Introduction to Time - Varying Electrical Networks
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Convolution integral for LTV systems

So, let us get started, good evening and welcome to advance electrical networks, this is lecture 26. So, in the last class we were talking about why time varying systems are important and why it indeed makes a lot of sense to study them and it is only surprising that we never get to study these things for some strange reason or the other. So, let us get so today we will begin in right earnest and it is always much easier to understand things when we try to relate it to somethings that we already know and the closest material that you know is stuff about linear time invariant systems.

So, we would like as we keep going along we would like keep drawing parallels between time varying systems and time invariant ones so that you can use what you have learnt to kind of a use that as base line and see where you extend the concepts and hopefully this will build intuitions.

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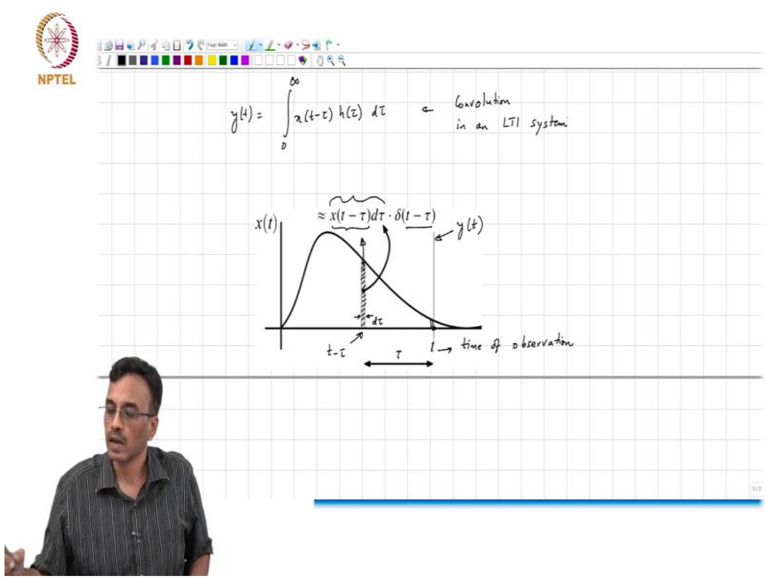

So, a quick refresher about time invariant systems well, so you have an input x of t and a time invariant system is characterized by an impulse response h of t and the output of the system therefore is got an using the very familiar convolution integral which I will simply derive from scratch just so that we can do the same thing when we go to time varying system. So, this is x of t and let us say you are interested in finding the output voltage, output quantity at certain time t and the signal tau before, this is the time of observation, t is the time of observation and this thin lever here that occurs, that is occurring at a time tau before the time of observation. This time therefore it is t minus tau.

This can be approximated by a direct impulse of with strength x of t minus tau times, this is delta tau and so, that is the strength of and that this impulse occurs at a time t minus tau. So, basically this is the strength of the impulse and this impulse occurs at t minus tau so that is basically we. The response to this impulse is the response to this impulse at time t is simply given by h of I get t, I get tau, I get tau minus tau.

It is simply you apply the impulse at t minus tau you observing it at t, so the response to this impulse is basically x of t minus tau times h of tau d tau and you integrate this to find the total response at time t you kind of keep sliding that tau all the way from 0, you start here I mean you assume that h of t is cosine I mean you assume that the impulse response is causal, so the system is not responding to inputs that can potentially occur in future.

So, you run you integrate this from tau equal to 0 to, when tau equal 0 you basically looking here and then x of t of course can extend all way from minus infinity, there is no reason why x of t must start from 0 though in this picture it does. We could still be responding to an input applied in 3000 vc nothing prevents that from happening. So, this goes from tau equal to 0 to infinity. So, this is the very familiar convolution integral, there is really nothing much to discuss. Now let us see what happens in, so this is convolution linear time invariant system.

