Introduction to Time – Varying Electrical Networks

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Lecture 24

Input Referred noise in electrical networks – part 2

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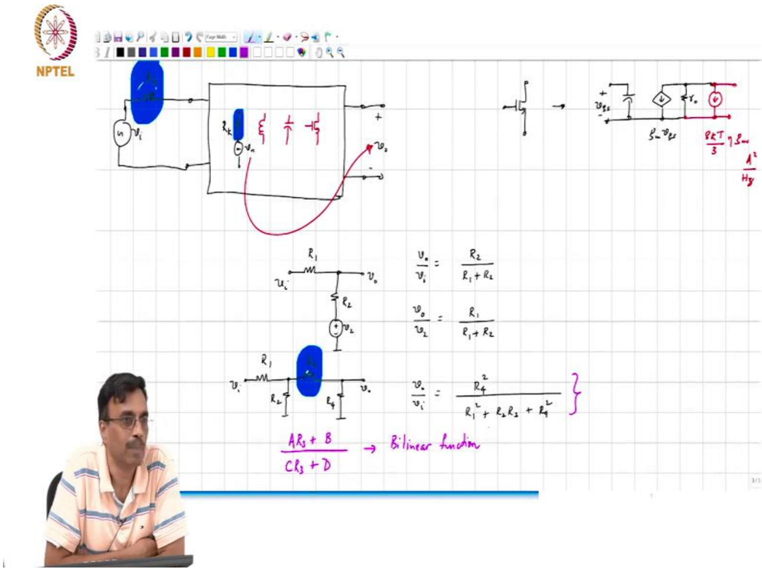
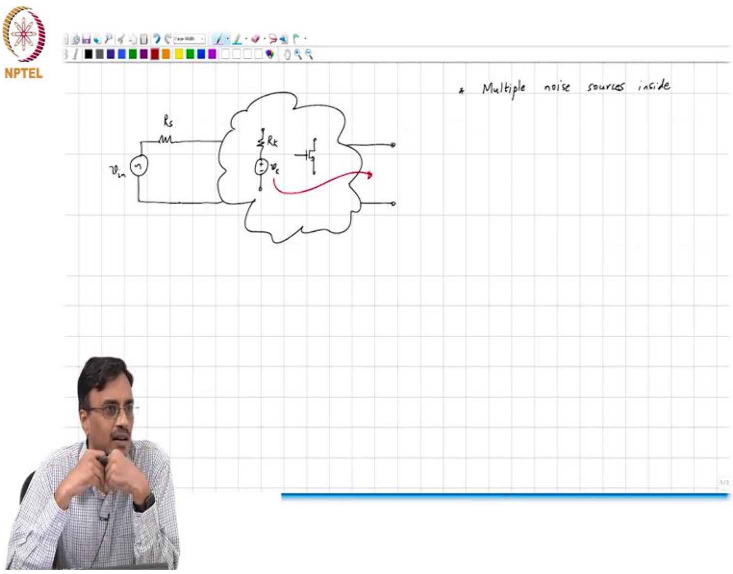
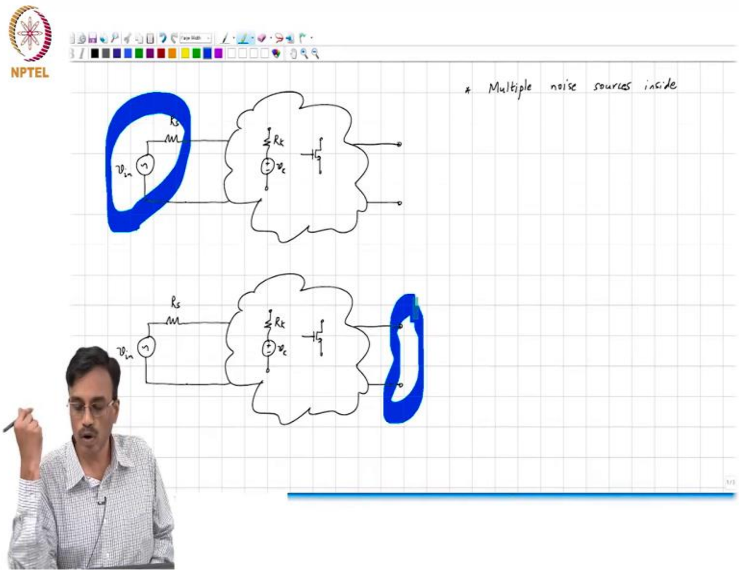
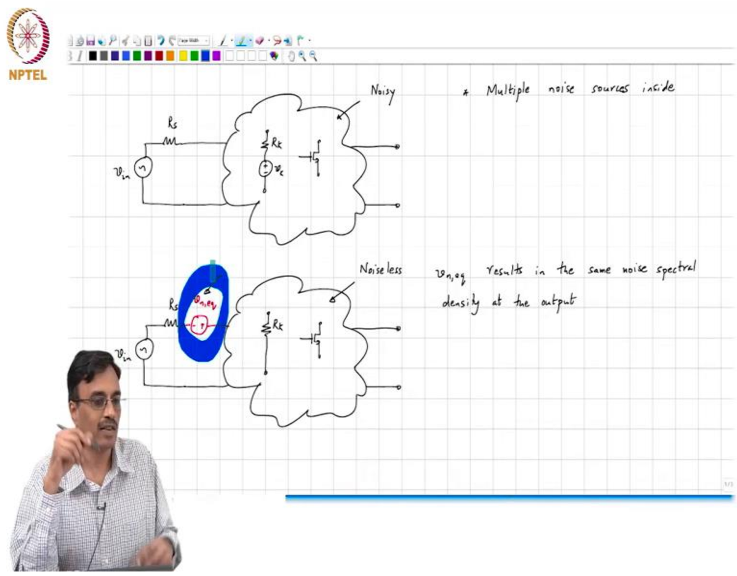
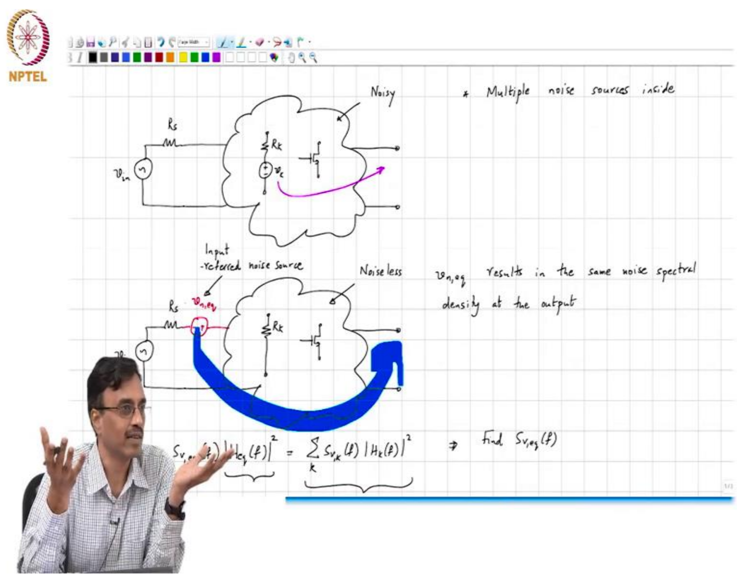





