Introduction to Time-Varying Electrical Networks
Professor Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture 17
Introduction to noise in electrical networks

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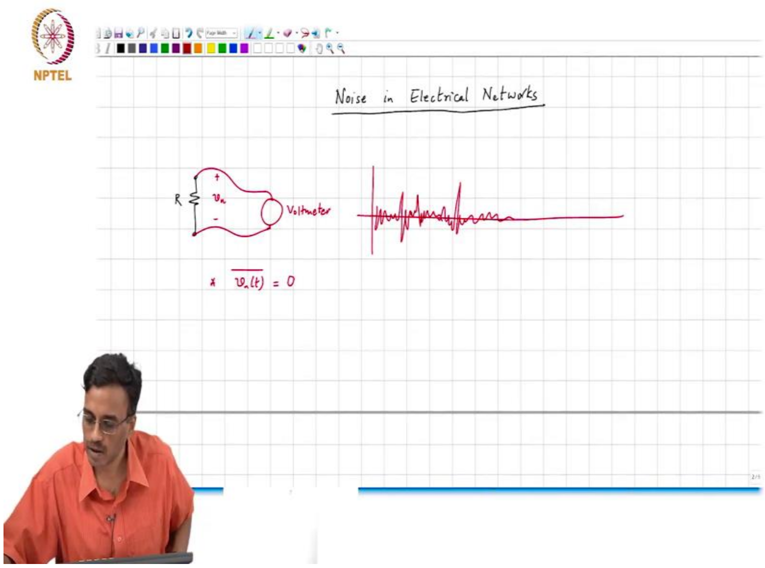

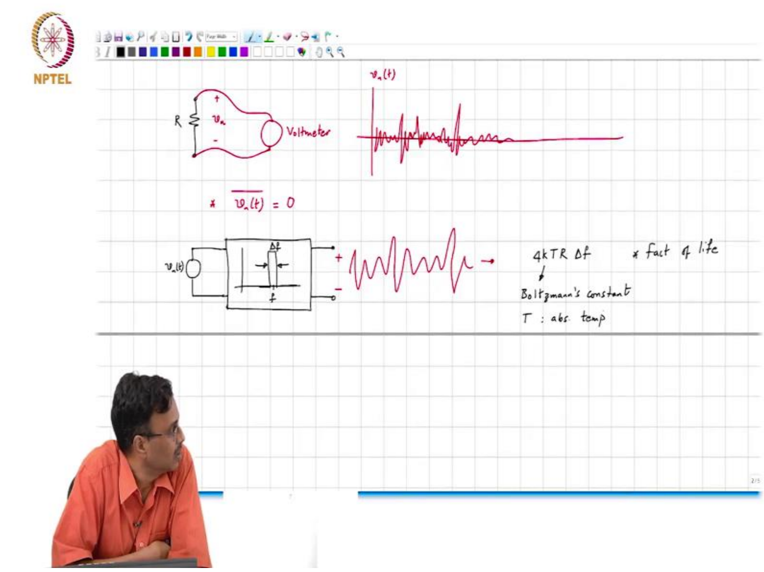

Basically, the next topic that motivates our next topic, which is, you know, we need to understand how, you know, we know that the individual elements, namely the resistors and transistors etc. are noisy. We need to understand, when we put all these elements together what happens to the output noise of a network.

So, that is the topic that we will study next. Once we study noise we will be in a position to come back to filters for a brief while and look at what the lowest, what do you call, signal you can put into the filter will be, in order to achieve a certain accepted signal to noise ratio at the output of the filter.

How many of you have looked at noi…I mean have studied noise in earlier, which course? And, what do you call, how many of you have looked at, have been introduced to noice from a communication class point of view? The BTechs have not seen noise at all. So, for those of you who have seen it in the communication class, we are not going to go into long theorem proof type results here. Our aim is, given a circuit can I predict how much noise there is at its output and what the characteristics of that noise are.

So, and again the, the idea is to work with, with practical circuits and be able to understand how to calculate noise, what to do to make to get better noise performance and that all. So, I mean to get to this stage in communications you would go through several classes of what a random process is, what ergodicity is and what autocorrelation is and then Wiener–Khinchin theorem and all that stuff. It is a, before you come to the calculations that we do. We are going to skip all that. Then, all that I am interested in this course is to give you a minimum bare bones toolset needed to be able to make useful nice calculations in circuits.

So, first I am going to mention some facts of life. There is nothing we can do about these things. So, it turns out that, you know, if you had a resistor in thermal equilibrium with its surroundings, with an absolute temperature T. It turns out that if you put a fictitious voltmeter across the resistor, it turns out that the voltage is not, you would ideally think that the voltage should be 0.

No current is flowing through the resistor after all. So, you think the voltage is 0, but it turns out that you will see a very, you will see a random waveform. It is almost impossible for me to draw that or draw a random waveform. So, but you will see some random wave. Without knowing anything else, what comment can we make about the mean of this random waveform?

Student: 0.

Professor: 0, why I mean why is it, why should be, how would you say 0, all in unison?

Student: (())(4:35)

Professor: I cannot hear you. Power?

