Introduction to Time - Varying Electrical Networks
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Inter-reciprocity in linear networks - using the MNA stamp approach

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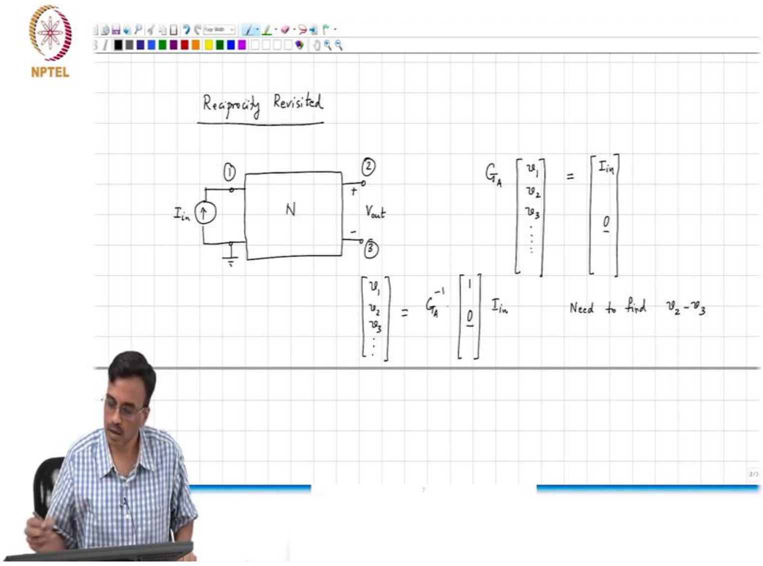

With this as I said background, let us take a step back and look at reciprocity again. So, the last time around, what did we use to prove reciprocity?

Student: (())(00:29)

Professor: Reciprocity and inter-reciprocity, how would we prove them?

Student: Tellegen's Theorem.

Professor: We use Tellegen's Theorem. I do not know about you guys. But I mean, the whole thing seems like pulling a rabbit out of a hat. Because I mean, Tellegen's Theorem to begin with is, is not particularly intuitive, especially when applied to other networks. It almost seems like we knew the answer and we were getting it. Fortunately, it is possible to derive it from first principles and that is what we are going to do next. So, for example, let me take.

So, let us say, we have a network here and I going do simply a current input voltage output system. You can go ahead and do the same math for voltage input, current output and any of those other combinations that we often encounter. So, this is, say Iin, this is the network N and this is Vout.

And, this of course network has a whole bunch of nodes and this is a free country. So, I am free to choose any node as the reference, I am going to choose that node as a reference. I will call this node number 1, I call this the node number 2 and this will call node number 3 and how will I go about solving the network? I mean, let us say I know the internals of the network what will I do?

Student: (())(02:28)

Professor: Basically, just go and write the MNA equations for the entire network and you will find that you will get an equation of the form some augmented conductance matrix. Where, the augmented conductance matrix will contain entries corresponding to the conductances and the controlled sources and all that stuff, which are potentially inside this box and the unknowns will be all the node voltages and?

Student: (())(03:08)

Professor: The currents through, all the controlled sources or the 0 voltage sources inside the box. So, the unknowns therefore, will be of this form, they will be v1, v2, v3 blah blah blah blah. And GA is an augmented conductance matrix, which we can, we are now in a position to determine by simply adding up the MNA stamps of the individual elements and this must be equal to, what must be there on the right?

Student: (())(03:52)

Professor: All the independent sources, what are the independent sources here?

Student: (())(03:59)

Professor: There is only one independent source namely?

Student: Iin.

Professor: Iin, so that just comes, that goes into node 1, the rest of this vector is 0 and we are interested in, what are we interested in finding?

Student: (())(04:25)

Professor: We are interested in finding the transfer function from?

Student: Vout to Iin.

Professor: Not Vout to Iin, Iin to?

Student: Vout.

Professor: Vout. So, what will you do? How do we will go about doing this? I know it is easy, but please tell me. What will we do?

Student: We will find v1, (())(4:50).

Professor: We will find, we will first find v1, v2, v3. I mean, I am not asking you to invert the matrix at this point, but all I am saying, this is what we will do. This is GA inverse times Iin followed by all 0s, which is equivalent to saying 0, 1 followed by a string of 0s times?

