Introduction to Time – Varying Electrical Networks
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Frequency response of an LTV system

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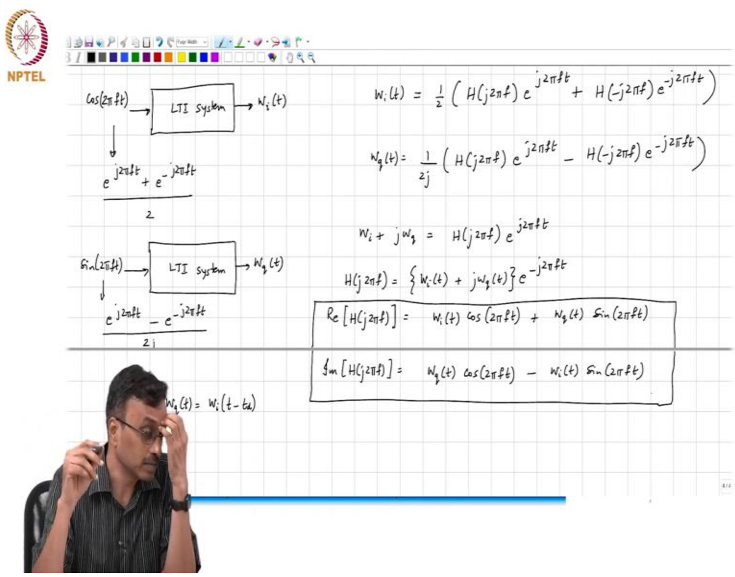

The next thing again let me keep drawing parallels between LTI and LTV so that, so let us say you are given an LTI system and I want to find a H of you are in the lab I want to find H of j 2 pi f. What will I do? Of course, in the lab you cannot say give me a complex exponential because it is what am I going to do to produce e to the j 2 pi f t. So, what do I do? What do you do?

Student: (())(0:54).

Professor: So, you say I will excite this with e cos 2 pi f t and cos 2 pi f t is nothing but e to the j 2 pi f t plus e to the minus j 2 pi f t by 2 and I could also excite it with at least conceptually with sine 2 pi f t and let us call this w i of t and w q of t. So, what will I get here? What is w i of t? Sine is e to the j 2 pi f t minus e to the minus j 2 pi f t by 2 j. So, what is w i of t? It is H of, it is half of h of j 2 pi f times e to the j 2 pi f t plus h of minus j 2 pi f times, as j two pi and likewise what common can we make about w q of t? 1 over 2 j H of j 2 pi f times e to the j 2 pi f t minus H of minus j 2 pi f minus j 2 pi.

So, if you have w i and w q what do you think we should do to get H of j 2 pi f? Wel,l you say w i plus j times w q gives you H of j 2 pi f e to , j 2 pi f t. So, how do I so what is H of j or so what is so H of j 2 pi f is nothing but w i of t plus j w q of t times e to the minus j 2 pi f t. So, what is the real part of H of j 2 pi f w i of t times cos 2 pi f t minus or plus, plus w q of t sine 2 pi f t and the imaginary part of H of j 2 pi f is w i is it minus should I just make it minus. So, cos 2 pi f t w q I hope you are not made in w i so plus w q w i, thank you, sine 2 pi.

So, if you are given a linear system you are excited with the cos and sine you get these outputs in steady state, remember all these are steady state referred to steady state solution and you look at this w the i output and the q output and this will give you the real part and this gives you the imaginary part and from which you can get magnitude and phase if you so do that. I mean some of you may argue I mean this must be this is a really dumb way of doing this, so what is the smarter way of doing this?

Well, I mean the other I mean before we go to step I mean one suggestion was why do not you just simply put a step and then find the step response from which you can get the impulse response and then do the Fourier transform but that seems to be an awful lot of work because you need to actually, well you are finding the frequency response at all frequencies. If I am only interested in the frequency response at one frequency it seems much easier to do, but I mean what I am talking about is even in this setup do you see some redundancy?

Student: (())(7:44)

Professor: So, what he is saying is that well there is a time invariant system and there is nothing really new that the sine wave is giving you because when compared to a cosine, a sine is simply a shifted version so I need not have calculated w I mean I did not have excited the system with a sinusoid and actually record w q, I could have simply taken w i delayed it by 90 degrees and that would be my w q. I mean in other words, w q is simply nothing but w i of t minus td, where td equals 1 over, what is the time delay between sine and cosine guys?