Student: (())(4:37)

Professor: What conservation? Where?

Student: (())(4:41)

Professor: Well, there is a voltage, so there is definitely power.

Student: (())(4:49)

Professor: Well, I mean the easiest way to defend this is, if, if the average was not 0, then we will be able to, I mean, there is no need for us to sit here. We could go, we will be making power and then selling it. So, you know it sounds too good to be true. And remember, in life anything that sounds too good to be true is, is always too good to be true. And therefore, the average batter be 0. No matter what else is true, the average better be 0. So, first fact of life. The voltage across the resistor, v n of t on average is 0.

The next fact of life is…this is v n of t. The next fact of life is that if you to take the v n of t, you recorded that v n of t, pass this through a bandpass filter, an ideal brickwall bandpass filter, centered at a frequency f. And, had, has a bandwidth of a small bandwidth, delta f. What you think the bandpass filter will do? This is an ideal bandpass filter, perfect brickwall, with the bandwidth of delta f and centered at f. What will the bandpass filter do? It will…

Student: (())(6:30)

Professor: It will remove all frequencies. I mean this is a random waveform. There is some waveform. So, we did not treat as some spectrum. You pass take this waveform and then pass this through a bandpass filter. The bandpass filter is only going to pick up those components which are centered at f and in a neighborhood of delta f around f. So, if you look at this waveform on a scope, what do you think this will look like? It will look like a…. It cannot look like a sine wave of f, of frequency f. If you had to get the…it cannot be a sine wave with frequency f.

Because if the output is a sine wave, pure sine wave then, this is linear time invariant filter. So, input must also be must be a, must be sine wave but the input is kind of randomly varying. So, what does it…Because it is a bandpass filter, the output will look like a sinusoid, but it is, because it is being driven by a random input. The out…the magnitude and phase of that sinusoid, the amplitude and phase of the sinusoid will keep varying. So, you will basically see a sinusoid like waveform whose, whose amplitude and phase are varying.

So, you can measure the mean square value of this waveform. There is a waveform. It is a free country. So, I can, nothing is preventing me from measuring the mean square value of, of this waveform. And, if you do, if you, so make the measurement it turns out that that mean square value will be 4 KTR times delta f. Again, this is, what you call a fact of life.

And those of you who are interested in figuring out why this is so, feel free to go, and it has got to do with a quantum mechanics and blackbody radiation all that function. So, what is kind of strange about this formula? Do you find something strange at all? Oh! By the way, 4 is 4, K is Boltzmann's constant, T is absolute temperature, R of course is the resistance value, and delta f is the bandwidth to the filter. So, what do you find as somewhat puzzling about this?

Student: (())(9:33)

Professor: It does not have F at all. So basically, this is telling you that no matter where the center frequency of the bandpass filter is, the output mean square noise always happens to be 4KTR times delta f.

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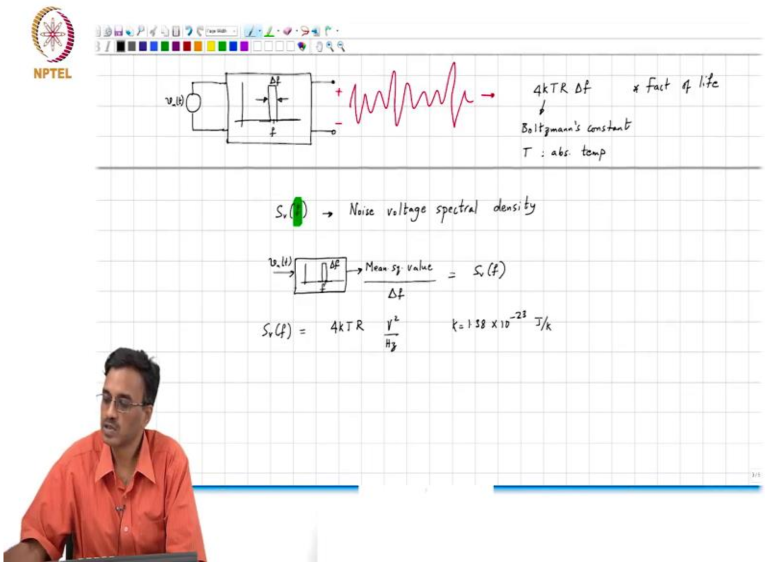