Student: Iin.

Professor: Iin, Iin is a scalar and this will give me, the output will give me?

Student: (())(05:30)

Professor: The node voltage, all the node voltages and all the currents that are flowing through the control sources and the 0 voltage-voltage sources needed in any of the controllers. Does that make sense? But what am interested in?

Student: v2 minus v3.

Professor: I am only interested in?

Student: v2 minus v3.

Professor: I need to find v2 minus v3. So, how will I find v2 minus v3?

Student: (())(06:13)

Professor: We have this common vector v1, v2, v3, and so on. We want v2 minus v3. What do you think?

Student: (())(06:22)

Professor: You multiply by?

Student: Row vector.

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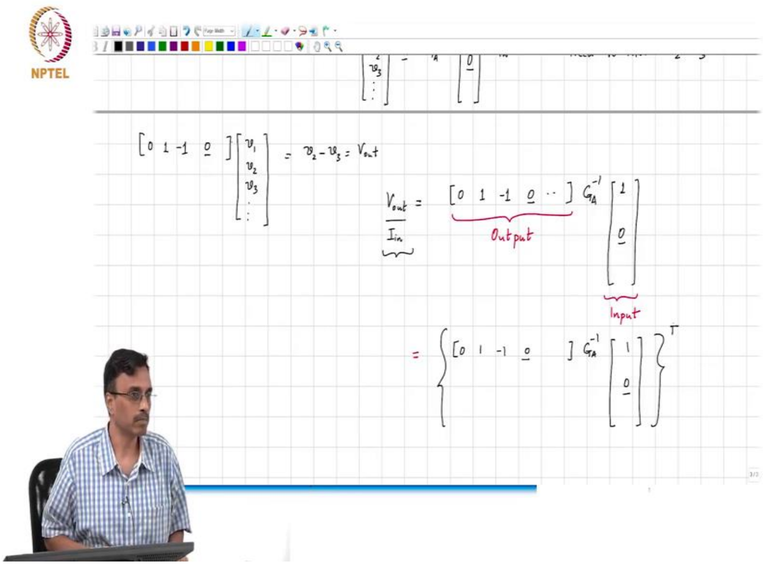

Professor: A row vector, so v1, v2, v3 blah, blah, blah, times?

Student: (())(6:37)

Professor: 0, 1

Student: Minus 1

Professor: Minus 1 and followed by all 0s, must be v2 minus v3 which is Vout. Does it make sense people? So, what is Vout? Putting these two equations together, it is simply nothing but 0, 1, minus 1, 0 times v1, v2, v3 blah, blah, blah and what is that v1, v2, v3 blah, blah, blah?

Student: GA inverse.

Professor: It is GA inverse times?

Student: 1 followed by all 0s

Professor: GA inverse times?

Student: 1 followed by all 0s

Professor: 1 followed by

Student: all 0s

Professor: All 0s, times? Iin. So, if I bring Iin out here, the transfer function from Iin to Vout is basically of this form. Now, is this a scalar or vector?

Student: Scalar.

Professor: This is a?

Student: Scalar.

Professor: Scalar, and what does this quantify? What information does that vector, what information does that incorporate?

Student: (())(08:28)

Professor: I understand. So, this quantifies where the input is applied? The input is applied and the circuit generates, this vector of node voltages. But we are not interested in all the node voltages. This you can think of as the, we only interested in measuring the difference between selected nodes. So, this measures what do you call quantifies across which nodes the output is taken? Does that make sense? And this is a scalar.

So, nothing will change if I, yes what happens? I mean if it is a scalar what, how can I write this as? hat is the expression for the scalar, a scalar is the same as its transpose. So, I can write, I can think of this as simply equivalently, this must also be equal to 1 1 minus 1 0, GA inverse times 1 followed by all 0s, this transpose.