NPTEL
* Multiple noise sources inside
Rs
Rx
vc
vin


NPTEL
* Multiple noise sources inside
Noisy
Noiseless
Rs
Rx
vc
vin
vn,eq
vn,eq results in the same noise spectral density at the output


NPTEL
* Multiple noise sources inside
Noisy
Noiseless
Input -referred noise source
Rs
Rx
vc
vin
vn,eq
vn,eq results in the same noise spectral density at the output
= Σk Svk(f) |Hk(f)|²
Find Sv,eq(f)

In the last class, we were talking about how 1 would represent the noise of a large network. . So as you're definitely aware, we are always dealing with networks with whole bunch of components resistors, some transistors, whatever and presumably the network has an input, which I will call this resistance RS and couples to network in some fashion like this, this is the output.

And so there are multiple noise sources inside and therefore there is a, associated with each noise source, there is a transfer function from the noise source to the output and all these (())(01:34) noise sources go and influence the output in some way and we know how to calculate the spectral density at the output due to all these noise sources. Now, as we were discussing yesterday, when you are giving this block away to somebody else for use, I mean, the other person is probably not really interested in the gory details of what is going on inside the box.

So, he is more, he or she is more interested in figuring out, how does this, this amplifier or filter or whatever affect the signal to noise ratio of my signal, my signal is there, it is being processed by this, this animal and the output is of course consists of two parts. One is the signal that is processed by the transfer function that it is supposed to, that is the within codes, desired output. On top of it, there is noise, let us add in from within the box and an obvious question to ask would be, how badly is my circuit block degrading the signal to noise ratio of my input signal?

So, for instance, if you had an amplifier, the job of the amplifier is to take a small signal and make it large, in some sense. Now and the reason why you need a large signal is because this, the circuitry following this amplifier, basically is able to discern what is going on. But, in the process, the amplifier also adds its own noise. So, the signal increases at the output port of the amplifier.

But likewise, there is also a lot more noise at the output than there was at the input and why is there a lot more noise there are two reasons for this one is that the noise that was inherent in the signal itself before it hit the amplifier will get amplified. Because the amplifier does not know what is signal and what is noise?

On top of this amplified noise from the signal source itself, there is internal noise that is generated by the amplifier and therefore, the total noise the output will in the best case be simply that which is amplified that of the signal source which is amplified by the amplifier and if your amplifier is really bad, then well it does a great job of amplifying my input signal, but also adds so much noise in the process that in effect, even though the magnitude of the output signal or the power of the output signal at the, is very large.

The fidelity of the signal is degraded so much because in addition to this large signal you have more than proportional noise amplification. So therefore, when you are characterising amplifiers and so on or filters for that matter or any circuit for that matter. You would like to be able to estimate or figure out how, within quotes, how bad is my signal to noise ratio, getting I mean, how badly is my signal to noise ratio getting affected? Because of, whatever signal processing I am doing inside this inside this box.

And since we are not really interested in the gory details of what is happening in the box, a reasonable question to ask would be, well, what if I need to compare the, how the I mean compared to signal to noise ratio of my signal source, which is that of this guy here, to what the signal to noise ratio would be here, an equivalent way of doing this would be the following. So, let us we asked the question, well if I found an equal and noise source here, where I went and disabled all noise sources inside the network. So, what do I do?