1 by 4 n, so technically speaking therefore I mean this is not strictly necessary but I mean we do that, we could have for instance used only the cosine and this is what you learn in your basic linear systems class. So, you excite with the cosine or a sine it does not matter or and the amplitude of the output sinusoid with respect to the input amplitude gives you the magnitude response and the phase shift between the two sinusoids gives you the phase response. So, that from which you can go and get real and imagine.

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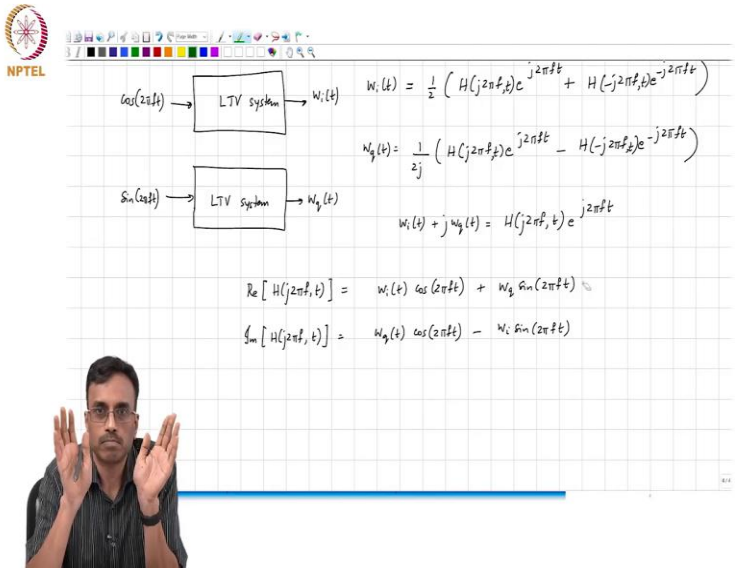

Now, let us see what happens in the time varying case. So, again now you have a time varying system and your task is to find H of j 2 pi f t. The question is how do we do this? One way is as he suggested well you can I mean now this becomes a lot more painful because you find H of t comma tau for all t and all tau which will give you that two dimensional function from which you can go and compute H of j 2 pi f comma t by computing the Fourier transform.

But if you are only interested in the response to a sinusoid at f that seems like an awful lot of work, the question is can we get away by simply, it seems reasonable that if I am only interested in the response at frequency f, why should I go through the effort of you know putting impulses all over the place. The impulse is got frequency content across all frequencies but I am only interested in the frequency and the response at f. So, evidently I do not need to work as hard.

So, just like how we did for the time invariant system we say let me try and excite, now what do you suggest? We have seen what we did with the time invariant case, what by the way what is the interpretation of this? What is H of j 2 pi f comma t, what does that mean in English? It is the complex gain, which is equivalent to saying the magnitude gain and the phase shift experienced by a sinusoid as it passes through a time varying system.

If the system was time invariant, the magnitude would be gained up by a fixed factor and the phase shift would also be a constant phase shift over time, if the phase shift will not change with time. Given that the system's properties are varying with time, it stands to reason that the gain experienced by the sinusoid will change, I mean the magnitude gain will change and so will the phase shift. So, what we are trying to find before is to find this that complex gain as a function of time when excited with a sinusoid at f, so what do you suggest we do I mean we obviously cannot put in e to the j 2 pi f t and then look at the output.

If you are in a lab the only thing you can do is you either put in a cosine or a sine or both or whatever. So, they are taking a q from the time invariant case if you put in cosine 2 pi f t what do you get at the output? Let us call this again w i of t what we get at the output? Well, that is easy. So, if I put in a cosine I obtain w what I call w i, I put in a sign I get w q and so though what is the only change I need to make to these equations?

Student: (())(14:02).

Professor: Pardon? Very good. So, basically all that I need to do is change this to this is now the gain is varying with time so this now becomes f comma t and we go through the same algebra except that now w i of t plus j w q of t equals H of j 2 pi f comma t times e to the j 2 pi f t and how this look like? So, what is H of, so what is real part of H of j pi f comma t w i cos, w q sine and the imaginary part is w q cos 2 pi f t minus and is this so what do you notice? It is the same thing, except that earlier when you did this with the time invariant system what you would expect to see when you do this is, is you would expect to see something which was independent of time.

Now you should expect to see something with which varies, but what you are doing is the same. Now can we now say well we do not need the sign because the sign is simply a delayed version of the cos, remember that this is a time varying system so, the response to a delayed input is not the same it will not yield the delayed response. So, the sign is a delayed version of the cos, but that does not mean that w q will simply be a delayed version of w i.