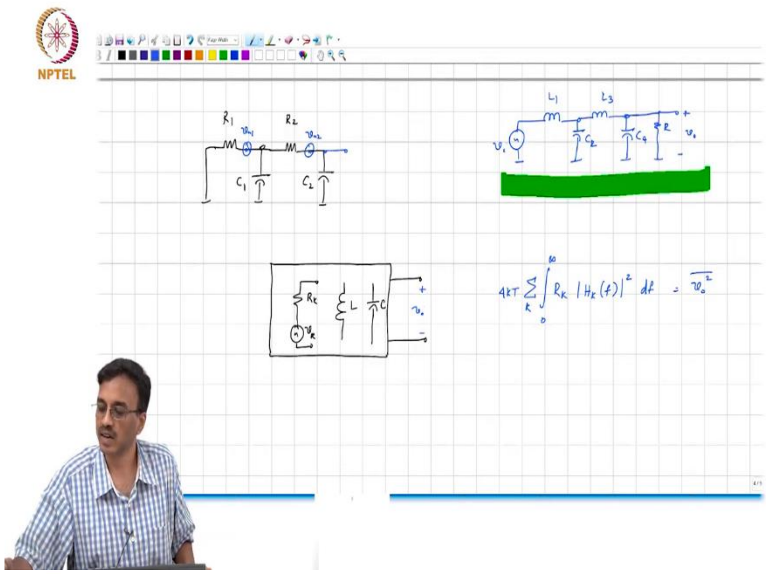
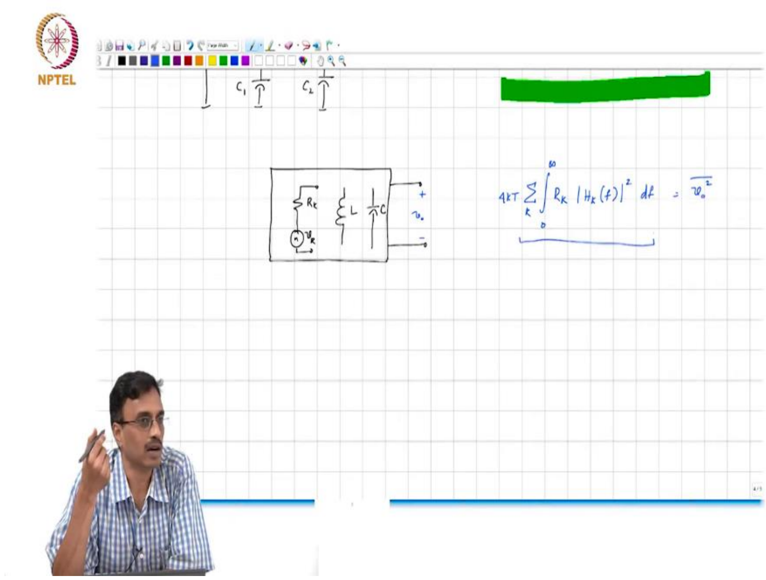
Introduction to Time - Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering,
Indian Institute of Technology, Madras
Lecture 21
Total Integrated Noise in RLC Networks

(Refer Slide Time: 00:17)

