Introduction to Time–Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Kirchhoff's Current and Voltage Laws, and the Incidence Matrix

All right good morning and welcome; this is advanced electrical networks lecture 2. So, let us gets in the last class we looked at the motivation behind the various topics that we cover in this course. And looked at why they are important in practice and today let us get started in earnest.

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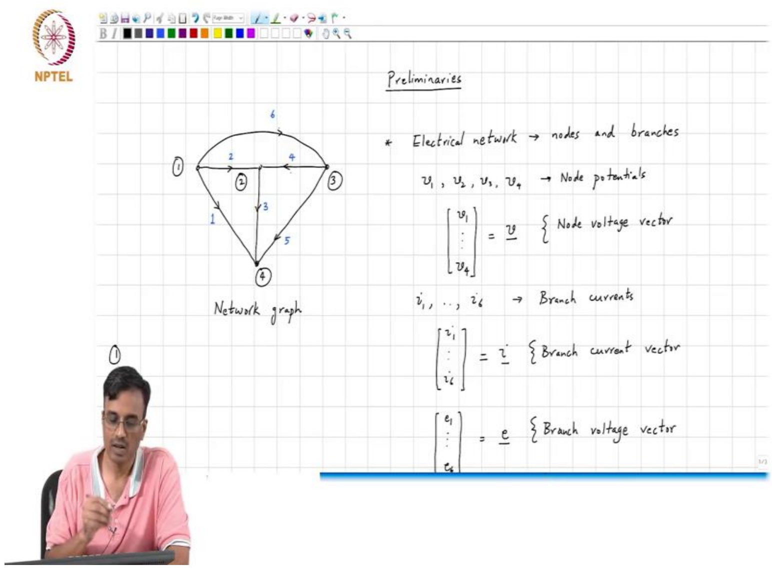

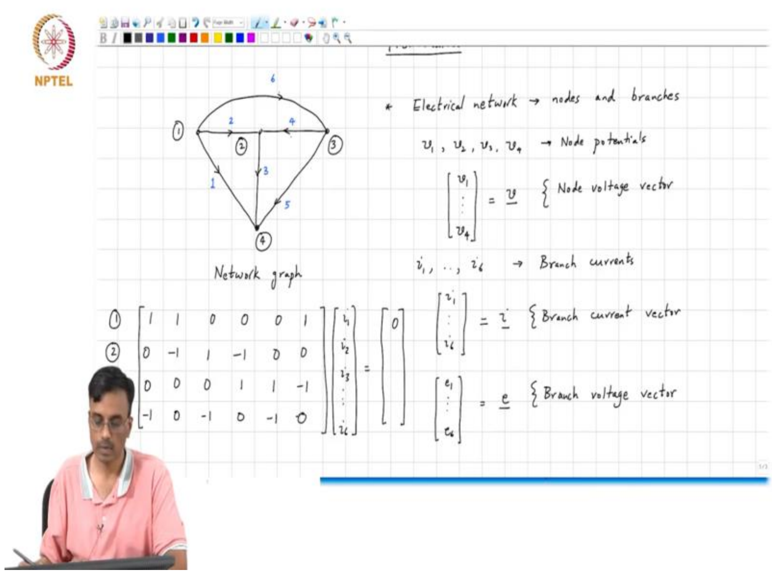

So, we will start off with some preliminaries. I am sure most of you know most of these topics and you have seen them at different levels in your earlier classes. But this will serve as a good starting point for all of us; so, and as I said in the last lecture this is a course about this is a course about electrical networks. And so, we are going to be dealing with with networks with nodes and branches; so let us number the nodes. It is going to quite arbitrarily choose the directions of; I am going to number the branches. So, an electrical network consists of nodes and branches.

In this example there are four nodes and 6 branches and every node is associated with the node potential; and we will call the node potential v1, v2, v3 and v4 are the node potentials. And we will call the column vector v1 blah blah blah v4; we will term this the node voltage vector. And every branch is associated with a node current, so i1 through i6 are the branch currents; and likewise, i1 through i6, we call this the branch current vector. And this picture is simply as you all know the skeleton on which the real network is formed. So, each branch could perhaps represent an element; the elements could be linear, could be non-linear, could be time invariant, could be time (invaria) time varying and so on.

So, so this is the network graph and every branch is not only associated with the branch current, but also with the branch voltage; and we are going to use e1 through e6 as the branch voltage vector. And what is the job of network analysis? We have seen this in the past. Well, the assumption is that you know the branch i-v characteristics; so, you know how the the current through a branch is related to the voltage across a branch. And what you would like to do is to solve the network namely given the branch relationships. When you connect the branches in this fashion what are the actual branch currents and branch voltages that that result.

