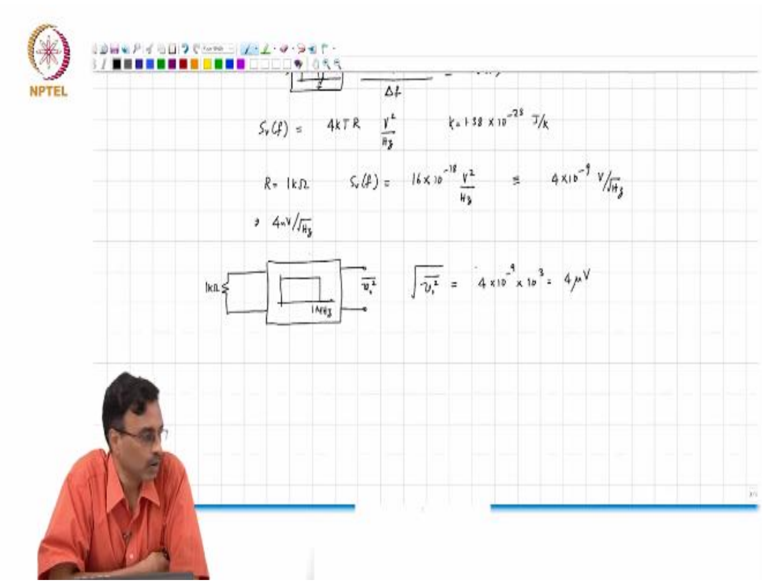
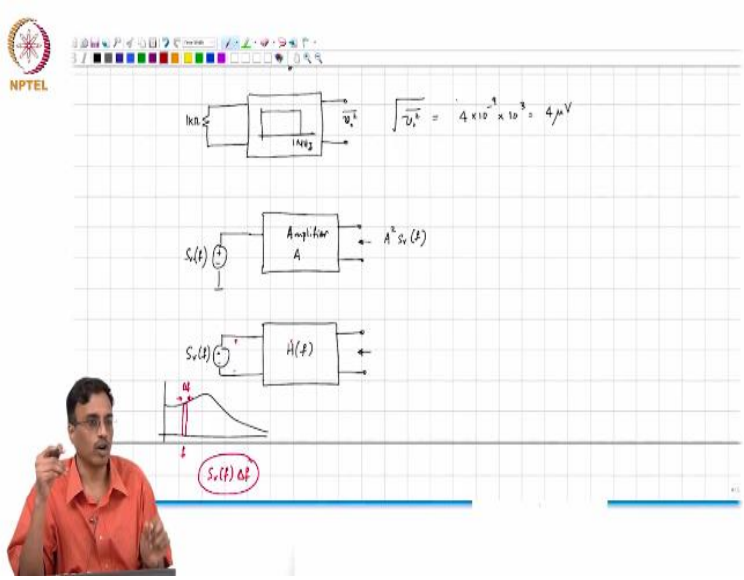
Introduction to Time- Varying Electrical Network
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture 18
Noise Processes by a Linear Time-Invariant System

(Refer Slide Time: 0:11)



So, we said this you know 1 kilo ohm and 4 nano volts per root hertz is a good number to keep in mind and also gives you an idea of you know what these numbers are like, good all right.

(Refer Slide Time: 0:31)



Now, let me take, give you another, let me ask you about, let us say this is simply an amplifier with a gain A, correct, what comment can you make about the mean square value of the noise now, if this voltage is Vn what do you think the voltage here will be a…

Student: (())(1:15)

Professor: A times Vn. So, what comment can you make about the mean square value of the output?

Student: (())(1:23)

Professor: This is simply will be A square times, correct. So, that was easy. Now, the next thing, if this was a noise voltage source with a spectral density Svn Sv of f what comment can we make about the spectral density of the noise here? Pardon?

Student: (())(2:02) A square.

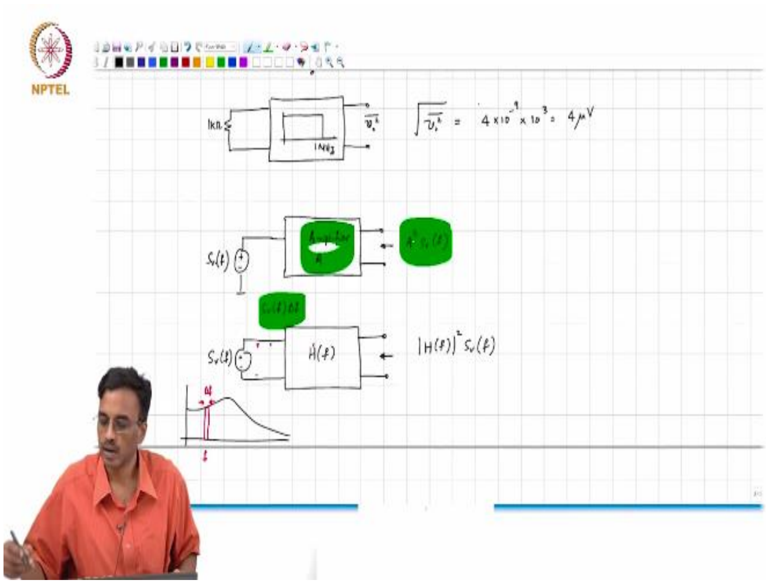
Professor: This is simply nothing but A square times the spectral density of the noise, input noise, easy enough?. Now, let us say I have a filter here with a transfer function H of f, I have a noise source, I mean though I keep marking plus and minus, I mean remember that the mean is 0 so, it does not really matter, Sv of f what comment can we make about the noise spectral density remember here?

