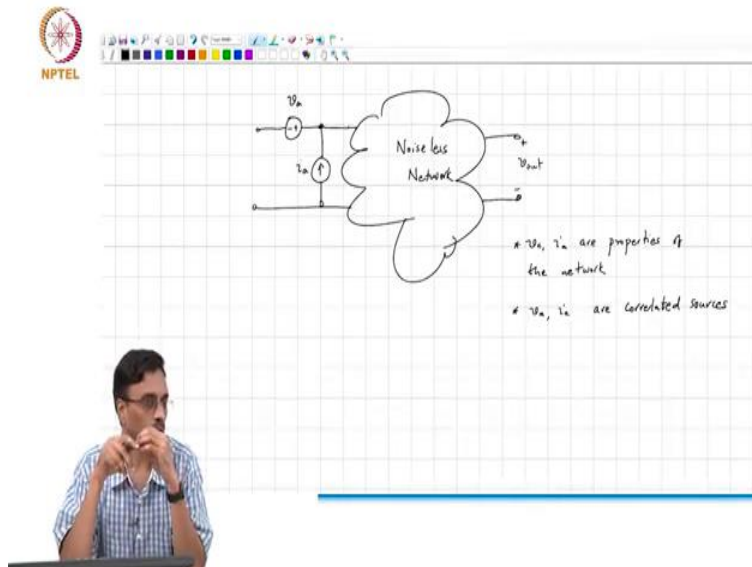


**Introduction to Time - Varying Electrical Networks**  
**Professor Shanti Pavan**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Madras**  
**Lecture 26**  
**Input referred noise and the noise factor**

(Refer Slide Time: 0:22)



So, in the last class, we basically concluded that if you have an amplifier or actually any linear system with internal noise sources, you can represent the effect of all the noise sources inside by thinking of this as noiseless and accounting for the effect of all the internal noise sources by using I think we call them  $V_A$  and  $i_a$ , the different notations to use, it does not matter what.

The key points to note are that  $V_A$  and  $i_a$  are properties of the network and the noise sources inside. And therefore, if you want to kind of hand this, this black box over to somebody else who might be using the box, you not only give them the 2 port parameters and so that they were able to figure out what the input impedance is and what they must drive this network with.

You also give them  $V_A$  and  $i_a$  and in principle, that should be good enough to be able to figure out what the output noise would be regardless of what source impedance you drive the network. Does makes sense? And in general,  $V_A$  and  $i_a$  are they independent, both of them are noise sources.

In general, are  $V_A$  and  $I_a$  independent? They are not because  $V_A$  and  $i_a$  are both derived from the same independent noise sources inside. So,  $V_A$  is some linear combination of  $V_1, V_2, V_3$

all the way through  $V_n$  and likewise  $i_a$  is some other linear combination of  $V_1$  through  $V_n$ . So clearly, they are related to each other. So, they are not independent. So, in general  $V_a$  and  $i_a$  correlated sources.

(Refer Slide Time: 3:11)

The slide features a circuit diagram on the left and handwritten notes on the right. The circuit diagram shows a voltage source  $v_i$  in series with a resistor  $R_s$ . The output terminals are labeled with voltage  $v_a$  and current  $i_a$ . An input impedance  $R_{in}$  is connected across the output terminals. The handwritten notes include:

- the network
- $v_a, i_a$  are correlated sources
- if source is an ideal voltage source
  - $\Rightarrow i_a$  is of no consequence
- if source is an ideal current source
  - $\Rightarrow v_a$  is of no consequence

And so, a couple of observations. Let us say for example, that we are driving the network with some  $R_s$ , and then here is  $v_a$  and here is  $i_a$  and this is the internal of the network. So, there is some  $R_{in}$  here that represents the input impedance of the network looking in. So, what comment can you make about the, what comment did we make about dividend voltage looking in here?

It simply  $v_i$  plus  $v_a$  plus  $i_a$  times  $R_s$ . So, if the  $R_s$ , if we were driving it with, if we were driving it with an ideal voltage source, then what comment can you make, the source is an ideal voltage source, what comment can you make which of these inputs referred noise sources is inconsequential?

Well,  $i_a$  is of no consequence. On the other hand, if the source is a, is an ideal current source, what is of no consequence? My  $R_s$  therefore tends to infinity and then what is of no consequence? The voltage sources of no consequence. So, in reality  $R_s$  is not, neither in practice  $R_s$  will either be 0 or infinity. And so, both at least in principle  $v_a$  and  $i_a$  will both be, will be important.

And if you, I mean this also basically points to the fact that there is a sweet spot as far as choosing, I mean remember that this  $(\omega)(6:24)$  voltage here is this and once you have that  $(\omega)(6:33)$  voltage that  $v$  times  $R_n$  by  $R_n$  plus  $R_s$  is going to get amplified by the or process by the rest of the network.

So, if you basically want to minimize within quotes "the noise" the noise evidently depends on, I mean this is a signal and this part corresponds to the noise. So, both the signal and  $v_a$  plus  $i_a$  times  $R_s$  are going to be processed by the same transfer function going forward. So, maximizing the signal to noise ratio therefore, means that you need to, you would like to make sure that the signal power to the noise power is as large as possible.

And so, and there is evidently a sweet spot let us say at a certain frequency the strength of  $v_a$  is very large compared to  $i_a$  times  $R_s$ , there are 2 quantities which cause noise. 1 is  $v_a$ , the other 1 is  $i_a$  times  $R_s$ . So, if  $i_a$  times  $R_s$  is very very small, let us say somehow it turns out that the input referred noise current source is very small in magnitude, then what comment can you make about  $R_s$ ?