So, this is a noisy network and, in the picture, below, I say Oh, the network, let us assume it is noiseless and I am trying to replace the effect of all these multiple noise sources inside the box with a single noise source, which for obvious reasons, I will call the equivalent noise source and in what way should the two be equivalent I mean, in what sense when we say equivalent what do we expect?

Student: (())(07:15)

Professor: Well, vn equivalent has the same or other results in the same noise spectral density at the output, does make sense. So, all that we are saying is or this is supposed to be an equivalent representation of this (())(07:57) noise sources inside the box and evidently, there is I mean, if we are able to club the effect of all these noise sources into one source at the input, that is what we call the input referred noise source. Now, let us discuss how we are going to find this input referred noise source. How I mean, can we say, can we think of how we are able to do this, what do you suggest?

Student: (())(08:30)

Professor: Exactly, yeah. So, basically the idea is very simple, we from vn equivalent to the output here there is going to be a transfer function and therefore, Sv equivalent of f will be whatever transfer function there is from vn equivalent to the output. So, that is basically you call that H equivalent times, this times H equivalent of f the whole square, must be equal to?

Student: (())(09:19)

Professor: Well, there are multiple noise sources inside. So, each one of them will yield Oh well, SvK of f times?

Student: (())(09:34)

Professor: Mod HK of f the whole square and you do all this over all the noise sources and which basically means that you can go and just do the math and find the spectral density of the equivalent noise voltage source that would result in the same noise spectral density at the output. Does that make sense? And I know as he in English all that this means is that this is nothing but what does this represent?

Student: (())(10:16)

Professor: The output noise spectral density and this represents?

Student: (())(10:23)

Professor: Well, recognise that the gain from vn and from vn equivalent is the same, this H equivalent of f simply represents the?

Student: (())(10:31)

Professor: The gain from the input voltage source to the output and therefore, it stands to reason that you find the total spectral density divided by gain square and this is the spectral density of the equivalent input referred noise. So, this is this is an equivalent noise source and this is referred to the input, input referred.

So, is the motivation and the jargon clear now. So, now let us get started a couple of things that I would like to draw your attention to. One is our job, therefore seems to be a, to find the transfer functions from each of those input sources to the output and also from vn equivalent to the output and well how, given a general network, we can only come up with some broad guidelines it is, without knowing the network, it is evidently not possible to come up with the transfer function.

But a couple of observations that I like to make are the following and to do that, as usual I will, what do you call use our KCL KVL type analysis, let us say I am interested in finding the transfer function from vK to vR.

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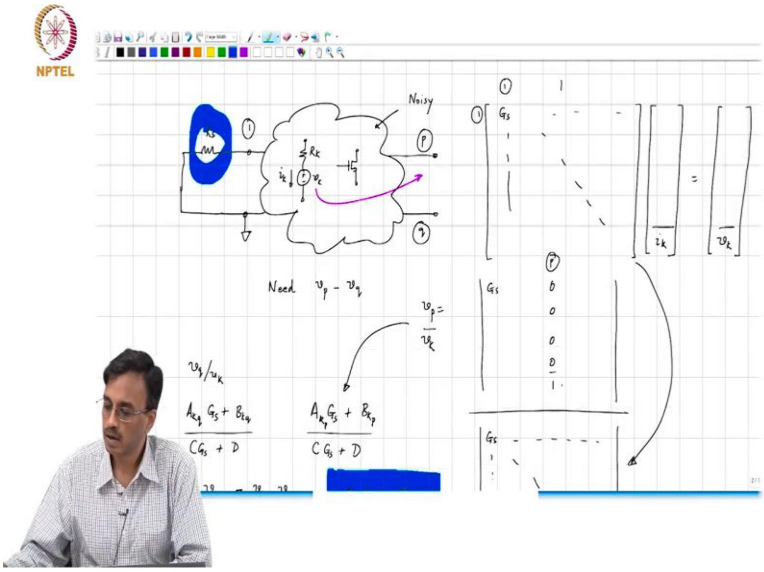

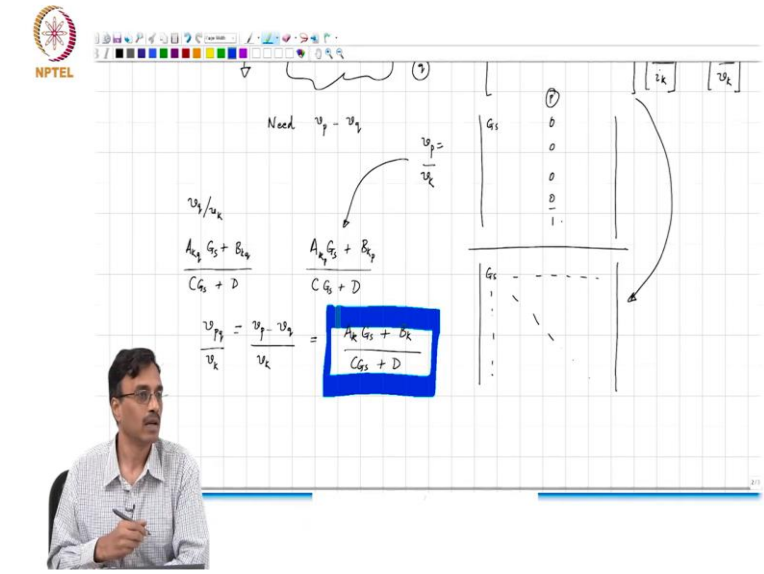

So, let us say I am interested in finding the transfer function from vk to the output and as usual, I choose, this node as my reference node that is ground 0, this is 1, this is I do not know node P and this is node q and we write the nodal equations, as we are used to and there is only one source here and so, if we write the nodal equations, what do we get? We get the M&A matrix. We know how to do this and let us and the unknowns will be, all the node voltages and?

