

**Introduction to Time - Varying Electrical Networks**  
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**MNA stamps of controlled sources - the CCCS & CCVS**

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Vector of unknowns: node voltages, currents through voltage sources

$$G_A V = I_s$$

Augmented conductance matrix

Vector with information on all INDEPENDENT sources

$$G_A = \text{Sum of MNA stamps of all elements}$$

Augmented conductance matrix

$$G_A = \text{Sum of MNA stamps of all elements}$$

Resistor stamp (nodes a, b):

$$\begin{bmatrix} a & b \\ a & g & -g \\ b & -g & g \end{bmatrix}$$

Dependent current source stamp (nodes a, b, c, d):

$$\begin{bmatrix} a & b & c & d \\ a & g & -g & 0 \\ b & -g & g & 0 \\ c & 0 & 0 & 0 \\ d & 0 & 0 & 0 \end{bmatrix}$$

Professor: In the last class, we were looking at how to write the equations of a network in a systematic way. and we concluded that, in general, you will get equations of the form  $G \times V = I_s$ , where this is the augmented conductance matrix and this is the vector of unknowns, and what are the unknowns? What are all the unknowns?

Students: (( ))(01:12)

Professor: all the node voltages and currents through the voltage sources and likewise, this  $I_s$ , is basically a vector with information on all the independent sources. does makes sense

people and even though it is called the conductance matrix, as we saw yesterday, all the entries there need not have dimensions of conductance, it is only that core part on the top left corner, which will have all the conductances that are there in the network if you had voltage sources and so on some of those entries may be dimensionless. Now and how do we form go about forming the conductance matrix? This is simply nothing but the sum of, sum of what?

Student: ( ) (02:44)

Professor: Very good, this is simply the sum of the MNA stamps of all the elements and yesterday, we saw what the MNA stamp of a current source would be, what an MNA stamp of a voltage source would be, what the MNA stamp of a conductance would be?

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$G_g = \text{sum of MNA stamps of all elements}$

Conductance  $g$  stamp:

$$\begin{bmatrix} a & b \\ a & g & -g \\ b & -g & g \end{bmatrix}$$

Voltage-controlled current source  $\mu(v_a - v_b)$  stamp:

$$\begin{bmatrix} a & b & c & d \\ a & g & -g & 0 \\ b & -g & g & 0 \\ c & 0 & 0 & 1 \\ d & 0 & 0 & -1 \end{bmatrix}$$

Voltage-controlled voltage source  $\mu v_a$  stamp:

$$\begin{bmatrix} a & b & c & d \\ a & 0 & 0 & 0 \\ b & 0 & 0 & 0 \\ c & \mu & 0 & 0 \\ d & 0 & 0 & -1 \end{bmatrix}$$

Current source  $i_k$  stamp:

$$\begin{bmatrix} a & b & c & d \\ a & 0 & 0 & 0 \\ b & 0 & 0 & 0 \\ c & 0 & 0 & 1 \\ d & 0 & 0 & -1 \end{bmatrix}$$

Professor: So, let us quickly write down that stuff. So, this is node a, and this is node b, and this is a conductance g and, what do we have? We have, this is the conductance matrix or the augmented conductance matrix. We allow for potential auxiliary equations and unknowns, but the only thing we need to be aware of is ab and ab. and what do you have here? g minus g, minus g, and so on. So, then we saw what a voltage-controlled current source would behave like.

Let us call this gm times ba minus bb and how does the stamp of this look like? What do we do? The only thing that changes is, this becomes c and d. Is that clear? Then, we were discussing the voltage-controlled voltage source, mu times va minus vb and how does the MNA stamp of this look like? We need an auxiliary variable now. The auxiliary unknown is

the current flowing through the voltage source, which is  $i_x$  and that is there in the unknown vector and what comment can we make about the MNA Stamp? They are in the rows?

Student: c and d.

Professor: c and d, and what are the entries there?

Student: 1 and minus.

Professor: 1 and minus 1, and in the last row, in the a, b, c, and d, what will you get? So, you will get minus  $\mu$ ,  $\mu$  1 and minus 1. So, this is where we stopped yesterday, I believe.

The next control source is the current-controlled current source and I am going to, remember a current-controlled current source always measures the current through voltage, a 0 voltage, voltage source, so this is 0 volts and so this is  $i_x$  and this is  $\mu$  times  $i_x$  and what comment can we make about the stamp of this character? How many extra variables do we need now? Do we need an auxiliary unknown or is there an extra unknown or do we?

Student: (08:26)

Professor: Why? What are all the unknowns that are there in the network?

Student: (08:37)

Professor: So, all node voltages, remember, please remember that all node voltages and currents that are flowing through voltage sources are all unknowns. So, now the question I am asking you is do we have an extra unknown now? When you have a current-controlled current source, is there an unknown, extra unknown that is introduced, that needs to be introduced? Yes, and what is that?

Student:  $i_x$

Professor: That  $i_x$  is unknown, is flowing through the 0 voltage source and therefore, it also needs an extra unknown. So, you will need an extra row and an extra column for that. and after saying that, what comment can we make about the MNA stamp of the current-controlled current source?