As I said this basically means that  $R_s$  can be made arbitrarily large. But on the other hand, I mean, but if  $i_a$  is not 0, but it is finite but small. In the beginning for some value of  $R_s$  let us say  $i_a$  times  $R_s$  is very small compared to  $v_a$ . And you say, Oh, well, great. Let me go and crank up  $R_s$ . As you go on increasing  $R_s$ , what will happen eventually? I mean, that basically, if I increase  $R_s$  basically means that I can use a poorer and poorer source. That is what it means. So, if I go on cranking up, yanking up  $R_s$ , what happens eventually? Yes? What happens? All the  $i_a$  times  $R_s$  starts to become very large.

So, I mean, there is evidently a sweet spot for, so given the strength of  $v_a$  and  $i_a$  and in fact you know that they are also correlated given the properties of  $v_a$  and  $i_a$ , you will find that there is a sweet spot for the source in this, in our example, source assistance in general source, I mean, all these are basically, we just assumed that there is no memory and then I did everything, finally, everything is going to be a function of frequency. So, all that means is that this is that  $(\omega)(9:37)$  of  $g$   $\omega$  at a certain frequency, there is some, there is some sweet spot for the source impedance which minimizes the total amount of noise.

So, if we choose for example, too small source impedance  $v_a$  will dominate. If  $R_s$  is 0 it is only  $v_a$  that is consequential, if you use too large  $Z_s$  it is the  $i_a$  will dominate. But perhaps it is possible to choose something, somewhere in the middle, where this  $v_a$  plus  $i_a$  times  $Z_s$  is small, it depends on the relative values of  $v_a$  and  $i_a$ , as well as the correlation between. And those of you who have done an RF class, basically, who have seen this in that context. But having doing that now, basically is not particularly useful.

(Refer Slide Time: 10:53)

The slide features a circuit diagram of an amplifier with an input noise source  $v_a$  and a noise factor block. Handwritten notes describe two cases: Case 1 (nulling internal noise sources) and Case 2 (all noise sources active). It includes equations for noise power spectral densities and the definition of noise figure in dB.

NPTEL

$v_a$  is of no consequence

Noise Factor (Noise Figure)

Case 1: Null all internal noise sources  
Output noise  $\rightarrow v_{n1}^2$

Case 2: All noise sources are activated  
Output noise  $\rightarrow v_{n2}^2$

$v_{n2}^2 > v_{n1}^2$

$\frac{v_{n2}^2}{v_{n1}^2} \geq 1$

$10 \log_{10}(NF) = \text{Noise figure}$

Quantifies the SNR degradation caused by the amplifier

The next thing I would like to talk about is, is this concept of, again this is largely (10:56), it is a nice factor or also called noise figure. And the idea is the following. So, we have our, let us say we have our source and we have our amplifier or filter or whatever you have, there is a whole bunch of internal noise sources and you look at the output and there, let me.

So, let us again assume that the input perhaps is coming from some kind of antenna, the antenna has got some source resistance, and this source resistance adds noise. So, this noise is basically, noise spectral density is  $4 kT R_s$ . So, in the first case, what I am going to do is simply look at the noise spectrum, let us call or assuming there is no memory  $v_n$  1 square that the mean square noise is the output of the amplifier.

Next thing I am going to do is, all noise sources are active. And let us say the output and I again measure the output noise, which will be, which will be larger it is a no brainer. So, basically,  $v_n$  2 square on a mean square value that will be much greater than  $v_n$  1 square. So, this ratio is a

measure of, what physical meaning can be attached to this quantity, the mean square value again, as I said this is we are assuming that there is no memory inside the amplifier. So, at this point is just simply the mean square noise integrated over some bandwidth.

So, but what physical significance can be attached to this quantity? What is the number quantify? Well, this basically is telling you how badly, I mean  $V_n^2$  is simply the input noise that is processed by with amplifier and that input I mean, the input signal is also processed by the same transfer function because the noise source corresponding to  $R_s$  and the input voltage  $V_i$  have the same transfer function.

So, this noise, with this noise basically in case 1 corresponds to the noise that would have been there at the output, even if the amplifier was perfectly noiseless. The second case is, what you actually see, which is simply consists of 2 parts, 1 part is the input noise processed by the amplifier, on top of it there is extra noise that the amplifier adds on top of its own.

And clearly the best you can do is if  $V_n^2$  is the same as  $V_n^2$  and so this number must always be greater than or equal to 1. And this is a, this quantifies the SNR degradation caused by the amplifier. Remember, if the amplifier is perfectly noiseless, what comment can we make about the input SNR and the output SNR?

Well, it is the same thing. You take a noisy waveform and you amplified it up, you have amplified the signal and the noise by the same factor and therefore, the SNR does not change. Now, what are you doing? Now, you are amplifying the signal and the input noise by the same factor, on top of it you are adding your own noise. And therefore, you have effectively degraded the signal to noise ratio.

And that is equivalent to simply saying, the degradation in SNR is simply found by just taking the ratio of the noise that you see at the output divided by the noise you would have seen provided the amplifier was noiseless. So, and this is therefore, this is what is called the noise factor or the nice noise figure.

So, it is common to, I mean there is also, I mean there is some confusing notation here. This absolute number is often called the noise factor, people I also like to do, I mean something else that you will see is  $10 \log_{10}$  of noise factor is noise figure and some people use it

interchangeably and so on. But, if they say the noise factor is 3 DB it basically means that, the noise factor is 3 DB what does it mean? The output now SNR is, I mean or  $V_n^2$  square by  $V_n^2$  square is a factor of 2 which basically means that the amplifier is adding as much noise as the input noise source after going through the amplifier.