Student: (())(12:58)

Professor: Well, there is only one voltage source, so the unknown basically is say sum i sum K and what do we get on the right-hand side?

Student: (())(13:12)

Professor: Yeah, and which are the independent sources here?

Student: (())(13:17)

Professor: There is only vK that appears here and what else do we know? Well, we know that there is a resistance RS from node 1 to, 1 to ground. So, if you look at this M&A matrix this is node 1, and this is the first row. So, this will be GS and then you have the rest of the matrix. So, we are interested in finding we are interested in, what are we interested in finding, this is the set of equations and what are we interested in finding?

Student: (())(14:10)

Professor: We are interested in finding vP minus vq Critic and how do we do that? Well, good old Kramer's rule. So, what do you do? So, vP for instance is nothing but yes, what do we do people? I cannot hear you at all man.

Student: (())(14:48)

Professor: Very good. So basically, you replace you find the determinant of this matrix. In the numerator, what do you do you have of course, you have GS here and in the Ph column, what do you do?

Student: (())(15:04)

Professor: You replace the staff with, you basically you will get P, all these will be 0, does it make sense people and what do we have in the denominator? It is simply the determinant of that matrix. So, this will be GS and what we had. So, of course, this basically means that you can take BK out as common and therefore, you have vP over vK which is the transfer function we are after as the ratio of determinants, which is like, which is like this.

So, if you expand the determinant, what comment can you make about the numerator? It will be of the form what comment can we make about? Can we make any comment about the form of the numerator? There will be a whole bunch of terms when you expand the determinant out to calculate it. A term can either consist of GS or not consist of GS. Further, if it consists of GS, GS will appear only in the?

Student: (())(16:45)

Professor: In the first (())(16:49). Does that make sense? So, in general therefore, the determinant correspondent to his matrix can be written in the form, let us say sum A sub K. Because we are doing this with respect to the Kth source? A sub K times G of S or G sub S plus, this AKth times GS, basically that AK term simply clubs all those terms with, with GS in it and therefore, and that is possible only because GS appears in the first (())(17:36) and then you will have terms without GS and what comment can you make about the denominator?

Well, it will also be of the same form, obviously the, you would not have the same AK and the same BK. So, you call this C times GS plus B and as we were discussing yesterday, such a form is called a bilinear form. So, if vP by vK is of this form, what comment Can we make about vQ by vK? Yes, people.

Student: (())(18:29)

Professor: You will get a similar form except that it will be some, whatever I am going to call this AK just to make running out of subscripts here, so this is AKq times GS plus, yes, Prashant, BKq, what comment can you make about the denominator?

Student: (())(18:58)

Professor: CGS plus d and why is the denominator the same or well the matrix below is the same, I mean the determinant below is the same. So, the key takeaway is that therefore if you do vpq which is vP minus vQ by vK what do you get? What do you think this will the form of this will be? What will it be? It will also, it will be again of this form. It is AK times GS plus BK divided by CGS plus D.

I hope you guys are convinced about this. So therefore, the transfer function from any internal noise source to the output will be of this general form where all I have done is basically brought out the, the explicit dependence on, the explicit dependence on the source resistance and so therefore, if we want I mean, let us now at this point not worry about those sources being noise sources.

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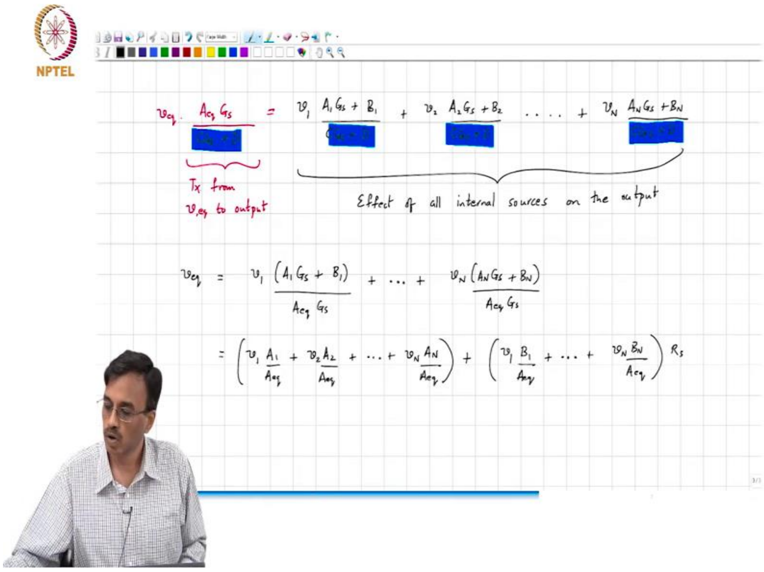