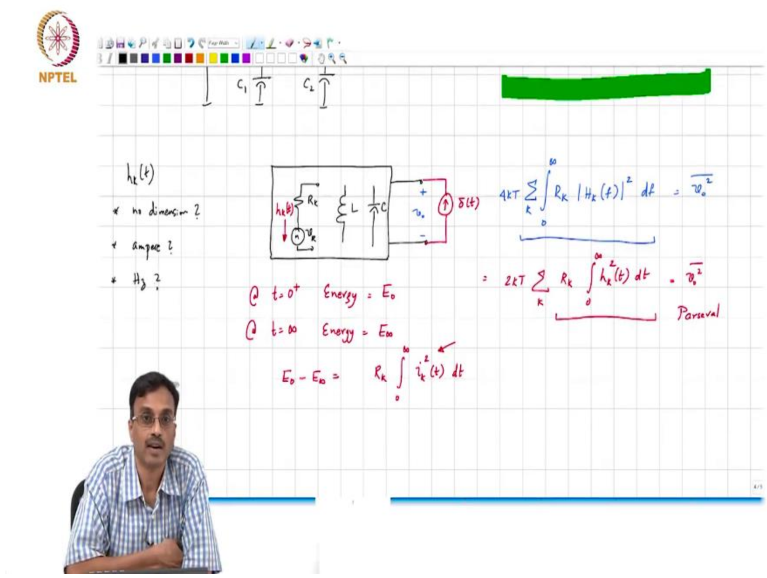


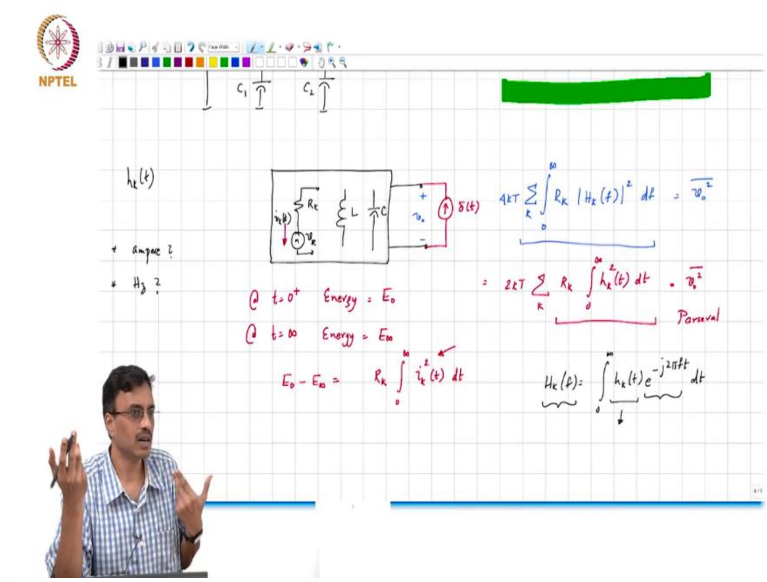


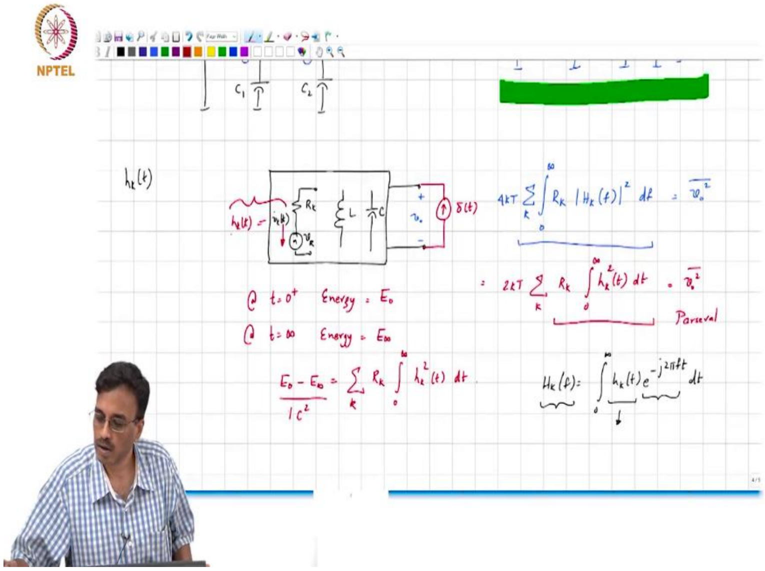
Professor: Alright, so let us try and see how we can do this. So, again, this is R sub K, this is V sub K, alright and we have L, we have C, and we are interested in finding what is the noise spectral density at the output it is 4KT, R sub K, R sub K mod H sub K of f the whole square is the output noise spectral density, corresponding to the Kth resistor, and what are we going to do, what do we need to do, of course we need to sum over all of them, and or we integrate this over, what if you are interested in finding the total integrated noise, what are we supposed to do? you integrate this from 0 to infinity, and sum over, this is V, V Square, this is clear.

So, in other words, the key aspect of the whole problem is, how do we calculate this integral in a, in a simpler way. I mean, one is to say, go and go and find all the transfer functions and do this, but you can see that it is a lost (())(2:12).

(Refer Slide Time: 02:25)







So, again, we have not really exploited, the fact that you have only RLC elements inside the network, and as we did with Nyquist theorem, we can always exploit reciprocity. But now I will do this in the, in the time domain. So, let us say, I put an impulse current here, by the way before I do that, so this is the frequency domain, and this can be written as 4KT sigma over all K. I can pull the R sub K out of the integral, and you do HK of f the whole square the f, that is V square, does makes sense.