Remember, what is Sv of f? It is simply, let us say that this noise spectral density is doing something like this, the noise spectral density remember quantifies at a certain frequency f and if you have a bandwidth of delta f and this is f, the mean square noise is what? Here in that bandwidth what is it? Yes?

Student: Sv of f.

Professor: Sv of f times delta, so that is the mean square noise in that band, correct center right f. Now, what is the gain of the filter at frequency f? Mod H of f, correct. So, this is getting, this spectral density is being passed through an amplifier whose gain is dependent on frequency and at a certain frequency f the gain of the amplifier is H of f. So, what comment can you make about the spectral density at the output or the mean square noise at the output at a frequency f, do you understand the question?

(Refer Slide Time: 4:06)

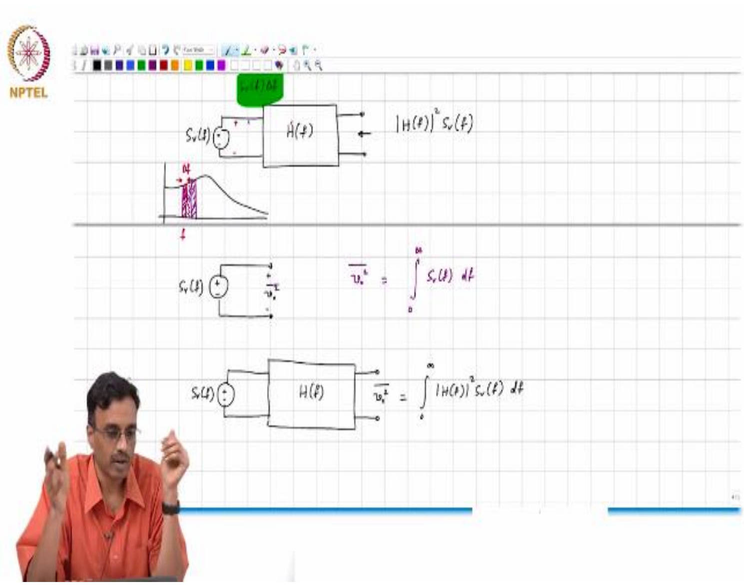


At a frequency f the mean square noise here will be Sv of f times delta f. And earlier when we had an amplifier whose gained did not change with frequency, all of you immediately said that the mean square, the spectral density the output is A square times Sv of f, now what do we need to do? Well, the game is changing with frequency, this consists of components at a frequency f and narrow bandwidth delta f, the game corresponding to that frequency is H of f. So, what comment can you make about the mean square noise of the waveform coming out at frequency f?

Student: (())(4:57).

Professor: Very good, it is simply, so therefore, the output noise spectral density is simply mod H of f the whole square times Sv, this is also very straightforward.

(Refer Slide Time: 5:23)



So, this is a, and what comment can you make about if you have a noise source with spectral density Sv of f what comment can we make about that mean square noise of the noise source? In other words, if I put a filter there whose bandwidth is in finite, what do you expect the mean square noise to be? Well remember that the mean square noise if you put a bandpass filter of bandwidth 1 hertz or delta f hertz is Sv f f times delta f, correct. So, if I put a filter with the bandwidth, if I do not put a filter at all what would be the total mean square noise?

