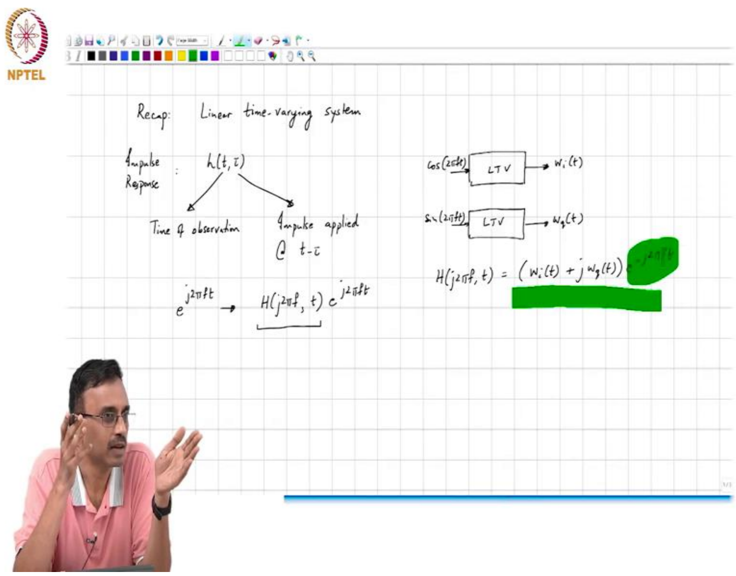
Introduction to Time – Varying Electrical Networks
Professor. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Linear Periodically Time – Varying Systems (LPTV)

(Refer Slide Time: 0:19)



In the last class we were studying about a linear time varying system and we said that the impulse response is defined or written as h of t comma tau where t stands for time of observation, while tau is the impulse is applied at a duration tau before the time of observation that is at time t minus tau.

And after we went through the math we said that perhaps not surprisingly, if you take a linear time varying system and you excite it with e to the j 2 pi f t and why are we exciting it with e to the j 2 pi f the? Well, we used to doing this from our linear time invariant system days. In a earlier time invariant system, we interpreted e to the j, we found that if we excite the system with e to the j 2 pi f t, the output is a complex number multiplied by e to the j 2 pi f t and that was, that complex number was what we call the frequency response of the linear time invariant system.

Here, we found that the output of the system is basically e to the j H of j 2 pi f comma t times e to the j 2 pi f t and all that this is saying is that if you want to think of it that way you can think of the frequency response as varying with time. So, the gain experienced by a sinusoid varies with time, it obviously depends on the frequency of the input sinusoid but it varies with time and that makes intuitive sense because, well the properties of the system of varying with time.

So, just like when I show up at 8 o'clock in the morning you know I find majority of you are like charged up and when we see 5 o'clock class (())(3:00) class on Friday evenings you are all kind of half asleep, but then I see people who are also time invariant who are seem to be asleep all the time. So, it seems to be reasonable that well if the system properties are changing with time the gain experienced by a sinusoid will also change with time.

Another thing to keep in mind is that we have some LTV system and if we want to find the H of j 2 pi f t, what do we do? Well we excited with cos 2 pi f t you excited it with sin 2 pi f t and you basically will get w i and w q and what do you do? H of j 2 pi f comma t is simply w i of t plus j w q of t times e to the minus j 2 pi f times t. Intuitively, if you want to kind of give a mechanically analogy, well first think of it in terms of linear time invariant system.

What is this e to the j 2 pi f t? It is basically you can think of it as an arrow which is rotating counter clock wise, with the frequency f. So, you push this e to the j 2 pi f t into a linear time invariant system and what is happening, the input arrows is rotating counter clockwise at a frequency f, the output arrow basically will be a stretched and phase shifted version of the input arrow that is also rotating at a rate f.