And what are the the cornerstones with which you can solve the network; the the well known Kirchhoff's current and voltage laws. So, let us write KCL for each node; so rather than write we can write this in shorthand notation or in matrix notation as follows. And again I am sure you have seen this in the past, but let me just do that anyway. So, this is i1, i2, i3 blah blah blah i6; so we arbitrarily say if a branch is leaving a node, the current is is is positive. So i1 for example, if you write KCL at node 1 what is the branch equation that you will get? You see that you will get i1 plus i2 plus 0 times i3 plus 0 times i4 plus 0 times i5, plus one times i6 is 0.

We do this at the second node and we will see that the matrix is minus 1, 0, three is plus 1; four is minus 1; 5 and 6 are 0. And similarly, at node 3, we see that 4 is plus 1, 5 is plus 1 and 6 is minus 1; and at node 4 we have minus 1, 0, minus 1, 0, minus 1 and minus 1, 3 and 5 so sorry.

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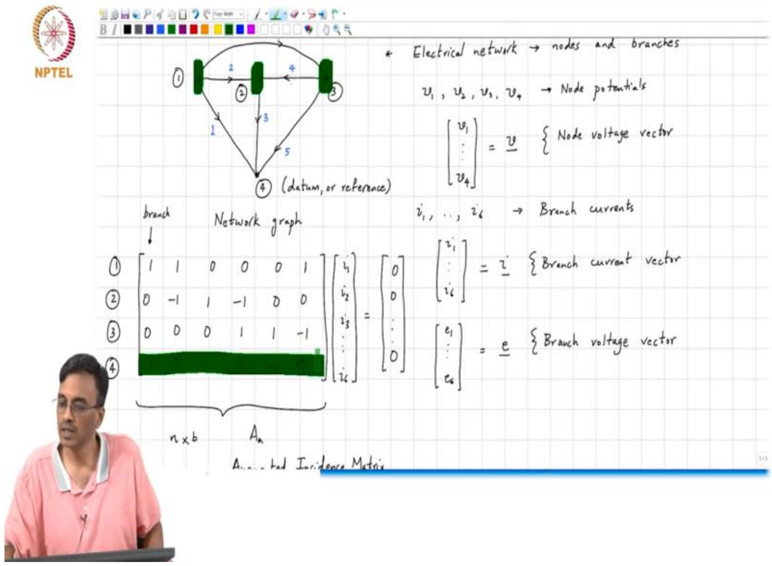

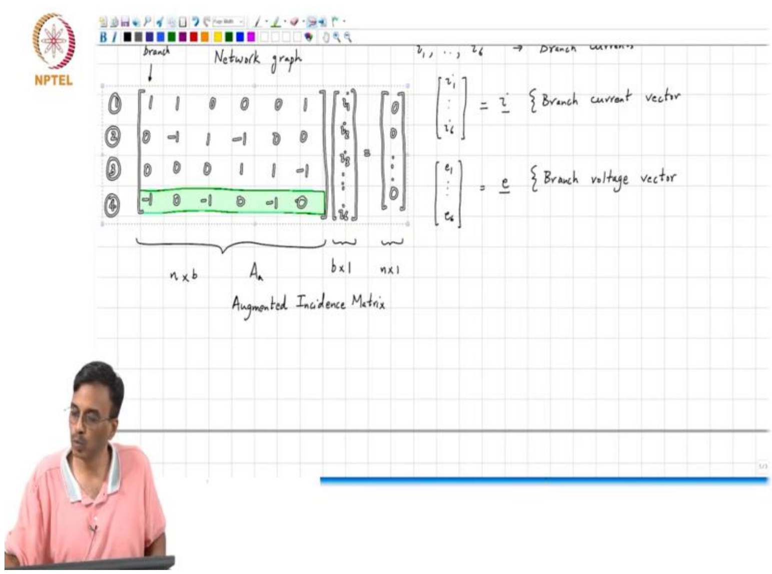

So, this what comment can we make about the size of this matrix; so, for each branch you have every every branch is represented by a branch current. And therefore, you have you have a column for each branch, and you have a row for each node. So, the size of this matrix is the number of nodes is the number of rows across the number of branches is the number of columns.

So, this is an n cross b matrix and this is often what is called the augmented incidence matrix; we will see why this name makes sense. Well, incidence matrix it is a matrix well, so you better call it a matrix. Incidence, why do you think it makes sense to call this incidence? Why does that word make sense? It is telling you on which nodes a particular branch is incident with the direction plus or minus 1. So, it makes sense to call it an incidence matrix; why does it make sense to call it an augmented incidence matrix. Augmentation basically means that adding something extra, and actually if you stare carefully at this matrix.