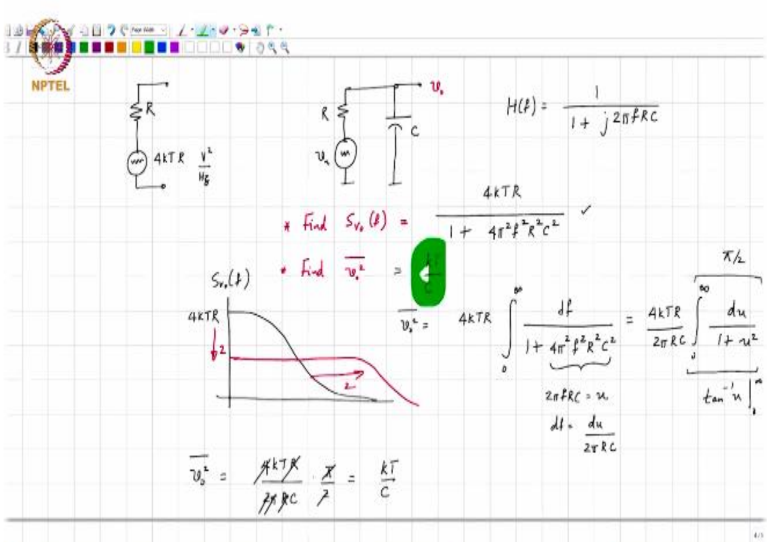
Student: (())(6:18).

Professor: It is simply nothing but, well it is the mean square value of this component plus the mean square value of that guy plus the mean square value of this chap and so on. So, this is nothing but integral Sv of f df from 0 to... So, now, I know we just put everything together that we have seen together. So, well let us say you have a noise source with a noise spectral density Sv of f, this is H of f, what is the output, what is the mean square output noise? What is the output noise spectral density? Pardon?

Student: (())(7:31)

Professor: Mod of H of f times Sv of f, so what are the mean square output noise? Integral 0 to infinity mod H of f the whole square times Sv of f. That makes sense people? You can see all this I mean, you know, the random process guys will, you know will throw a fit if, you know, they see derivations like this. But if we had to sit and do stochastic processes and do this, we mean you know by the time we build a circuit we will be dead, so these seem intuitively reasonable. And this is what we are going to use. This is good enough to for us to make useful calculations like the one we will do right now.

(Refer Slide Time: 8:34)



So, let us start applying the stuff that we have seen to real circuit. So, basically every resistor is accompanied by a noise source. The bottom line is that every resistor is accompanied by a noise source whose noise spectral density is.

Student: (())(9:01)

Professor: 4 kT R volt square per hertz, you can think of it as there being a voltage, so I mean there is a voltage source in series with every resistor and the noise spectral density of that voltage source is this. Now, let us try and see if we can make some calculations. So, this is, let us take our first simple circuit, this is a resistor and a capacitor in parallel.

So, this is Vn, this is a voltage source, it just happens to be a noise voltage source. So, this is a voltage source, what comment can you make about the transfer function from the voltage source, we are interested in finding the noise spectral density at the output as well as the total mean square noise at the output. So, how do we, how do you propose that we go and find out the noise spectral density at the output?

So, we want to find Svo of f and find the mean square noise. How do you propose that we find Svo of f? Well, that voltage Vn is there, I mean you know there is an RC filter, so what comment can you make about the transfer function from Vn to Vo? So, the transfer function H of f is 1 over 1 plus J2 pi FRC, so the output spectral density is the mod H of f the whole square times the input spectral density, the input spectral density is nothing but 4 kT R times, what should we do? 1 over divided by 1 plus 4 pi square f square R square C square and how does this look?

What will the spectral density be at DC? DC it will be 4 kT R and why does it make intuitive sense? At DC the capacitors open, the transfer function is 1, so the low frequency output noise spectral density will be the same as that of the (())(12:18) and why does the shape make sense? Well, it is a low pass filter, at high frequencies the capacitors behave like a shot. So, the high frequency components of the noise are getting attenuated by the filter and that is what the filter is supposed to, so that is clear.

Now, the next thing is to find the mean square noise, the output, the total mean square noise the output, how will you figure that out? Well, it is simply 4 kT R integral 0 to infinity 1 over 1 plus 4 pi square f square R square C squared df and well we use 2 pi fRC as u and df is nothing but du by 2 pi RC. So, this is nothing but 4 kT R over 2 pi RC integral 0 to infinity du by 1 plus u square so, an and now (())(13:48) integral what is it?

Student: (())(13:52)

Professor: Tan u and that has to be valid to the limits of 0 and infinity which basically is, what is that? Pi over 2, so the mean square noise for all this is 4 kT R times pi over 2, you might get this JEE or GATE exam, everything cancels out 2, 2, 4, R, R what do you get? KT by C.

So, what is this telling us? That the mean square noise is (())(14:51) kT over C. Now, we kind of little bit puzzled and this integral seems pretty complicated. Perhaps you made a mistake? How is it possible that the mean square noises, after all the noise is coming from where? From a resistor and but the total integrated noise is got no R in it at all.