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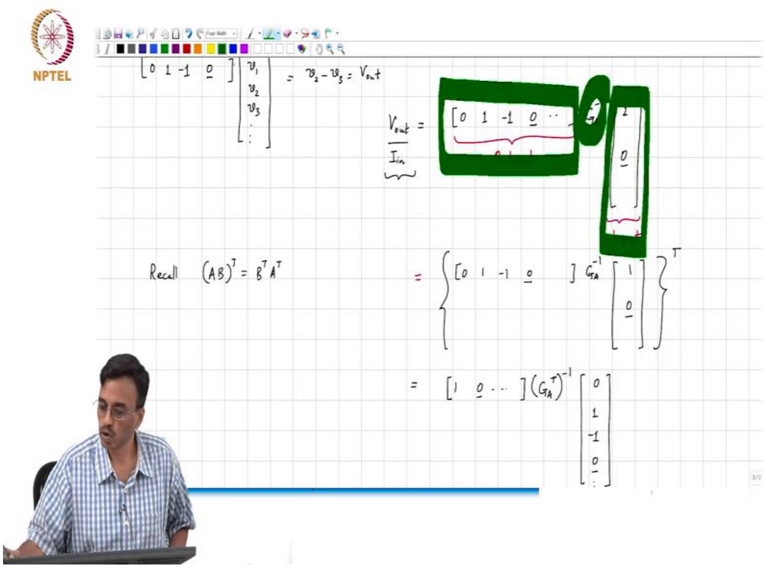

And what is the transpose of this? Recall, AB whole transpose is what?

Student: B transpose.

Professor: B transpose, A transpose. So, what do you get here? Yes, come on people.

Student: 1 followed by all 0s

Professor: 1 followed by,

Student: All the 0s

Professor: All 0s, times?

Student: GA transpose inverse.

Professor: GA transpose inverse, whether you invert first transpose or do it the other way around it is the same thing, times?

Student: 0, 1, minus 1,

Professor: 0, 1, minus 1, followed by all those. Is this clear people? So, now how can I interpret this result as? Remember this fellow here, this column vector is the input and this is the, this goes through the circuit and results in some node voltage vector whose output you are measuring. So, likewise, this can be thought of as. How can you interpret this expression?

Student: (())(12:01)

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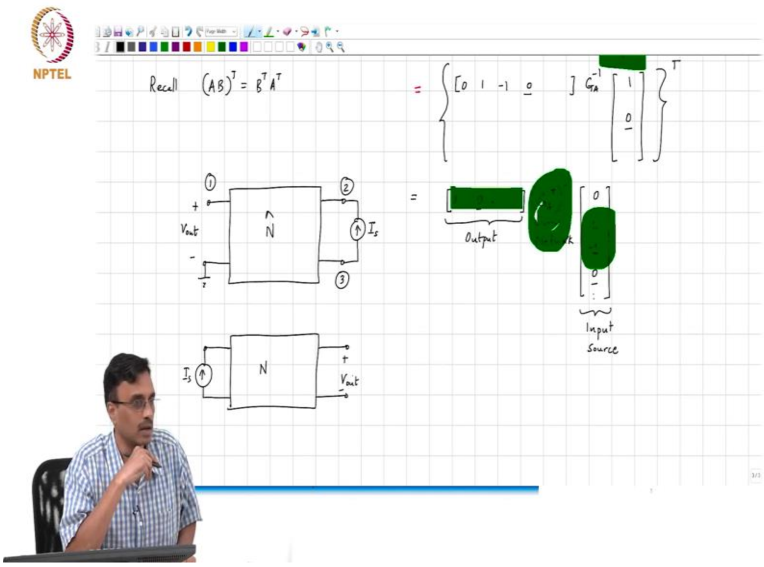

