Introduction to Time - Varying Electrical Networks
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Lecture 20
Noise in RLC networks

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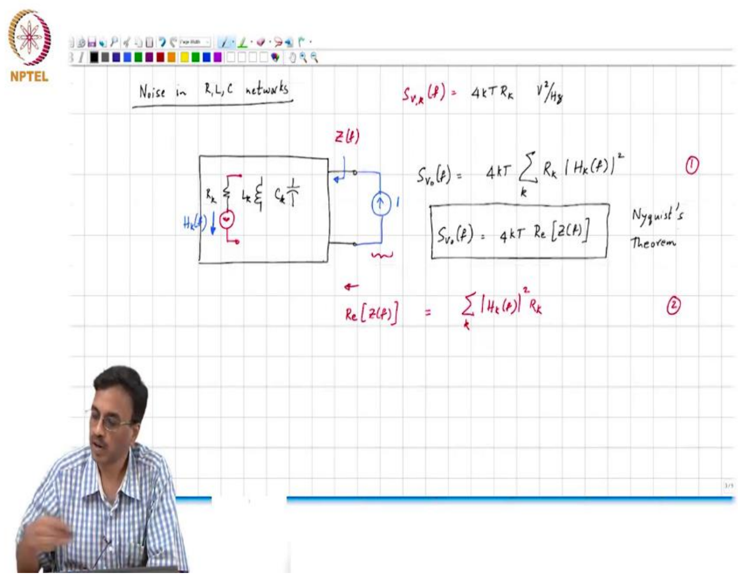

The next thing is to talk about noise in RLC networks and this occurs quite often in practice. So in other words, we have already seen what happens with noise in a resistor, it turns out, as I was mentioning the other day, capacitors and inductors are noiseless, so if you if you have an RLC network, what it basically means that that you have a network with any number of R’s, L’s and C’s and every resistor is associated with a noise source which we denote by Vn sub K and this is the output, correct, so what is Svn, let us call this Svk of f is what, is simply 4KT times R sub K, the units being volt square per Hertz.

So, it seems reason i mean it is obvious that, now if you have a whole bunch of resistors and you have this black box and then you get two terminals out the voltage if you put a voltmeter across these two terminals what do you expect to see, what would you expect to see, you put a voltmeter across those two terminals you will have obviously noise because, well there are a whole bunch of noisy elements inside the inside the box and how will we find the noise spectral density at the output? well its very straight forward, what do we do I mean step by step approach, is to simply find the transfer function from every noise source to the output, let us call that H sub K of f and that and what is the spectral density corresponding to the Kth noise source is 4KT, R sub K times mod H sub K of f the whole square, this would be, this will be the in English what is this stand, what does this mean, what is that quantity there that 4KT, R sub K, HK of f the whole square, what does that represent?

Student: Noise Spectral.

Professor: The noise spectral density at the output due to the due to the,

Student: Kth resistor

Professor: Due to the Kth resistor, and we also discussed that the noise from different resistors is independent, so if you want to find the noise spectral density due to all the resistors, it is simply what, is the sum over all resistors. So simply sum over K RK HK of f the whole square. So, well this one aspect of the whole problem that we have not exploited yet what is that, well this is true for any network.

If you have multiple sources you find the noise spectral density at the output due to each source and you add them, what is new, there is nothing new here, this one aspect of the problem that we have not exploited yet and what might that be, we have the network consists only of RL and C and therefore what we have not exploited is once you have a network like this there is also this you can one aspect is that it turns out to be reciprocal.

And so basically that is what we are going to exploit next, and so to see that, so let us say this is a current the current i, what comment can you make about the transfer function from here to the current in the Kth resistor? Well, if this is i this will be or rather let us say this is a phasor of angle one angle 0 what will be the phasor here? this phasor is simply H sub K of f times, is this clear, so the next thing I like to draw your attention to is the looking impedance here, let us call that Z of f.

So, what comment can we make about the energy supplied or the power supplied by this current source into the box what comment can we, I mean if you have a current driving into an impedance Z of f, so what comment can we make about the energy supplied by the current source into this box? Pardon?

Student: i square times.

