So, this is g0 very good, what comment can you make about the second column in the first row of this matrix? What is the second term here will have a frequency of. Sorry, this is not is capital k; so, this will be f minus K minus 1 times fs. So, that will have if that has to result in a tone at f minus K fs. What is what should you multiply it by g? The input tone is f minus k minus 1 times fs. So, the frequency is jumping by plus 1 fs, so that the only way that can happen is if you multiplied by g1 g minus 1 minus; sorry g minus 1. And so on and so on and so on, finally the last term will be g.

Student: (())(20:34)

Professor: g minus 2K; let us go to second row. What comment can you make? So, this would be the second entry there would have a frequency of f minus K minus 1 times fs. And this is f minus k fs, so this has to go is the output frequency greater than input frequency or the greater. So, what should we have? g plus 1; and then this would be the diagonal the second or second column will be g0. And what is the last, so I guess you see the pattern now g minus 2K plus 1. So, now it does not take a genius to fill out all those other entries. So, what will be the last entry here? g 2k; this would be g. And by the way g of t is a real function; so, g of l must be equal to. g of t is a real function, so the Fourier coefficients must satisfy conjugate symmetry; so gl, g of minus l star.

Student: (())(22:58)

Professor: That makes sense?

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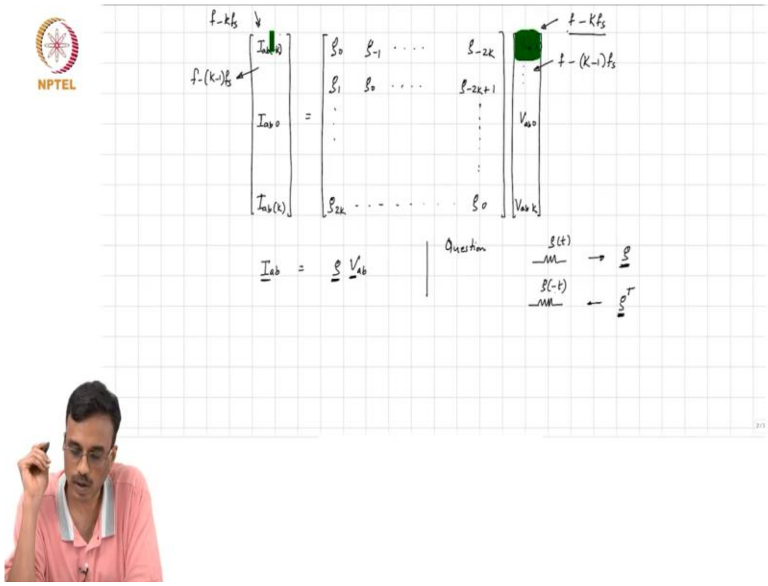

So, now I am going to call see what happens. The I phasor therefore is in the time invariant case; the current phasor was simply g times the v phasor. Now, the I phasor is now a vector and that is g matrix multiplied by Vab. Does make sense case? So, apart from those underscores below each one of those terms; now the equation looks pretty much the same as it did for the time invariant case. Now, out of the question I am going to ask you is this g of t had this conductance matrix g; now, if I transpose g. Remember this what kind of matrix is this, this g matrix this small g conductance matrix for this conductance.

What kind of matrix is it? It is square with size 2 k plus 1 cross 2k plus 1. Now, the question I have asked I am going to ask you is if I transpose this matrix; evidently that corresponds to some other conductance. The question is what does that correspond to? G of minus minus t. Just something to keep in mind; we will use this later. But it is a simple observation that excellent.