Now you want to figure out what that amplitude and phase shift is? The amplitude scaling factor and the phase shift is, so what do you do, you sit on a disk which is centered at the origin which is also rotating at e to the j 2 pi f t in which case what happens if you sit on that disk which is also rotating at the same speed. How will that arrow look like? This arrow is rotating this way at a frequency f, you are sitting on a disk which is also rotating at a frequency f. So, how will the arrow look like?

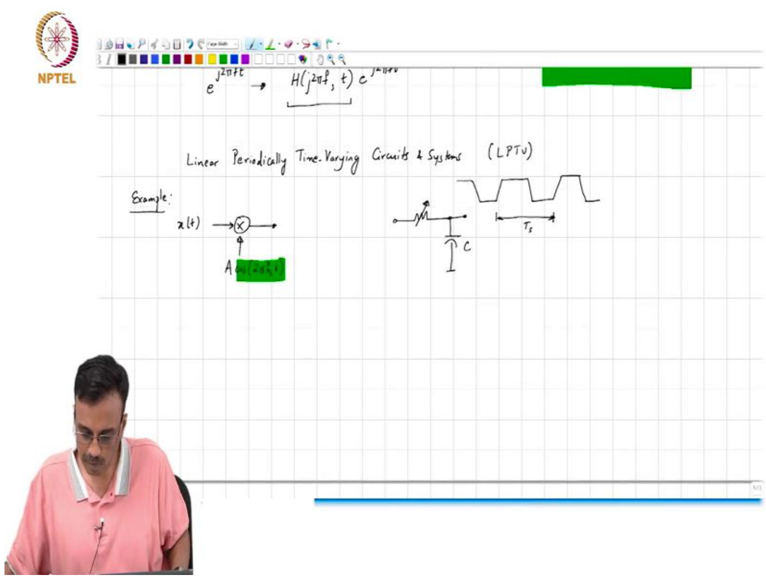
It looks stationary and then basically you find the x coordinate of that arrow in your stationary reference frame or the y component and that is basically the real and imaginary part of h of j with 2 pi f t. And this sitting on that disk and moving at the same speed as the arrow is what this e to the minus j 2 pi f t means, because if that is rotating at f and you are also rotating at f the relative difference between the angel of velocities is 0. So, that multiplying by e to the minus j 2 pi f t is if you want to think about it in a mechanical way you can think of you sitting on disk which is also rotating in the counter clockwise direction with the same speed as the arrow. Now if you have a time varying system what do you think will happen in this analogy?

Student: (())(6:46).

Professor: Basically that arrow in the time invariant system, that arrow would be fixed in your rotating frame. Now, what will happen because the system properties are varying, the magnitude of the arrow is changing and so is the bearing of the arrow with respect to your rotating frame which is also rotating at f.

So, that is an easy way of remembering of how you get, if you always get confused whether you need to multiply by e to the minus j 2 pi f t or plus j 2 pi f t. In the time invariant case you would like to make sit in a frame which is basically moving with the same angle of velocity as that e to the j 2 pi f t phase. Does it make sense?

(Refer Slide Time: 7:45)



Now it turns out that all this stuff is good but in practice what is of most relevance is class of systems which are called linear periodically time varying circuits and systems and we have seen an example already of what practical utility such a system has got. This an example of a multiplier where you have x of t and you multiply it with some A cos of say omega 2 pi f naught or 2 pi f s times t. If this cos 2 pi f s times t was not there and if it was just simply 1, then A is simply I mean x of t is simply experiencing a gain which does not change with time.