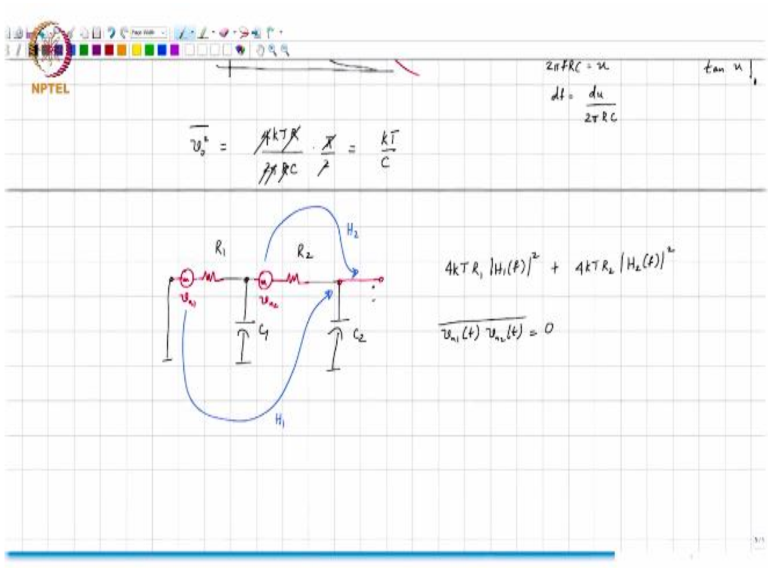
That would sound to me like hope so maybe we made a mistake, then I trust your judgment and I trust on your competence with integral calculus, so appears right, so kT by c, so now we got to figure out, well, the answer as we know, turns out to be Kt by c, why does this make sense? And we can say what can I say comes down to the math, the question is there, you know, some intuition behind the design.

So, as you can see, if I reduce the resistor, what is happening to the spectral density of the noise produced by the resistor? Let us say we reduce the resistor by a factor of 2, what happens? The noise added by the resistor is smaller. I mean, the spectral density of the noise added by the resistor is small. But there is another important thing happening, what is that?

Well, the bandwidth is doing, has become quite the spectral density has gone down by a factor, let us say I reduce the resistance by a factor of 2, what has happened to the spectral density the resistor, of the noise? It is gone down by a factor of 2, but what has happened to the bandwidth? Well, the bandwidth has also gone up by a factor of 2.

So, while it is true, that reducing the resistor has the beneficial effect of reducing spectral density of the noise source, well the total noise does not change because, the bandwidth over which way it has to be, it makes a contribution is now large. Now, that does not mean that this pattern that factor exactly cancel, in this case, it just so turns out that, you know, the reduction in spectral density exactly cancels out (())(17:30), does make sense? So, that is the intuition behind this kT by C.

(Refer Slide Time: 17:47)



So, now all of you look sufficiently bored about doing this, this is C1, this C2, this is R1, this is R2 of course, we are not going to do this in the class. So, we have 2 resistors, so we have two noise sources and let us say we are interested in finding the mean square noise across, we intend to find the spectral density and the mean square noise across C.

What are we going to do? I think fantastically new, what do we do, we need to find the transfer function from the Vn1 to the output, let us call that H1, the transfer function from Vn2 to the output, let us call it H2 and so what would be the total output, what would be the noise spectral density at the output? What is the noise spectral density at the output can I get a clear answer?

Student: (())(19:36)

Professor: So the spectral density of Vn1 is 4 kT R1 times mod H1 of f whole square plus 4 kT R2 mod H2 of f whole square, by adding power that is only true if the cross terms, I mean see Vn1 will result in some output noise here, Vn2 will result in some other output noise waveform and while we are saying is that the mean square noise of the sum is the same as the sum of the mean square noise of the individual components. That is only true if average value of Vn1 of t times Vn2 of t, these averages… and that it turns out that that is indeed true.

Physically, it is because well this noises because of random movement of carriers in the resistor, in the conductor, there is no way carriers and one conductor can know what is happening in the other one. So, it is like saying if you have a whole bunch of classrooms, every class is noisy, but then why is noise in one class is completely unrelated to the noise that is happening in another class, because different people are sitting there hopefully yakking about different subjects.

So, what do you hear is basically no correlation between that and these are independent. So, that is very convenient because all that we need to do is, we need to find the transfer function from these noise sources to the output, I mean you are already expert doing this because you can write them in a matrices and go. And then once you find the transfer functions, you basically find the amount of the mean square and then you integrate and then.

So, if you have 400, resistors and 300 capacitors, what do you do? Well, you know, the same thing 400 times, so you need to find, so this is where you want to find the mean square noise at the output of the network. You can see now that you need to find transfer functions from multiple input sources to the same output.

And this as you know, I mean, if you could do this by superposition and you know that is as we have seen, it takes a very long time, simply because you are doing the same old calculations over and over again. So, now that you know everything about reciprocity and (())(22:43) joints, what do you think you will do? Why do you think those concepts are useful? To find H1 and H2 and H300, you do not need to do it one source at a time, you just (())(22:58) the output with a current source and find, evaluate the whole network just once and find the currents through all the individual sources.