Now, let me rewrite this in a form that you are hopefully will, will ring a bell, I write this as 2KT times minus infinity to infinity, integral mod H whole square df. Is there something, is there some other way of writing this? Pardon?

Student: Parseval's theorem.

Professor: Parseval's theorem, very good, what is, that is, what does that say, the mean square integral in the frequency domain is the same as the mean square this thing in the time domain, so this is simply equivalent to saying, well, this is integral 0 to infinity, we know that the impulse response, corresponds to hK of f is going to be (())(04:23), so this is Parseval. So, now we are reducing to we have reduced a problem to finding this integral, and we still have not used reciprocity, so since here it seems like in the time domain, it seems reasonable that, use the time domain for reciprocity. So, let us say I put an impulse current here.

As soon as I put an impulse into the network, what will happen, where is that if any of this impulse current goes into the network, where is it going to go? It is only going to go into the capacitors, it cannot go into the inductors, because inductors are going to be an open circuit at 0 plus, and the resistors will have an impedance which is much larger than the capacitor. So, all the current goes into the capacitor.

So, at t equal to 0 plus. Let us call I mean since there is some voltage across some capacitors inside the network at equal to 0 plus, the network has got some, some energy. Because these capacitors, by hook or by crook or we charge by this, this impulse current source to some voltages, and, as a result, the initial t equal to 0 plus, the energy in the network is nonzero.

And let us call it E 0. Now, at t equal to infinity what comment can you make about the capacitors. Well, it is not necessary they have been discharged, but there will be some, I will show you examples where it is not necessary that the capacitors be discharged. But there will be some redistribution of the, well the impulse has come, charged up some capacitors to some voltage and gone. Then, well the network does its thing.

And these capacitor voltages redistribute themselves, it is entirely possible as you guys pointed out, that the capacitors all get discharged. But at any rate t equal to infinity, there is some energy left in the network, that energy could be 0. So, let us call that energy at infinity, let us call that E infinity, does that make sense, all right.

So, what comment can we make about E0 minus E infinity? Remember power and energy are different things. What comment can we make about E0 minus E infinity. It better be positive, otherwise you would not be here, that is 1, that is a great piece of insight what next is do we have, E0 minus infinity E infinity which is a positive number, which has got dimensions of energy, must all have been dissipated in the resistors, and so, what comment can we make about the energy dissipated in the resistors? what is the power in the resistor? I square R, the energy is simply nothing but the integral of the power.

So, it is the E0 minus E infinity is nothing but RK in the Kth resistor, is RK integral 0 to infinity. IK square of t dt, is this clear people? And what is IK? By reciprocity, oh well, what is the reciprocity saying, it is the saying that the voltage transfer function from VK to the output is the same as the current transfer function from the output from an impulse injected at the output to the current through this voltage source VK. So that is the small heads-up K of t. So, by the way, what are if in all these calculations you have to be very careful about dimensional consistency. Let me ask you a question therefore, what is the dimension of physical units of HK of t? Pardon?

Student: No Dimension.

Professor: Let us see, how many answers I get, no dimension.

Student: Depends on Input and Output.

Professor: Input and Output quantity are known.

Student: Ampere.

Professor: Ampere. Pardon, 1 over yes, Alfred, 1 over second, that is Hertz, come on people, can we decide? HK of of of j2pif or HK of f is nothing the Fourier transform of HK of t, e to the minus j2pift dt.

So, what is the dimensions of HK of f, it is, it is a gain, so it is dimensionless is unitless, what are the units of e to the minus x? It is a number, so it is got no units. So, what must be the units of this? This better be, this better be what, 1 by time which is Hertz. So, definitely ampere is not correct, it is Hertz.