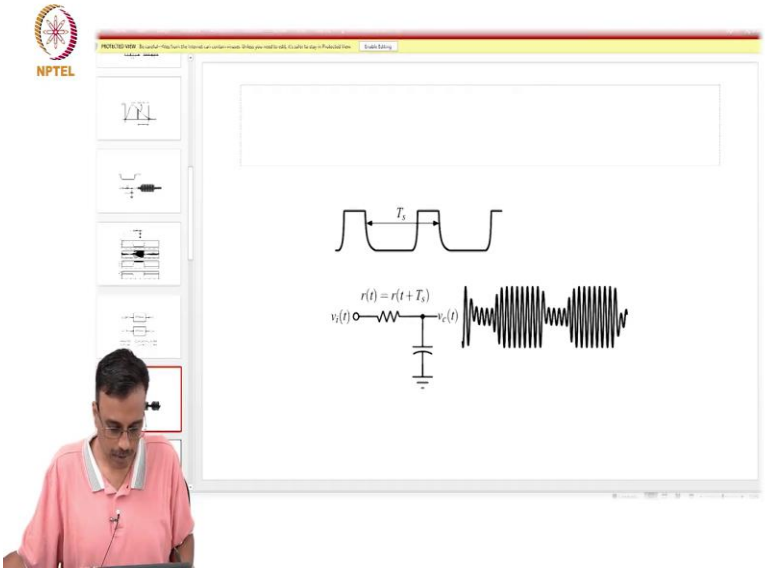
Now what is happening, you can think of it as x of t is being is experiencing a gain. However, the gain is varying with time and how is it varying with time? It is not merely varying any old way with time, that variation of the gain is happening periodically. So, on other words, so this is an example a trivial example because there is no memory at all in this system. Well, it is entirely possible to have to complicate our lives with by having elements with memory and as you may imagine if linear circuits were all resistive circuits without capacitors and inductors you basically be able to do nothing and likewise, a linear periodically time varying circuit where everything was only time varying resistors would basically also probably not be very useful.

So, here is an example where, let us say you have resistors which is varying periodically with time. So, can somebody think of a practical example of a resistor that varies periodically with time? A periodically operated switch is an example where the resistance varies from small value, ideally 0 to some large value ideally infinity. But that may not be the only form of periodic variation any other periodic variation also will.

So, here is an example of a linear periodically time varying circuit and if you excite this with a sinusoid before we go into the math, what do you expect to see at the output? So, somebody said the sinusoid with varying amplitude, what about. Well this a varying amplitude, basically not nearly amplitude, amplitude and phase. So, basically all that you expect to see is that I mean we will do the math and figure that out anyway but all that you expect to see is that, the properties of the systems are vary periodically.

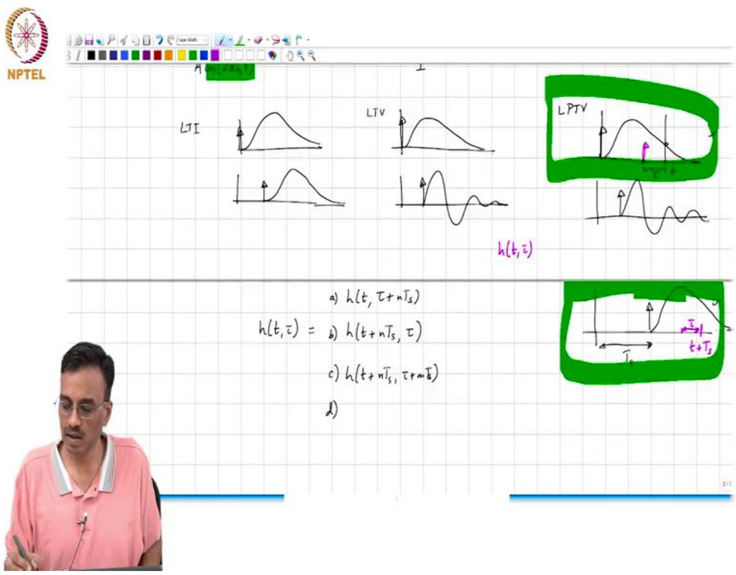
So, it seems reasonable that, the gain experienced by a sign wave at frequency f will also be varying periodically and with what period, this period is t s. What comment can you make about the periodicity of the gain? You also expect it to be varying, the gain to be varying periodically with the same period. So, in other words, let me see if I have a nice picture.