You can see that it is not necessary physically speaking it is not necessary to write KCL at all nodes. If we wrote KCL at all but one node, then KCL at the other node is is implied. How? Because if the current flowing through all these nodes net nodes is 0; you can think of that as a super node, and therefore writing KCL at node 4 is actually redundant. And this also reflected in the matrix, and if you observe carefully what comment can you make about the sum of of these of these rows? So, basically you can see that if you add all the rows up. The sum of all the rows turns out to be turns out to be 0, which is telling you that the rows are not independent.

And therefore, this also agrees well with our notation that if you have a network with n nodes; you do not really need to find the pote… the potential of all the nodes. You assume one of them to be the reference and the potential of all the other nodes with respect to the reference is what you need to find. So, in other words, this row for example if we choose this as the datum reference; then this row for instance can be neglect, if the data of course no new information, correct. So, this matrix which remains which I am going to copy and paste, and by the way what is the size of this matrix?

How many rows and how many columns? b rows and 1 column; and what should be the the number of what are the dimensions of the vector the 0 vector on the right? Very good, n rows cross 1. Now, what we realize that we do not really need all these rows; so, we can do with one fewer row.

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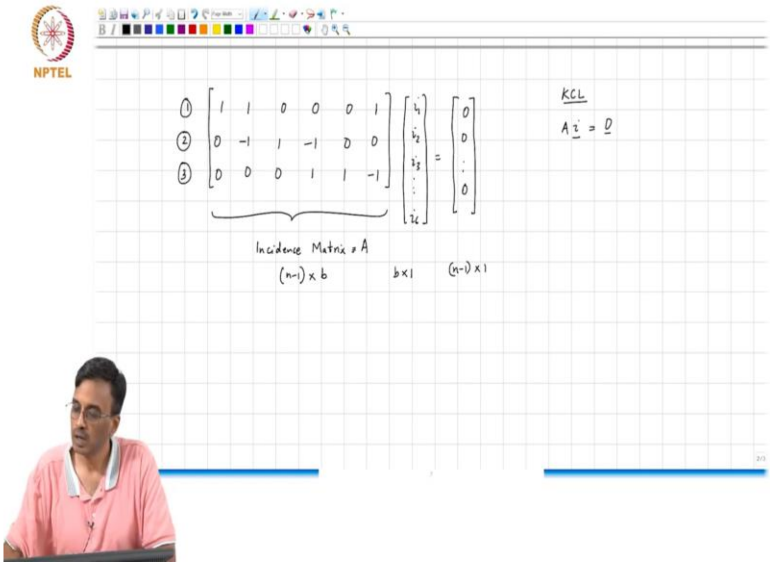
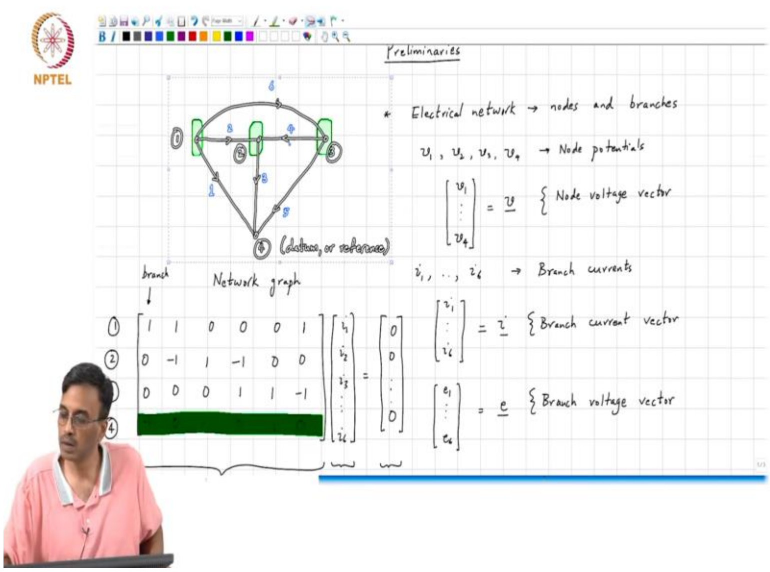
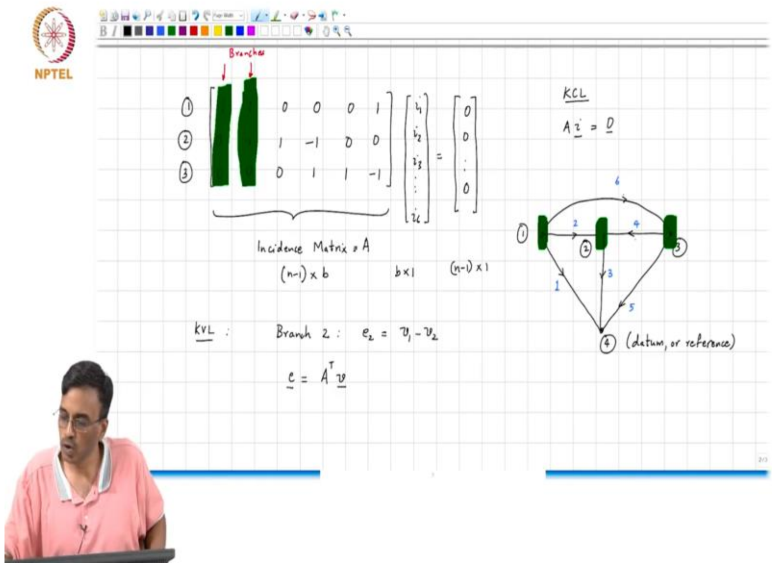
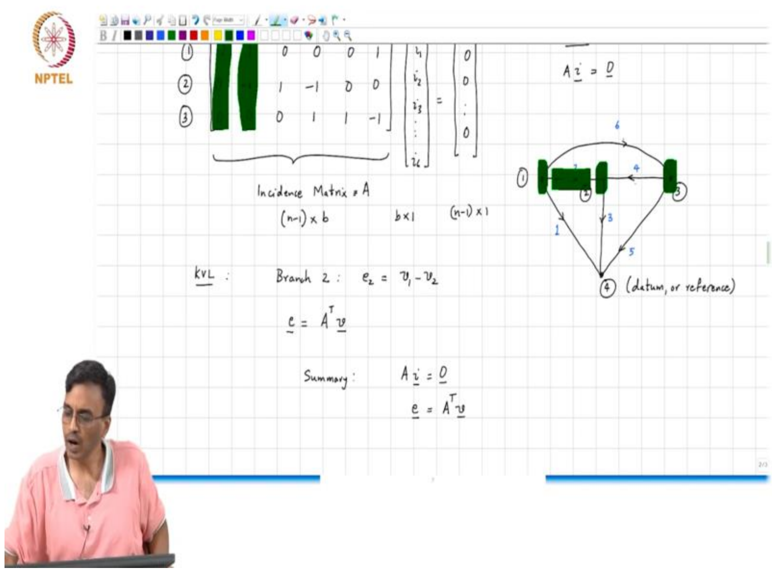