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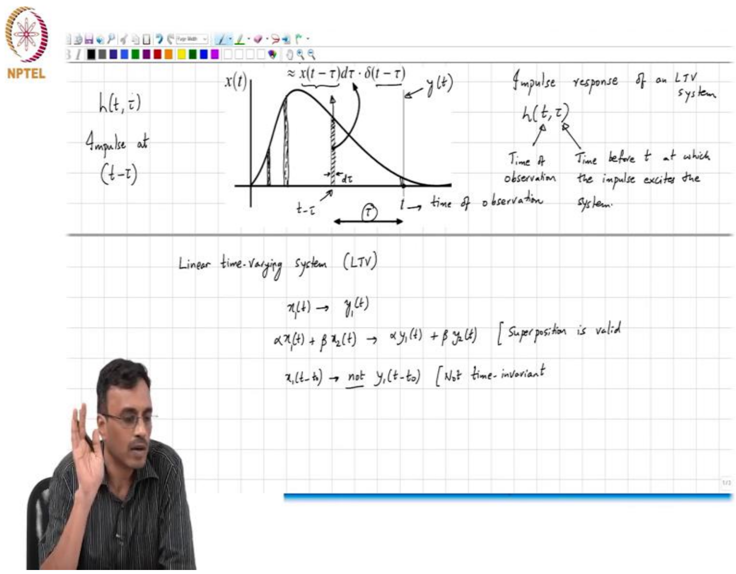

Now, a linear time varying system or LTV, the only difference between linear time invariant system and the time varying system that, if you get x of t let us say you got y of t and if you put it is still linear so alpha x1 of t plus beta times x2 of t will yield alpha times y1 of t plus beta times y2 of t and so super position it is still valid. However, x1 of t minus t naught will not necessarily yield y1 of t minus t. So, this is not time invariant. So, in other words, if you apply an impulse so let us say again we have faced with trying to figure out what the output of a linear time varying system is to an input.

We again, we resort to the notion of the impulse response and just like how we work with a linear time invariant system what do we do? There we split up input x into a whole bunch of impulses and then we know the response to each impulse and we added up the response to each one of those thin levers and then that is how you get the convolution integral.

Now we are going to do the same thing except that, so let us say we break this up into impulses. So, this is a lever, this is yet another lever each of which is approximated by an impulse. When we had a linear time invariant system, the response only depends on the time difference between the point at which the system is excited with the impulse and when you are observing the output. It did not depend on the fact that, you launched it particularly at t minus tau and you are observing it at t, if you pushed t minus tau and t both to the right for instance by the same amount, the response would still be the same. That is what time invariance means.

A time varying system is where it depends both on the actual time of observation as well as the actual time at which the impulse is launched. In other words the impulse response is of the form h of t comma tau, where t is the time of observation and tau is, so time before t at which the impulse is launched. In other words, h of t comma tau basically means that the impulse is launched at what time? At t minus tau.

This is just a matter of notation, there is nothing holy about this. It is also quiet common to have a notation where the tau denotes the time at which the impulse is launched. You excite a tau, you observe a t. That is another alternative notation I just chose to use h of t comma tau as where can I say h of t comma tau it is the response at time t due to an impulse applied tau earlier. So, the impulse is applied at t minus tau. Does it make sense?

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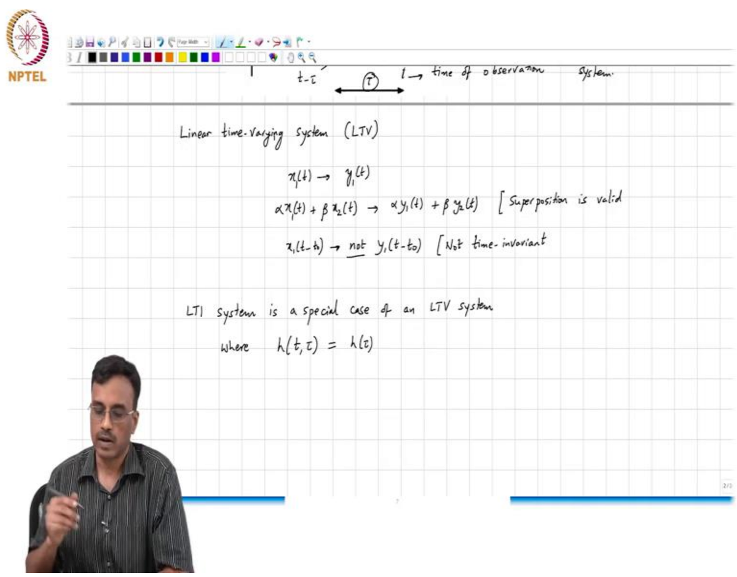

So, an LTI system is a special case of an LTV system where h of t comma tau equals it is independent of t, it is only a function of tau.