Professor: i square times the real part of, very good. So basically, the power going in, is basically i square fortunately that happens to be 1 times real part of Z of f. So, where is all the power inside the box being dessi dissipated?

Student: Resistors.

Professor: It is being dissipated in the resistors, and well do we know the current through the resistors? That is nothing but HK of f, so what is the power dissipated in the Kth resistor? yes mod HK of f the whole square times RK. So, what is the total power dissipated in all the resistors its simply the sum overall K and that must be equal to, that will make sense people.

So, so now can you stare at these two equations and tell me what conclusion you can draw from this? Pardon? Yeah, so what common can we make about the output noise spectral density? So SVo of f is simply put 1 and 2 together you get 4KT times the real part of 0 and this is what is called Nyquist theorem, I guess it must be called one more of Nyquist theorems.

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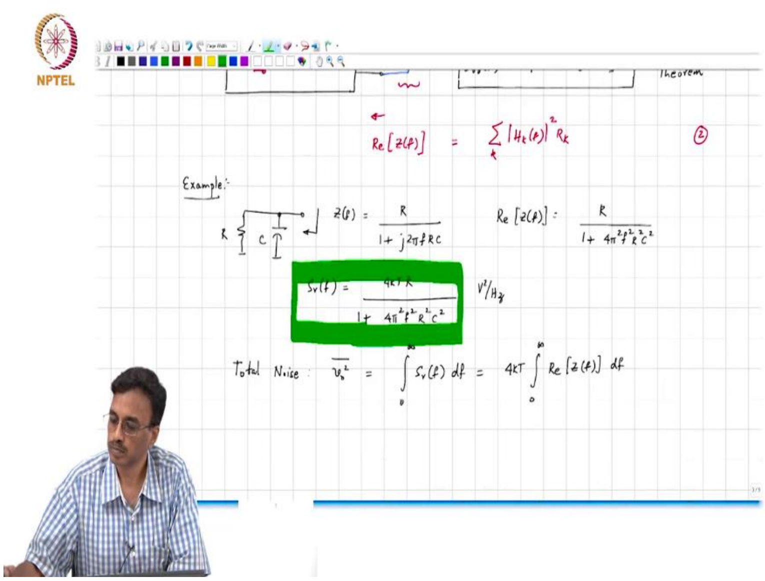

And so for example let us come back to our familiar example here we already know the answer and, so what comment can we make about Z of f, R by 1 plus j 2pifRC real part of Z of f is R by yes people, 1 plus 4pi square f square R square C square, so Sv of f is simply 4KTR by 1 plus 4 pi square f square R square C square, well this and this is volt square per Hertz and we knew this answer already where earlier we had actually calculated the transfer function from the noise source to the output and and done the math, it turns out that, we could have done this this way.

This clear people, so so what comment can we make about the total noise, that is Vo square is, is simply the integral of Sv of f df and which is simply nothing but 4KT integral, what people, real part of Z of f and as we have already seen the real part of Z of f df is this and, and if you integrate this we did the integral the last time around and we found it to be KT over C.

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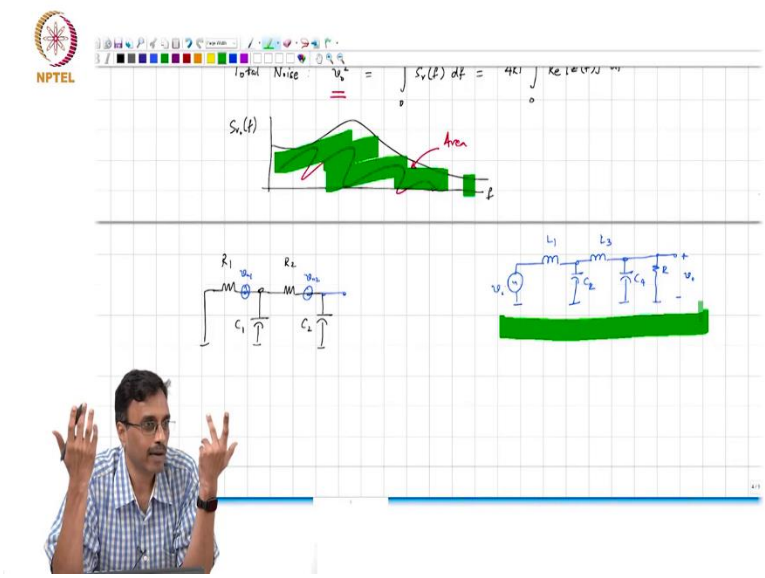