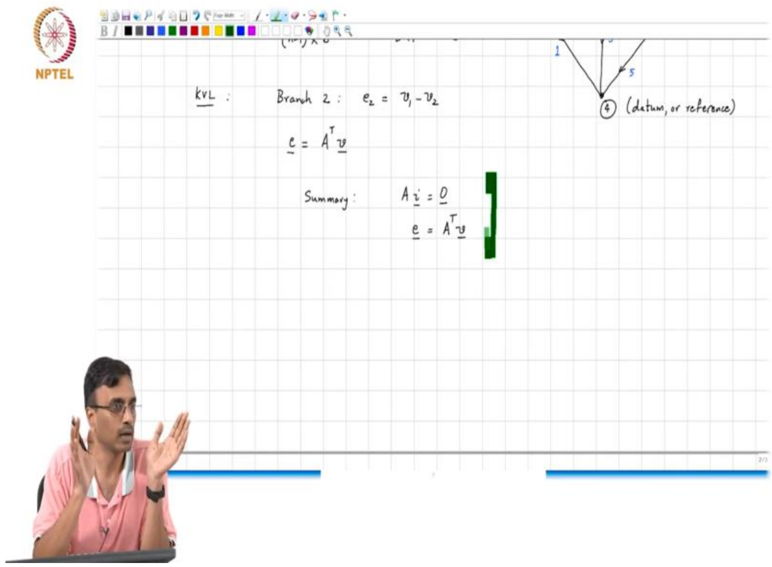




NPTEL
Branches
[redacted] 0 0 0 1
[redacted] 1 −1 0 0
[redacted] 0 1 1 −1
i1 i2 i3 : i6 = 0 0 : 0
Incidence Matrix : A
(n−1) x b   b x 1   (n−1) x 1
KCL
A i = 0
(datum, or reference)
KVL : Branch 2 : e2 = v1 − v2
e = A^T v



NPTEL
[redacted] 0 0 0 1
[redacted] 1 −1 0 0
[redacted] 0 1 1 −1
Incidence Matrix : A
(n−1) x b   b x 1   (n−1) x 1
A i = 0
(datum, or reference)
KVL : Branch 2 : e2 = v1 − v2
e = A^T v
Summary : A i = 0
e = A^T v



So we eliminate we can eliminate a 0 here. So, this is now if you remove the augmentation what you get? What do you think you will call this matrix? Well, the augmented incidence matrix minus the augmentation basically means that this is the incidence matrix. And this is an n minus 1 cross b matrix, this is a b cross 1 matrix, and this is an n minus 1 cross 1.