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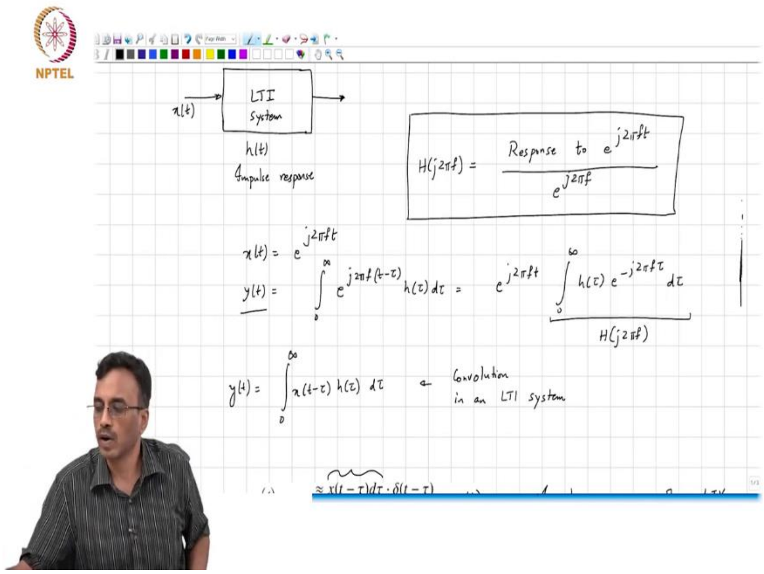

So, next thing again I will keep back and forthing between a time invariant system and a time varying system so that we see the parallels. Now, as you all know the complex exponential is a very important signal when you work with linear systems and that is because is an icon function of a linear system. Let us see, let us refresh our memories as to how a linear time invariant system reacts to a complex exponential e to the j 2 pi f t. So, let us say, x of t is e to the j 2 pi f t and then it starts at minors infinity and then keeps coming all the way.

So, y of t therefore is, you plop x of t into the convolution integral and then you basically get 0 to infinity e to the j 2 pi f t minus tau h of tau d tau which is e to the j 2 pi f t times integral e to the j 2 pi f, we may write this as h of tau e to the minus j 2 pi f tau and this is only when you perform this integral of course the tau is going away so this is going to be a function of f, so basically this is nothing but h of j 2 pi f and what is the name of this.

So, a couple of things to note, if you put in a complex exponential, the output is also in other words the output is also sinusoid except that the sinusoid is gained up by a complex number. The gain, so in other words you can think of h of j 2 pi f of the frequency response as the response to e to the j 2 pi f t divided by y of t is the response to e to the j 2 pi f t, so if you want to find, if you want to interpret h of j 2 pi f, what how do you interpret this in English? Is the ratio of the response to e to the j 2 pi f by e to the j 2 pi f, you knew this already.

So, in other words this h of j 2 pi f if you want to think about it, it is simply the gained experience by a sinusoid where it passes through the LTI system and of course this gain is a complex number in reality what it means is that not only do you see a change in the magnitude we also see a phase shift.

Now this system is time invariant and what does time invariance mean? Time invariance means that nothing changes as far as the system is concerned because if you excite the system now or if you excite the system a day later, you get exactly the same response. Since the system is not changing with time, it seems reasonable that the gain experienced by the sinusoid also does not change with time. Now let us do the same thing, let us see how time varying system fairs with the sinusoid input. So, we need to figure out the convolution integral first.

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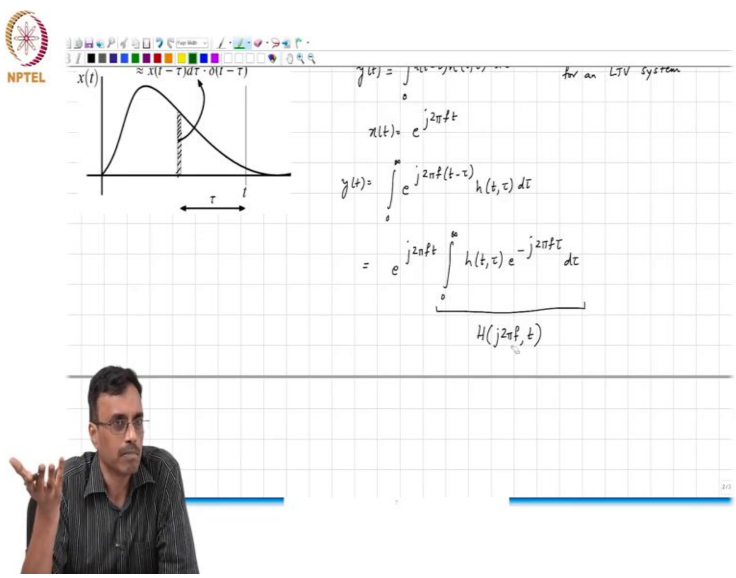

So, let me, so the response at time t due to this impulse here is what is nothing but h of t comma tau d tau x of t minus tau and then what do we run the integral over? Tau 0 to infinity. So, this is the convolution integral for an LTV system and as you can see sanity check if h of t comma tau is the same as h of tau which is what it would be if the system is time invariant, then you know you get back the familiar convolution integral that you are used to from your study of time invariance. Is clear people?