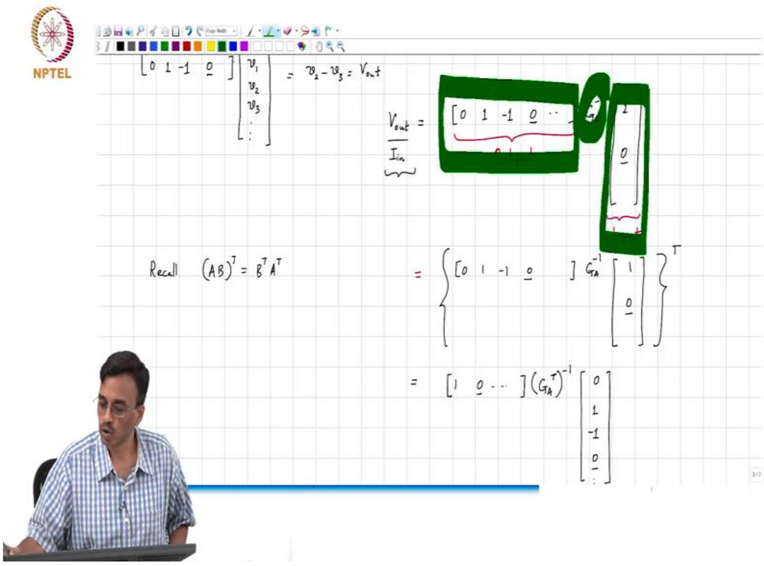

Professor: You can now think of this as the input source. This as the network and this as the output measurement.

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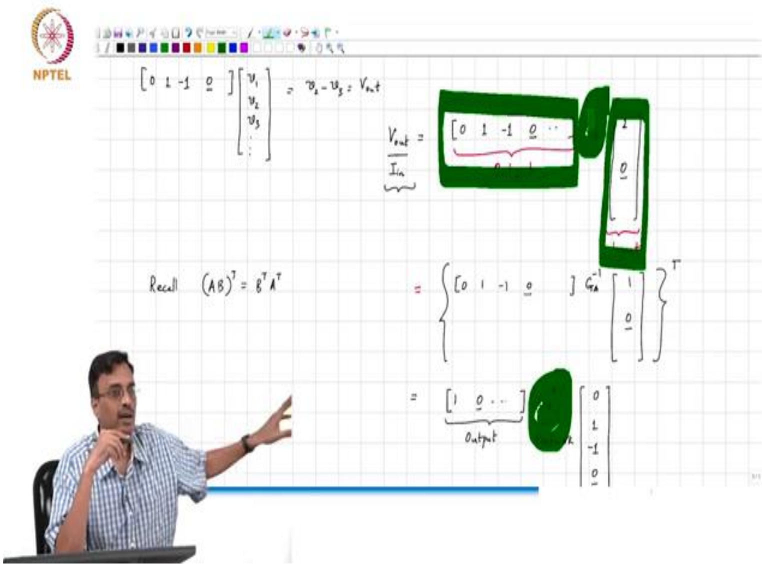

So, now, a couple of comments, is this network the same as this guy?

Student: (())(12:41)

Professor: Is it the same as the original network?

Student: (())(12:48)

Professor: In general, I mean, they have got different augmented conductance matrices and therefore the networks are not the same. However, evidently, the MNA matrix of this network, the lower network is simply the?

Student: Transpose.

Professor: Transpose of the MNA matrix of the original network. So, that is and the output is this. So, in other words, we can interpret this result as we have another network N hat, whose MNA matrix is simply the transpose of the MNA matrix of the original network and what comment can we make about, where we are injecting the input now? We are now injecting an input.

Student: (())(13:54)

Professor: And, what kind of input is that? There is something between 1 and minus 1, I mean between nodes 2 and 2. So, remember how did we number the nodes? This is node 1, this was ground, this was node 2 and this is node 3. So, if I gave you this matrix and told you, where are the independent sources, what kind of independent sources is there? And, where is it occurring? What would you say?

Students: It is leaving.

Professor: It is leaving.

Students: (())(14:27)

Professor: So basically this is, you can think of this as IS and, so this is a network N, whose MNA matrix is simply the transpose of the MNA matrix with the original network and where do we sense the output?

Student: (())(14:51)

Professor: What are the output matrix saying? Simply taking the node voltage vectors and simply measuring.

Student: The voltage of node 1.

Professor: The voltage of node 1. So, this is the node.

Student: (())(15:11)

Professor: Yes, it is clear people? So, if you want, in other words what is the moral of the story? If you have an original network N and you inject current IS here and measure the output here. You can get the same transfer by injecting current into the output port of a network which is not necessarily the same as N. But whose MNA matrix is simply the transpose of?

Student: MNA matrix.

Professor: Of the MNA matrix of the original. Is this clear so far?