Now, it turns out that in a lot of practical situations, we are only interested in the total mean square noise, in other words, so far I mean lets kind of backtrack a little bit and see what we have done so far to determine the total noise, what have we done? Yes can, yes can you remind me what we what we need to do?

Student: (())(12:35)

Professor: Very good, so we have gone and found the transfer functions from each noise source to the output then, there must be noise somewhere know,

Student: (())(12:47)

Professor: the transfer function is is got nothing to do with transfer function is transfer function, so we need to multiply you we need to find the transfer function from every noise source to the output, multiply find the magnitude squared of that transfer function, multiplied by the spectral density of the noise source and then that will give us the output noise spectral density due to one noise source and you find the sum over all the noise sources that will give you the noise spectral density at the output.

Now, you have to integrate this noise spectral density across the entire frequency range 0 to infinity, to be able to get the total mean square noise, is this clear people. So, I mean, in other words if you find if you found the total noise spectral density let us say like this you are basically finding the area under the curve, so this is Svo of f and this is f and basically what you are doing is finding the area.

Now, does somebody see this as being kind of wrong wind I mean long winded and perhaps unnecessary? see we are only interested in finding the, the total area under the curve, the exact nature of the curve is irrelevant, so it does not make sense to find the exact spectral density which is a lot of work, why is it a lot of work we had to calculate the transfer function from every noise source to the output and then find the square transfer function and I mean just in case since most of you have this, why is this so difficult look on your face, so let us call this R1, C1, R2, C2. The transfer function from this noise source to the output will have its a second order denominator, so the denominator magnitude square will be a,

Student: Fourth Order.

Professor: Will be a fourth order polynomial, and likewise the transfer function from Vn2 to the output, let us say this is the, we are interested in finding the total noise across C2, that will also be a fourth order polynomial and then you need to find the integrals of these fourth order polynomials across one over those fourth order polynomials across all frequency, and this is only for a second order network, let us say you had five capacitors and four inductors and then you have all of a sudden you have a ninth order transfer function, which means is a squared magnitude response will have 18th order polynomial in the denominator.

So, all the tricks you learned in your JE and GATE coaching classes which basically are of no use. So, but the fact remains, that oh well, I mean, so here is an analogy, let me ask you a question with respect to the circuit on the right, what is the order of that transfer function? it is a fourth order transfer function let us say I want to find the dc gain from Vi to Vo, this is Vo, what comment can we make about the the dc transfer function from Vi to Vo, 1, how do you do, I mean I mean, can you give us some insight into the dramatic speed with which you are able to get the answer excellent, I mean so this is common sense what you would do is basically say, oh.

Well, the capacitors are open at dc the inductors shots the answer is 1, you could also do it this way which is not very uncommon by the way in exams you could, you could find the whole transfer function, fourth order transfer function, and then put s equal to 0 and get the answer to be 1.

So, this this finding the area under the curve, to find the area under the curve to write the magnitude response square it and integrate it from overall frequencies, is to do a whole lot of work to get the details of the spectral density at the output and then throw away 99 percent of that information and since you are only interested in the, in the area under the curve. That is exactly equivalent to finding the transfer function, I mean corresponding to this network, if somebody asks you what the dc gain was is to find H of s and plop then plop in s is equal to 0 and be very happy that oh most of the terms vanish.

So, again fortunately it turns out that this, I mean, if you are only interested in the in the total area under the curve or if you are only interested in the total mean square noise, it turns out that it seems reasonable that you do not have to work as hard to go and find the actual spectral density which basically means that you are working very hard to get a lot of information and then throwing most of that information away because you are only interested in the area, I mean all of you understand that there is a lot more information in the exact shape of the curve than in the than in the area is this clear people.