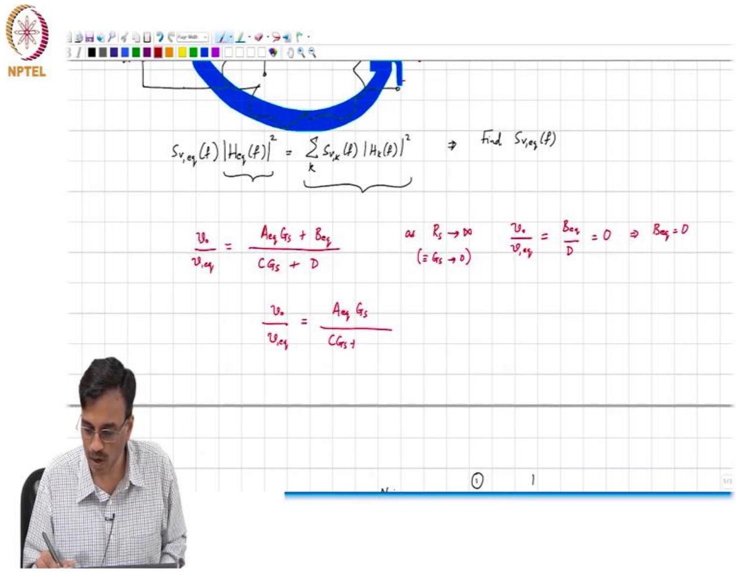

If there were deterministic sources, the total output because of these multiple sources would be v1 A1 GS plus B1 divided by CGS plus D. This is the first noise, I mean first source inside the box plus v2 times A2 GS plus B2 over CGS plus D, blah, blah, blah, until you are blue in the face. So this is, whatever we vN AN GS plus BN divided by CGS plus D. So this, therefore, is the effect of all internal sources on the output does it make sense people.

The key point to note is that the denominators of all these transfer functions remains the same and that this must also gel with your intuition. I mean, with your prior background, namely that, this is this is why the polls of any transfer function that you compute in the same network, is independent of where you put the input and where you take the output, it is a prop, inherent property of the, the network itself and that makes sense, because that is coming from the denominator, the determinant of that of that M&A matrix, which is a characteristic of it is got no sources inside.

The M&A matrix is just, got to do with the network and its topology. Excellent. So, now, we want to find a single voltage source, v equivalent whose effect at the output of the amplifier is exactly the same as the effect of all these internal noise sources on the output. This is the effect of all the internal noise sources, all the internal sources on the output and we would like to find what the equivalent input sources, which will have the same output. So, what comment can we make about, so therefore we need to find the transfer function from vN equivalent or v equivalent into output. So, what comment can you make about the transfer function from the v over v equivalent?

Student: (())(23:54)

Professor: Pardon. Yes, people.

Student: (())(23:59)

Professor: Well I mean, you will say well, it is the same form, what should be the denominator?

Student: (())(24:07)

Professor: CGS plus D very good and the numerator the numerator is some equivalent times GS plus B equivalent, but there is a twist. As RS tends to infinity, what comment can you make about vo by v equivalent?

Student: (())(24:34)

Professor: As RS tends to infinity, in other words, GS tends to 0. What comment can we make about the transfer function from that equivalent source to the output, which is the same as asking the question, what are the transfer function from the input to the output, when that RS becomes, RS becomes infinite? What do you, what comment can you make?

Student: (())(25:03)

Professor: It is B equivalent by D that is fine. So obviously, as G becomes 0, GS tends to 0 this is B equivalent by D, but simply inspecting the network tells you that if I remove the resistor there is no output to talk of and therefore, this must be equal to 0 and therefore, what does this mean? What does this mean?

Student: (())(25:28)

Professor: B equivalent must be 0, it is clear. So, therefore, while vo by v equivalent is of the same form, what we need to understand is that this must be, that B equivalent must be 0. So, this must be of a form a equivalent GS divided by CGS plus. So, with that in mind what is the single equal and voltage source that has the same effect as all these multiple sources inside, what should we put on the left hand side? Yes.

Student: (())(26:29)

Professor: Well, v equivalent times, A equivalent times GS divided by CGS plus. So, this is the transfer from v equivalent to the output. So, what do you see as happening here? Well, one thing you notice is that all these guys cancel out and therefore, we are left with v equivalent times, v equivalent is simply v1 times A1 GS plus B1 divided by A equivalent times GS plus blah blah, blah and this can be written as v1 times well, A1 over Aeq plus v2 times A2 over Aeq all the way up to vnth plus. By the way, is this damagely consistent? Well, A A A basically has the same dimension. So, this is indeed a voltage plus?

Student: (())(28:50)

Professor: v1 times B1 by A equivalent plus all the way up to vn times Bn over A equivalent divided, multiplied by divided by GS, which is equivalent to saying it is.