So, and the incidence matrix is often denoted by the symbol A; there is nothing holy about that notation. And therefore KCL expressed in matrix form is simply A times; how will you express KCL in matrix form? A times i equals to 0 vector; where all the matrices have appropriate dimensions. Now, remember that we also I am going to copy the graph over; so remember we wanted to solve this network. We assume that the branch relationships are (known); so we need to use KCL and KVL. KCL in matrix form is written in terms of the incidence matrix as follows, and the next thing is to write KVL.

And how do you express KVL in matrix form? What is Kirchhoff's voltage law? You can think of it in many ways. One way is to say is the familiar way of thinking about it as if you go around a loop; then the the sum total of all the voltage drops you see across the branches in the in that particular loop is 0. It is actually equivalent to saying that you can relate the node potential to the branch potentials by simply for example, let us say take branch 2. What is the the potential of the branch 2? We call that e2 and that is simply v1 minus v2. So, relating the node I mean calling the branch potential as the difference between the node potentials is equivalent to saying that when you go around a loop; the sum total is 0.

Because you are starting and stopping at the at the same; so, this avoids the hassle of having to find loops. So, KVL is therefore expressed conveniently as you denote the branch potential as the difference between the node potentials; that is an equivalent statement for Kirchhoff's voltage law. Does not it makes sense?

Now given the incidence matrix can you think of a way of finding the branch potentials in terms of the node potentials? Well remember what does each I mean remember the incidence matrix basically contains all the information regarding how many branches are there, how many nodes are there; and how are the branches connected between the nodes.

So, simply staring at the incidence matrix should be able to give us this information that we are after namely, how are the branch potentials related to the node potential; and let us stare at this at this matrix. Remember that each one of these columns corresponds to each one of these rows corresponds to a node, and each one of these columns corresponds to a branch. So, can you stare at the at the columns and take a guess as to how the branch voltages can be related to the node voltages. What is this 1 minus 1? What is this telling us? Let us take this column; this corresponds to which branch?

This corresponds to the second column which basically means that this corresponds to the branch number 2. And it says one for I mean that column indicates that this branch starts at which node and ends at which node. It starts at node 1 and ends at node 2; and therefore, what comment can you make about the potential difference? What comment can you make about the the branch potential in relation to the node potential? It is simply v1 minus v2. So, taking this forward what common can you make about the branch potential vector in terms of the node potential vector.

Similarly, so let me lead you to the answer; what about the potential of node 1 of branch 1? v1 minus v4. V4 we know is our datum or reference node who whose potential we arbitrarily defined to be 0; and therefore, it is simply v1. So, you can see that the the branch potentials in terms of the node potentials are simply given by A transpose.

So, the branch potential vector e is simply the transpose of the incidence matrix, multiplied by what? The node potential vector. You can see that in each column they are either what you call one or two entries. If there is only one entry, it basically means that that particular branch goes to ends at the reference node.

If there are two branches it means that that particular I mean there are two non-0 entries. It means that that particular branch does not touch the reference node; and the plus and minus basically tell you where the branch starts and where the branch ends (())(20:25) makes sense people. So, so this is KVL, so to summarize therefore once the topology of or the graph of the network is known; KCL is simply written as A times i equal to 0 and e which is the branch voltage vector is simply written as A transpose times the node voltage. Is that clear? Yes.

Student: (())(21:06)

Professor: Correct, this is not this is not enough for us; he brings up a good point. These are not enough to basically find the actual voltages and currents; we still need some more information. What is that information? We need to how each branch voltage is related to the branch current. In a linear network that relationship will be linear; in non-linear network that will not be linear. Yes.

Student: (()) (21:43)

Professor: Pardon.

Student: (()) (21:48)

Professor: Yes, I mean as so if current is flowing this way; we always assume that current flows from a higher potential to a lower potential. So, the direction of current is always you know if current is flowing this way; then the potential is is higher on the left is that clear. Now, this makes these two are simply KCL and KVL and make no assumptions whatsoever about the nature of the network.