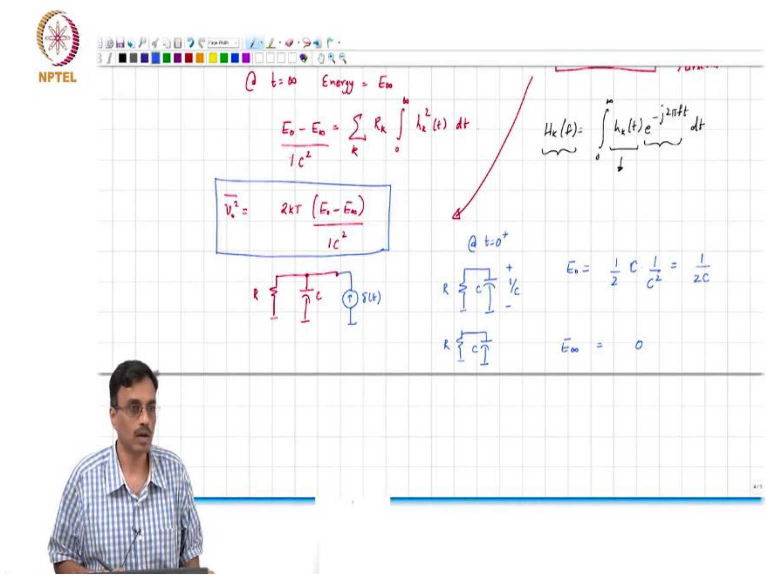
So, this, but so when you say you put delta of t here, you get IK of t which is hK of t, this is definitely not, this is not correct, is this correct or no. If I put an impulse there, my claim is that by reciprocity the current in V sub K must be HK of t, that is dimensionally inconsistent, that is incorrect. All you can say is the shape of the waveform is the same as HK of t, but I have to multiply it by, by what, HK of t is basically per second, IK of T is ampere. So, I have to multiply it by ampere second, where is that ampere second coming from.

Student: Charge from the capacitor.

Professor: it is the, is the not the charge from the capacitor, it is, it is basically the impulse. The input current, actually it is the input current, multiplied by the delta t in the convolution. And so therefore, it is incorrect to write this as IK is IK of T as HK. They are related, but they are not the, not the same 1 is apples 1 is oranges, so you cannot compare both of them. You have to multiply, HK of t with ampere second, to be able to get IK. This is subtle, but it is important. Otherwise, you will get results which are dimensionally inconsistent.

So, therefore, this is nothing but, what is IK square times, I mean, what do you call, what comment can we make about this guy now. This is related hk square of t or not? Yes. No. Yes. So, this is nothing but hk square of t dt, this divided by this times, you have to multiply that hK with ampere second which is coulomb. So, that is, if I divide this by 1 coulomb square. I basically get RK times integral 0 to infinity hk square of t dt. So, what I needed, I mean, this summation over K is obvious. Is this clear.

(Refer Slide Time: 15:26)



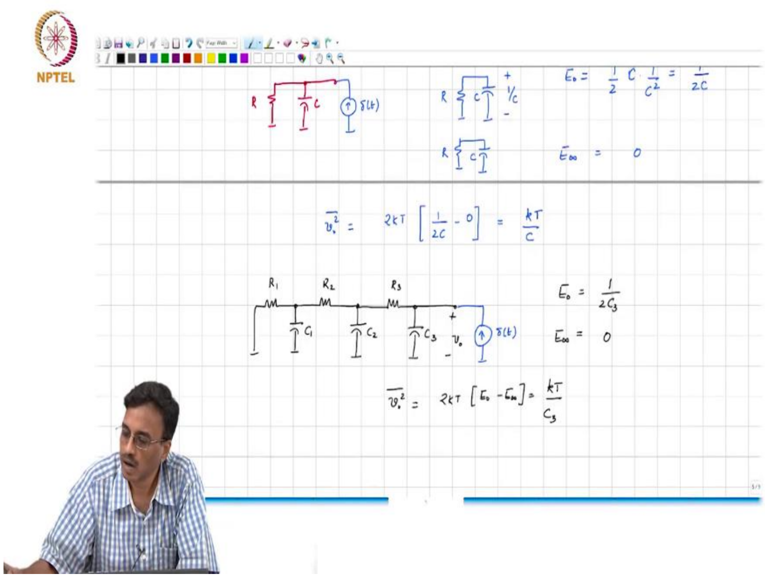
So, therefore what comment can you therefore make about the mean square value of o Vo, guys? mean square value is nothing but 2KT times E0 minus E infinity by (())(15:50). Let us start with our typical known case which is R and C. So, what do we do, we need to, what is E0, by the way, let us recall what is E0? The energy you inject an impulse current into the output node and find the energy at zero plus. So basically, this is R, this is C at 0 plus, what do we see, what happens to the impulse correct?

Student: It goes to capacitor.