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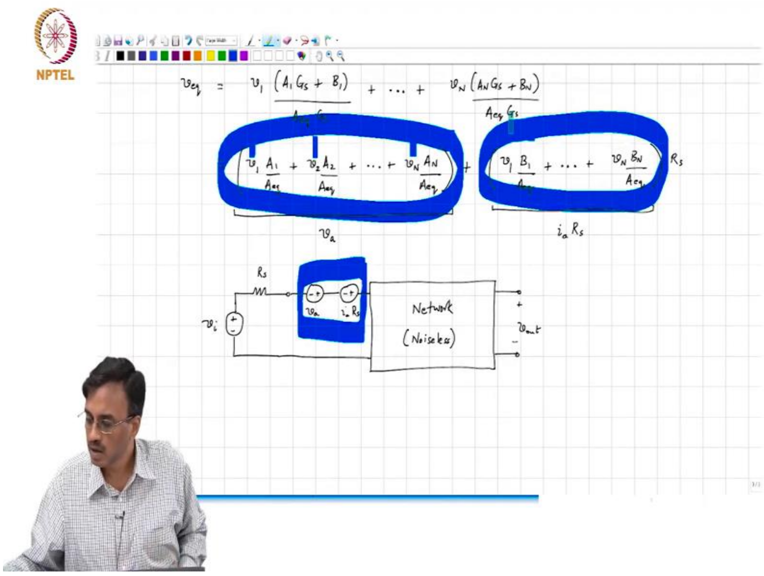

So, what are the dimensions of this?

Student: (())(29:25)

Professor: It is some voltage and obviously the strength of that voltage source depends on, it depends on the strengths of what are v1 to vn?

Student: (())(29:42)

Professor: or they are the internal sources. So, clearly it makes sense that the voltage source depends on the internal noise sources and A1 by Aq, A2 by Aeq keyword etc depend on the transfer functions from the internal sources to the output divided by these those A1 by Aeq basically quantify the relative strengths of the transfer functions from the internal sources to the output. What comment can you make? So, this is let us call this va and what comment can we make about the dimensions of this quantity here? This is current and this is i times RS.

So, before to come back to our equivalent noise source. So, we have vi, this is RS and our equivalent noise source consists of two components 1 is va and the other one is iA times RS and well this is my internal network, whatever it might be, but here I have gone and made sure that all my all my noise sources are null, in other words the network is noiseless and this is the output.

Now, does it make intuitive sense that this noise source this equivalent noise that you have here, does it make intuitive sense that depends on RS and one thing I say that what can I say, just comes down to the math but is their intuition, I mean does it make intuitive sense that we should have a term which is proportional to RS, any thoughts?

Student: (())(32:35)

Professor: Fair enough, I mean and the easiest way of understanding this is the following as RS becomes larger and larger, what comment can you make about the influence of vi on the output, well RS becomes tends to, starts becoming larger and larger, the connection between the vi and the network becomes weaker and weaker and therefore you should expect the transfer function from vi to vo to be, to kind of become smaller and smaller, which basically, so and but the noise due to the internal sources that is what it is.

I mean, if you remove, if you remove the input source and RS altogether, there will be, the internal sources will cause some noise of the output. But as RS becomes larger and larger, the noise caused by the internal sources remains largely the same at the output. However, the transfer function from vi which is also the transfer function from that equivalent noise voltage source keeps dropping.

So, if you want to achieve the same noise that you see as RS becomes infinity or as RS becomes larger and larger, the only way to do that is if the strength of the, of that equals input noise voltage becomes larger and larger and must be proportional to RS, because asymptotically the transfer function from vi to vo, will fall of as RS, I mean unless you consider, pathological cases like input impedance being infinity which is does not exist in practice.

Input impedance of a network being infinity only exists in a textbook. So, in reality there will be some capacitance between the input end and ground and, so basically as RS becomes larger and larger, you will find that the transfer function must basically become the transfer function from vi to vo keeps falling down and if you want the equivalent noise source to have the same effect as all these (())(35:25) internal noise sources, the strength of the noise source better go up in proportion.

Because otherwise, this sequel and noise source has to get multiplied by the transfer function to give the same output noise. So, if the transfer function is going down, the strength of the noise was better get keep going up. That is the intuition why you see iA times Rs. Now we are in a bit of a bind. Because remember, what was the aim of our whole project when we started off? What were we trying to do? I mean, when we started off this whole discussion on, equivalent noise source?

Student: (())(36:05)

Professor: What we were trying I mean, the big picture is that, here is a network that I am going to give to you, the consumer and just like how I give you the two port parameters of my amplifier, I would also like to give you something which has the same noise behaviour and in other words, that is the reason why we started off this, this input referred noise discussion, so that you, the user can take this information and put that in your, I mean you are presumably building a bigger system, you want to see how the noise of my block impacts your system.

So, what you would like to do is take the model of my two port, including noise and plop inside a bigger model. Unfortunately, the way we have it now, seems like and we can say, well here is the input referred noise or the strength of the input referred noise source, but there is a problem, what is the problem? You think?

Well, we ideally like that input referred noise source to be a property of the, of the network. But it, the way it appears here is that it looks like it obviously depends on RS and we also saw on why that makes sense. But we are now stuck with an input referred noise, spectral density or whatever voltage which depends not only on the network, through vA and iA.

Remember that vA and iA are only dependent on all these quantities only depend on the network, they do not depend on RS. Is that clear by the way, because that those A B C and D terms are all independent of RS. So, these are only network dependent though the one circled in blue. So, we are in a bit of a bind.