Now as usual it is a linear system we interested in figuring out what happens to the system when you excited with a complex exponential and what is so special about the complex exponential by the way. I mean why we are interested in complex exponential? Pardon. So, what does that mean in English? So what?

Student: (())(19:39).

Professor: So what?

Student: (())(19:43).

Professor: So, the motivation to use sinusoidal signal is that any signal can be decomposed into a sun of sinusoids via the Fourier transform. So, if we figure out how the system responds to one sinusoid, to a sinusoid with a frequency f you can figure out how it responds to an arbitrary input. So, x of t is e to the j 2 pi f t, so y of t is integral e to the j 2 pi f t minus tau h of t comma tau d tau and this is e to j 2 pi f t h t comma tau and again this is the same sanity check if h t comma tau was independent of t then you get in familiar Fourier transform that of the impulse response which is the frequency response that we all know.

Now what comment can we make about this guy here we integrate with respect to tau so this after integration what do you get? You get a function which is, this will be both the function of j 2 pi f as well as, does it makes sense? So, what do we see now? When you excite a linear time varying system with the complex exponential, the output is that complex exponential multiplied by a gain. The gain as you can see here, depends not only on frequency but also on time.

Now does this seems reasonable or does it seem unreasonable? Well this seems reasonable because, well the systems properties are varying with time so it seems reasonable that you excited to the sinusoid therefore, the gain the sinusoid experiences as it goes through the system will also vary with time.

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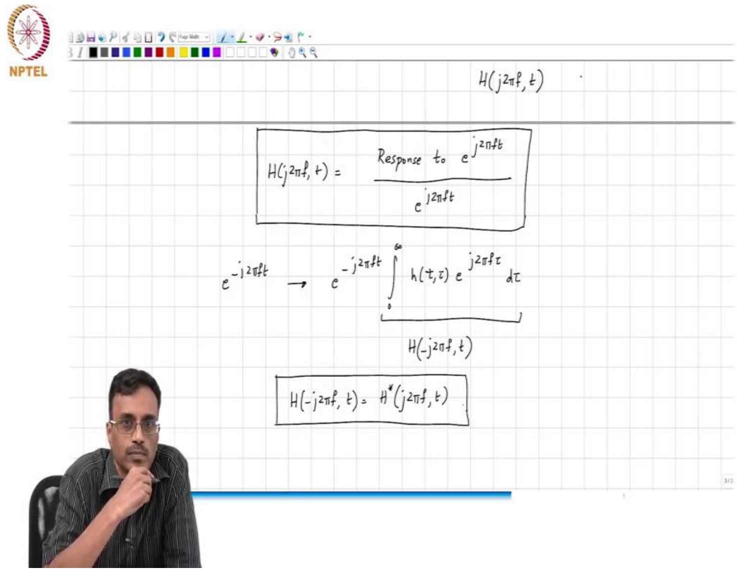

As before how did we interpret h of j 2 pi f in the time invariant case?

Student: (()) (22:53).

Professor: We interpreted h of j 2 pi f in the time invariant case is the ratio of the response to e to the j 2 pi f divided by e to the j 2 pi f and that is the same thing here. So, H of j 2 pi f comma t is simply response to e to the j 2 pi f t divided by e to the j 2 pi, like in the. And what comment can we make about h of minus j 2 pi f comma t in other words. If we excited it with if f was made minus f what would we get at the output.

So, when we put in e to the j 2 pi f t this is what we got. If we put in e to the minus j 2 pi f t what would we get? You get e to the minus j 2 pi f t times all that we need to do is replace f with minus f so that becomes 0 to infinity h of t comma tau e to the j 2 pi f tau and this therefore is what is this h of minus j 2 pi f comma t and how does this relates to h of plus j 2 pi f comma t? Is the complex one.

So, H of minus j 2 pi f comma t equals H star of j 2pi f comma t. And the assumption here is of course that H to t comma tau is real. Does this sound familiar or is there something sanity check? Well, if the system is time invariant, then the gain will not depend on t and we know that there must be odd symmetry about complex conjugate when you change f to minus f you must get the frequency response which is the complex conjugate. So, magnitude and phase will be even and odd function of frequency. Does it makes sense?