Professor: It all goes into the capacitor, so what is the voltage, 1 by C, I mean hey dimensions dimensions or we what is the dimensions of 1 by C? I mean 1 by C is, is what, what is the units of 1 by C is, what should it be, it is a voltage so it must be volts. So, are we sure we are dimensionally consistent? That 1 by C that 1 is a corresponds to 1 coulomb which is contained in that unit impulse.

So, when we say 1 by C here that 1 is 1 column by farad, so it is indeed volts. So, E0 therefore is what 1 by 2 half times C times V square which is 1 by C square, which is 1 by 2C. And that 1 is what are the units of that 1, it is 1 by C, people, what are the units of that, are we are we dimensionally consistent coulomb square. So, what is E infinity? what happens to this voltage as t tends to infinity, oh well the capacitor completely discharges, so what is E0 or E infinity 0.

(Refer Slide Time: 18:56)



And so, what is the mean square noise? it is, it is well 2KT times 1 over 2C minus 0, at 1 by coulomb square and this coulomb square gets cancelled. Remember it is E0 minus E infinity by 1 coulomb square. Once you do that, there is a coulomb square and 1 over 2C, so that gets cancelled. So dimensionally we are consistent, so what is this, this is nothing but KT over C.

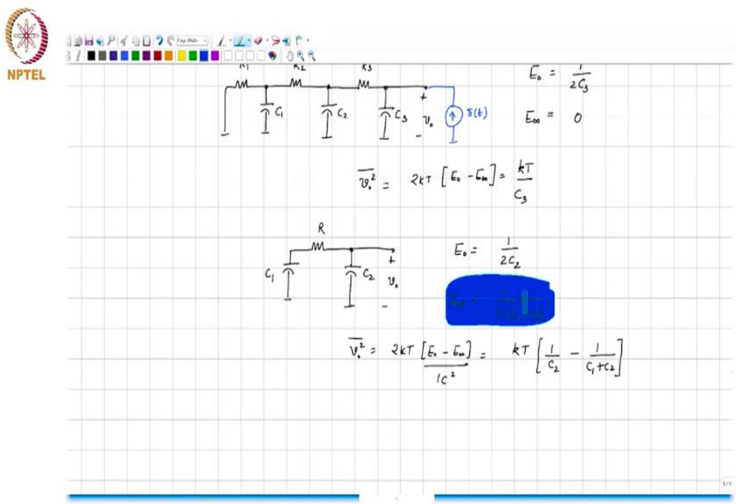
Now, let us try less trivial example, so this is R1, R2, R3, C1, C2, C3 and we are interested in finding the, the mean square noise across say, C3. So, as usual, we inject an impulse here. So, where does that impulse current go at t equal to 0 plus all the current goes onto C3. So, what is E0, therefore? Or E0 by 1 by, E0 by 1 column square is nothing but 1 over 2C3 E infinity is, well at t equal to 0 plus the vol there is a voltage on C3, after a long time what will happen? I mean all the voltage across C3 will discharge through the resistors, what about the voltage across C2 and

C1? well, yeah. so, at t equal to infinity, the steady state voltages across all the capacitors is zero, so what is the energy at t equal to infinity?

Student: 0.

Professor: So, what is the mean square noise? It is 2KT times E0 minus E infinity which is KT over C3. So basically, it shows, you do not need to, I am it is wasteful to kind of find the entire spectral density and integrate it and throw away all that information, if you are only interested in the, in the, in the total noise, it is much easier to use some techniques like this.

(Refer Slide Time: 22:31)



The final example. Let us say this is the output noise we are interested in, so, what is E0? E0 is nothing but 1 by 2C2. What is the E infinity? what is the voltage at E equal infinity? Pardon? 1 by C1 plus C2 because that t equal to infinity, all you need to do is simply replace the resistor with a shot, because no current flows through the resistor at t equal to infinity.

Whatever charge there was in C2 must be shared between C1 and C2, and therefore the voltage at t equal to infinity is 1 our C1 plus C2, so what is the energy at t is equal to infinity? 1 over 2 times C1 plus C2. Does that make sense people? And therefore, the mean square noise is 2KT times E0 minus E infinity by 1 coulomb square and that is 2KT times, KT times 1 over C2 minus 1 over C1 plus C2.

I mean this just is there to illustrate the fact that it is not necessary that, all the capacitors discharged to 0. And this shows, the example of a network where the capacitors do not discharge. And so we need to check if C1 goes to 0, C1 goes to zero infinity, C1 goes to Infinity, it is like having a short circuit there. You should reduce to KT over.