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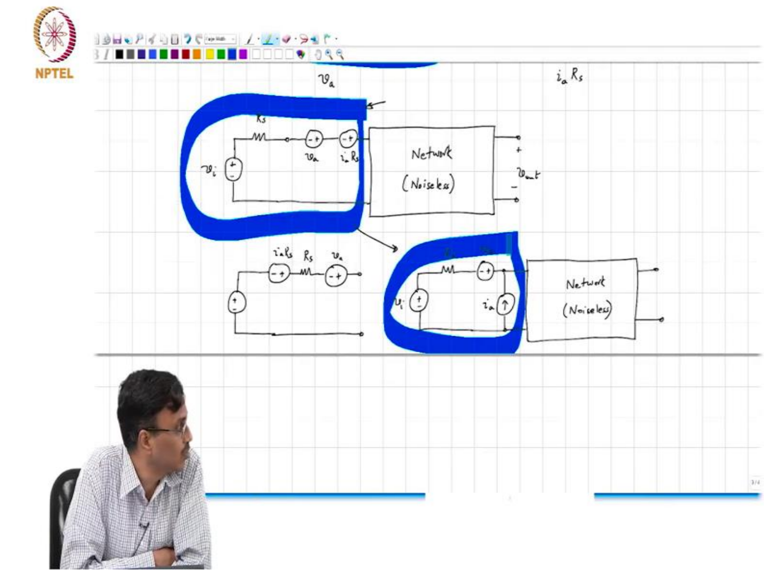

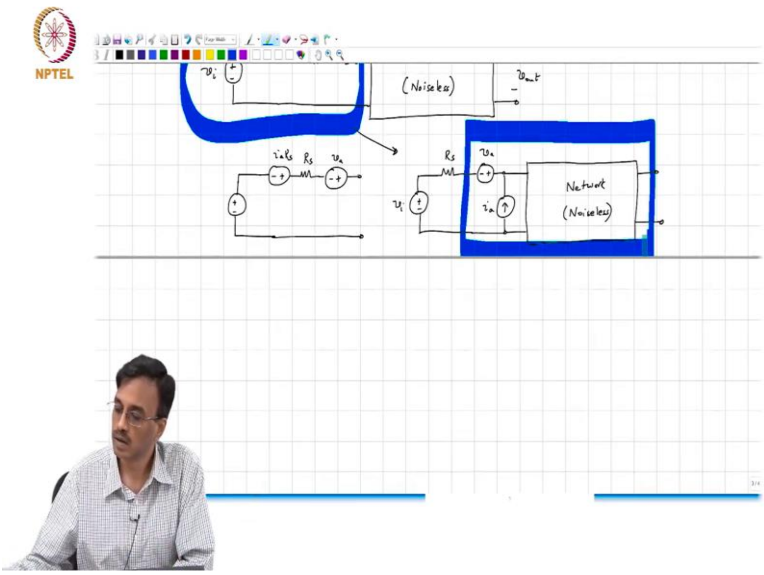

Fortunately, it turns out that you can, if you stare at this, you can, it turns out that I can move voltage sources around. So, if I make this iA times RS, RS and then vA and then you recognise that you can do a naught into seven and transformation there and you see that well you can get the same (())(39:17) equivalent for this box. If you did this. So, you have vi RS, then you have vA and you have iA and here is the network which is noiseless.

And how do we know that this is correct? Well if you look there what is the (())(40:12) equivalent? What is the (())(40:15) resistance.

Student: (())(40:18)

Professer: RS, what comment can we make about the (())(40:22) resistance here and what comment can you make about the (())(40:30) voltage on top?

Student: (())(40:33)

Professor: vi plus va plus iA times RS and what comment can you make about the (())(40:39) resistance I mean (())(40:40) voltage of the lower on the, in the lower circle, it is the same vi plus vA plus iA times RS. So, these two are evidently equivalent. So, now what can you say.

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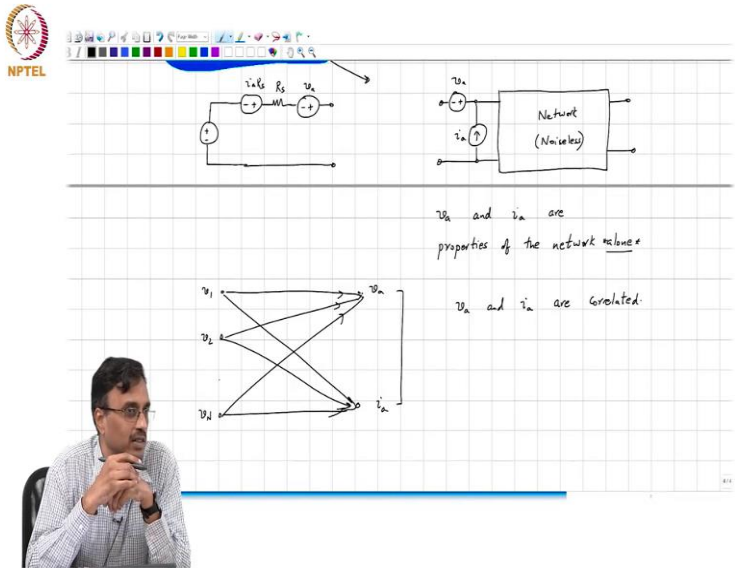