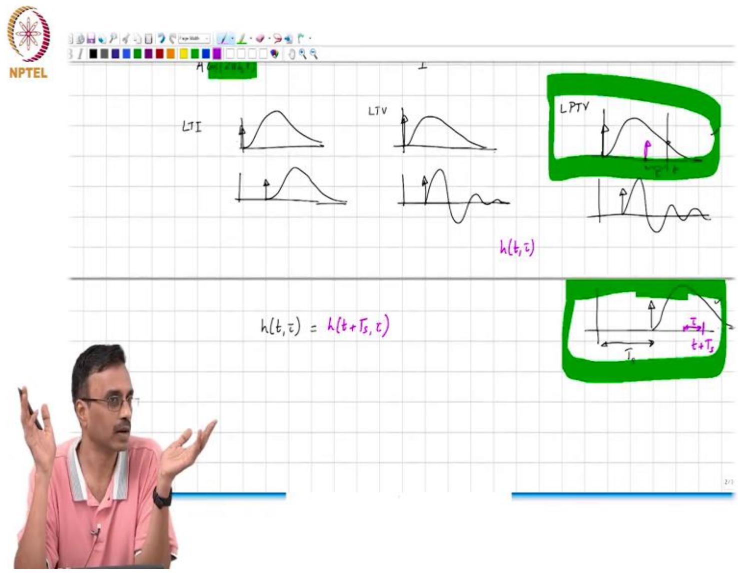
(Refer Slide Time: 12:20)



So, here is an example of something where the resistance is very periodically in some fashion and as you would expect the output is doing something like this well it certainly looks periodic but that does not mean that is. So, what do you call when the resistance is low if you think over in terms of bandwidth, then basically the bandwidth of this RC low pass filter is reducing I mean is increasing and therefore the output envelope will increase and likewise when the bandwidth is increased the output envelope will decrease.

(Refer Slide Time: 13:13)





Now, so in terms of impulse response what comment can we make? If it is varying periodically with time, what comment can we make about, this must be reflected somewhere in the impulse response. So, in a time invariant system let us take a step back, well you apply an impulse now, you get a response like this if you apply an impulse some t later you get exactly the same thing shifted one unit.

Now, if this was a linear time varying system, you apply an impulse here you get that, you apply an impulse here you get something like that. Linear periodically time varying system, you apply an impulse here, you get something like that, if you apply an impulse here you perhaps get something like this. But if you apply an impulse at time T s, which corresponds to the period with which the system is varying, then what would you expect?

Student: (())(14:40).

Professor: Same, I cannot here you guys. Which of those pictures, row or column?

Student: Column.

Professor: First row, last column. So, I am not exactly the same thing but, so, the connotation is that this is the and this are identical. So, if I just showed you these two pictures you would not be able to tell me whether the system was, if I just showed you those two pictures in the green boxes you would not be able to tell me whether the system was time invariant or periodically time variant.

So, what does this mean? So, now that we know what is this how to express this mathematically for an LPTV system therefore, h of t comma tau is the same as? No, h of, let us write down all the answers I got. What did you say? T comma tau plus n T s that is choice a, anybody else? H t plus n T s comma tau. Any other answers? Both are same.

Student: (())(16:32).

Professor: No. Let us write this also down, h of t plus n T s or tau plus m T s what else? All the above? Common folks. See, if T is, if this is the time of observation and you apply an impulse tau before, so let us say you apply the impulse here. What will you get? You will get h of t comma tau. Now, if I move a time T s away, so that I move my time of observation by some magic number T sub s and I apply an impulse tau before the time of observation, what will I get? I should get the same thing.

If you choose n equals to 1, you can generalize it but h of t comma tau therefore must be equal to h of t plus T s comma. So, if you move exactly by T s you do not know that anything is changed. It is like, I mean if you, what do you call, if take an particularly strong dose of anesthetic and you know, you do not know how long you have slept, you wake up 4 days later and then in the morning and then everything looks exactly as it does today.

I do not recommend that you do this but I hope you understand what I am saying. So, h of t comma tau is simply the same as h of t plus T s comma tau, which is obviously the same as h of t plus 2 T s comma tau and so on.