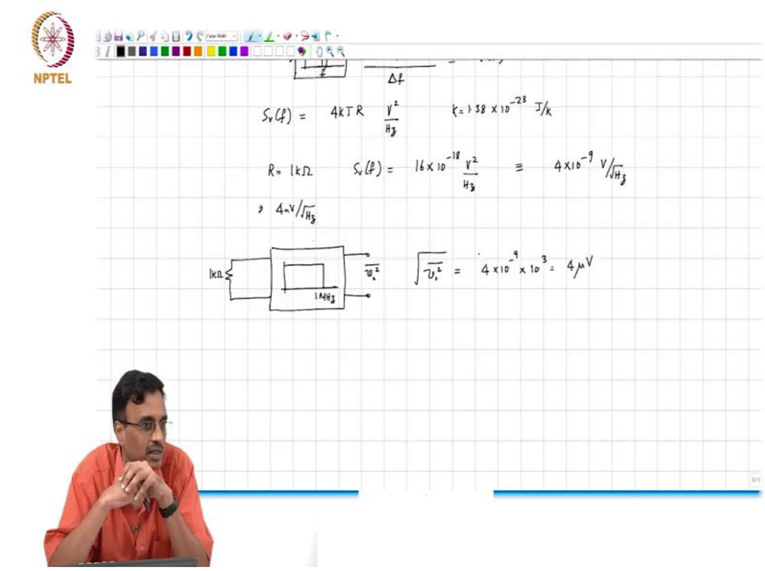

So, we therefore, what we now define is what is called the noise voltage spectral density. Those of you who have not done noise before, there is no need to get intimidated. All that this is saying is that if I take this noise source, pass it. Remember, this noise voltage spectral density is a function of f, the frequency at which you are making the measurement.

This is basically saying mean square value of the waveform, of the waveform that appears when you take v n of t, pass it through a bandpass filter of bandwidth delta f, and the center frequency f, and find the mean squared value here, and divide this by 12, if I make the filter by bandwidth narrower what do would you expect for mean square value of the output? What do you expect? If I make delta f 0, what do you expect the output? You get nothing. So, clearly the mean square value must be dependent on the bandwidth of the bandpass filter.

So, the mean square value divided by delta f is some measure of how much power there is in that waveform at that frequency. So, this is what is called the mean square…. I mean…the mean square value divided by delta f is basically the, this is the voltage noise spectral density as a function of f.

In English, all that it means is that if you take a voltage, I mean, a noise voltage with this spectral density, you pass it to a bandpass filter, centered at f, and a bandwidth delta f. You should expect that the mean square value of the voltage at the output of the bandpass filter is simply S v of f times delta f. That is all that there is to it. So, now that we have learned some jargon. Let us try to apply it. So, what comment can we make about the noise voltage spectral density of resistor? Regardless of where you put the bandpass filter, we seem to be measuring a mean square value of 4KTR times delta f.

So, this basically is 4KTR. And what will be the, what are the dimensions of this? What are units of this?

Student: (())(13:24)

Professor: Volt square per hertz. Remember that K is, what is Boltzmann's constant? 1.38 into

Student: 10 power.

Professor: 10 power

Student: Minus 23.

Professor: Minus 23. And unit is?

Student: Joules per Kalvin.

Professor: Joules per Kalvin, is this, is that at room temperature or at some high temperature? It is a constant. A constant is supposed to be constant regardless of temperature. That is why it is called constant.

And to put some numbers, so I get a feel for this. It turns out that if you put R equal to 1K, what comment can you make about S v of f. Please do the math. 16 into 18-volt square per herzts. And, this is often, also expressed, the square root of this is also expressed in what is called nano volts per, volts per root hertz. So, this as you can see is 4 into 10 power 9 volts per root hertz.

So, a good, a quick number to remember is that a 1 kilo ohm resistor has, has a noise spectral density of, noise voltage spectral density of 4 nano volt per root hertz. So, if you are still in early stages, let us, kind of see how to make some calculations. So, let us say, I take the voltage waveform across a 1 kilo ohm resistor and have a low pass filter of 1 megahertz bandwidth. How much, what is the mean square value that you see there?

How do we do the math? We know that the spectral density is 4 nano volts per root hertz. The bandwidth is, the RMS value at the output will be, the RMS value at the output will be simply 4 nano volts per root hertz times 10 power 3; which is 4 micro volts. So, the RMS noise of a 1 kilo ohm resistor in a 1-megahertz band is 4 micro volts.

So, this basically prompts the next question. If I make a, if I make a megahertz or gigahertz, what will happen? We can multiply by 30. If I make the gigahertz or terahertz what happens? Pardon?

Student: (())(17:01)

Professor: If I make the terahertz, 10 billion terahertz, what will happen? If I made infinity, what will happen? It will be…

Student: (())(17:17)

Professor: The formula is telling as it will be infinite. So, what is your comment on that? What do you think? I do not have to measure it. This is just telling you that if I just, if I do not have the filter at all….

Student: (())(17:32)

Professor: no, I do not have a filter at all. I just have a resistor. That is all. It is telling me, this math is telling me that RMS value can go to infinity. That basically means that air around it should have broken down and we will be seeing sparks everywhere. Clearly it does into happen. So, there must be something wrong with this. So, that is what is called the ultraviolet catastrophe and like, so this is all as I said, all comes back eventually to blackbody radiation And, I do not know if you still remember your high school physics, but, this h nu by a to the h nu by kt minus 1 and so on.

I do not know if you remember blackbody radiation spectrum right goes up and then kind of eventually falls off. But it turns out that as far as circuits are concerned, we are always working with frequencies which are at best a few 10s of gigahertz, and up to those frequencies it is as good as being constant. At, at ultraviolet frequencies, obviously, the spectral density is not constant. And beyond that, it actually falls off. So, the integral is actually finite. So, do not go right away and then start, open a start up saying we will generate energy by using resistors.