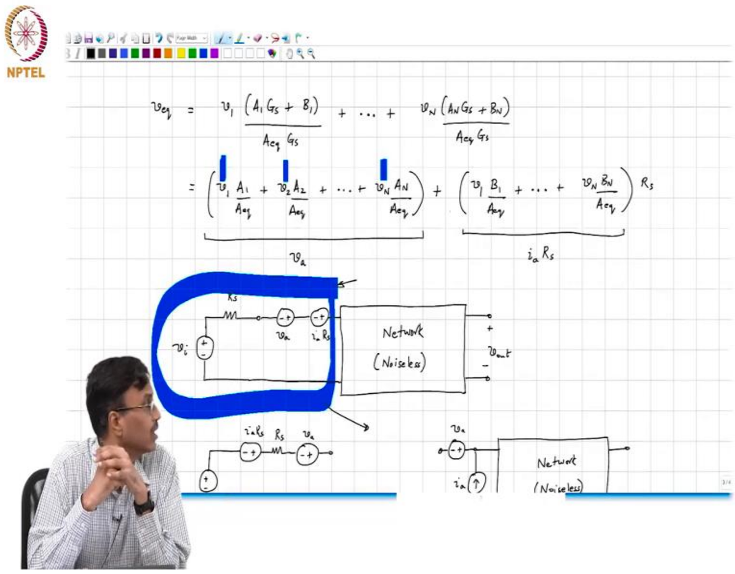

You say that this picture and then therefore what do you give the user?

Student: (())(41:05)

Professor: Well, you say that this is the equivalent noise representation of my network and so now this representation is independent of the source resistance and both vA and iA are properties of the network alone and the reason is that the vA and the iA depend only on v1 through vn which are noise sources that are internal to the network, they depend upon A1 by Aeq etc, which all depend only on the As and the Bs remember are terms which do not, are terms in the determinant of the M&A matrix or with appropriate column zero doubt, which do not depend on GS.

So, in other words there only properties of the network. Another thing that I like to point out is that remember, see finally vA and iA are within quotes noise quantities because v1 through vn are all noise sources, so the spectral density of vA can be easily found by, well Sv1 of f times mod A1 by A equivalent whole square and so on and so forth. Can we comment on weather vA and iA are independent or dependent? Pardon.

Student: (())(43:20)

Professor: Why are they dependent?

Student: (())(43:24)

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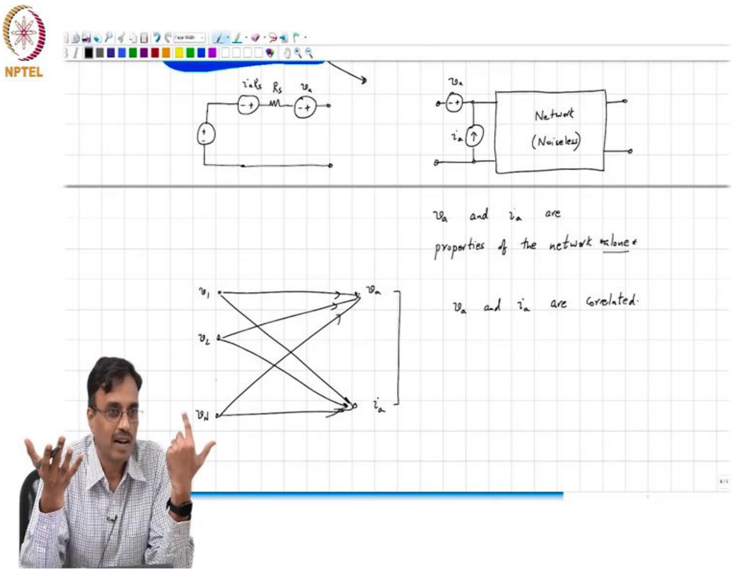

Professor: Well, remember that v1, v2 blah, blah, blah vn are they independent or dependent?

Student: (())(43:35)

Professor: They are generally independent because they are all noise from different resistors and different transistors, they do not know what is going on in the other transistors. But vA is some linear combination of v1 through vn, iA is?

Student: (())(43:55)

Professor: Some other linear combination of v1 through vn, so what comment can you make about vA and iA, I mean is there some relationship between the two or they are completely independent?

Student: (())(44:12)

Professor: Both are basically taking forming the linear combination of some bunch of independent sources and therefore these two will be, these two will be in general dependent. So vA and iA are dependent, in other words, the noise sources vA and iA are correlated. So, this is like, Indian marriage, arranged marriage You basically, each party is making inquiries about the bride or the groom as the case may be.

Now, what do you do? Well, you ask the neighbour, is this fellow okay? Or like, does this guy come home drunk at night at three in the morning and make a ruckus? Well, if you ask the neighbour the neighbour on the left side and the right neighbour on the right side, the information you get is most likely going to be, I mean this this fellow is a random phenomenon, but you are simply.

You are looking at you know, alpha times random and then beta times the same random. So, you ask both neighbours, you basically are not going to evidently get phenomenally new insight. If you ask his boss in the office and his neighbour, then each one is looking at an independent aspect of the person's performance. So, it is a basically, they are also they will be correlation because, it is still looking at the same person, but hopefully there will be lesser correlation.