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So, this is a voltage-controlled voltage source. This is the current-controlled current source. What comment can we make about the stamp? Yeah, what is the stamp? Do not jump to conclusions. So, in which rows will the last column have entries c and d, and what will be the entry on c, the cth row?

Student: Minus mu

Professor: Minus mu and dth row?

Student: Plus mu.

Professor: Plus mu and are there other rows on, I mean, where all, in what all rows will.

Student: (( ))(10:18)

Professor: You will have entries in all four a, b, c, and d because there is  $i_x$  flowing out of node a and into node b and  $\mu$  times  $i_x$  flowing into node c and out of node d. So, in nodes a and b you will have?

Student: 1 and minus 1.

Professor: 1 and minus 1, and node c minus mu and?

Student: Plus mu

Professor: Plus mu and what comment can you make about the last row, the auxiliary, the extra equation that we get, what is that relates the voltage source?

Student:  $v_a$  minus  $(\ )$ (11:04)

Professor:  $v_a$  equals  $v_b$ ,  $v_a$  minus  $v_b$  equals 0. So, you must get?

Student: 1 and minus 1,

Professor: 1 and minus 1, does it make sense?

Student:  $(\ )$ (11:13)

Professor: Pardon?

Student:  $(\ )$ (11:14)

Professor: The which is, I mean see, you always, I mean, the assumption is that in a circuit simulator what you always assume is that you measure current through a voltage, 0 voltage-voltage source. It is also likely that you can just have an arbitrary branch current and make this  $\mu$  times  $i_x$ , a multiple of that arbitrary branch current. In which case is straightforward simply introduces an artificially introduced 0 voltage-voltage source and add an extra node there and the formulation just simply works. Is this clear?

Professor: So, and I would just like to draw your attention to one fact and that is that, do you see any, if you stare at these two MNA stamps, what do you, what comment do you think you can make, of the voltage-control current source and the current-control voltage, I mean the voltage-controlled voltage source and the current-controlled current source, you stare at the MNA stamps I mean do you see anything at all?

Student:  $(\ )$ (12:30)

Professor: It is not the transpose, I mean it is, please note that the MNA stamp is not simply the transpose, you also have to?

Student: Swap.

Professor: Swap, swap the rows a with c and b with d. And this is just something that I draw your attention to we will use this fact later on. At any rate, it looks very similar, suspiciously similar to the MNA stamp of the voltage-controlled voltage source.

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CCVS

$v_x - Z i_x = 0$

MNA stamps transpose  $\rightarrow$  Flip controlled & controlling parts in VCCS and CCVS

VCVS : MNA stamp transpose  $\rightarrow$

The next thing is basically the current-controlled, we have seen the voltage-control current source, the voltage-controlled voltage source, the current-controlled current source and the last man standing is the current-controlled voltage source, and how will this look like? So, you have a, again, we have 0 voltage source and this is 0 volts, this is  $i_x$  and what are the, this is node c, and node d, and this is some z some impedance times  $i_x$ . So, how many unknowns do we have now?

Student: Two unknowns.

Professor: You have two unknowns now, because of?

Student: (())(14:23)

Professor: And the, remember that the current flowing to any voltage source is unknown or is an unknown, and therefore, we need two extra columns and two extra rows. And now you will tell me, what we need to do? Let us assume that this corresponds to  $i_x$  and this column corresponds to  $i_y$ , what do we do?

Student: (())(15:06)

Professor: So, between nodes, which of the rows will you have entries in the last but one column?

Student: a and b.

Professor: In a and b. So, that is 1 and minus 1 and what else? And what comment can you make about the last but one row?

Student: (15:48)

Professor: Last but one row,

Student: (15:56)

Professor: No,

Student: (16:00)

Professor: I mean, the last but one row will basically, quantify what is happening with that?

Student: 1 minus 1.

Professor: 1 minus 1. So,  $V_a$  must be equal to  $V_b$  and we are done with 1 row and 1 column. In the last column, what do we see?

Student: (16:18)

Professor: There must be 1 and minus 1, and what must you see in the last row?

Student: (16:30)

Professor: You see,  $V_c$ , (16:35) so  $V_c$  minus  $V_d$  equals or minus  $Z$  times  $i_x$  equals 0.

Professor: So, what do you see? What is that equation?

Student: (16:53)

Professor: 1 minus 1 and?

Student: Minus  $Z$ .

Professor: Minus  $Z$  and 0. I guess the entities which are not are all assumed to be 0, so they should be 0.

Now, if I flipped the ports, if I interchange  $a$  with  $c$  and  $b$  with  $d$ , what do you think will happen to the MNA matrix or the MNA stamp of this guy? First of all, what do you notice with respect to this matrix and this matrix? That is simply the transpose. So, if I flip the controlled and controlling ports, what comment can we make about the MNA stamp?

Student: (17:56)

Professor: What will happen to that last entry minus  $Z$ ? Which row will it enter now?

Student: (18:03)

Professor: It will enter?

Student: (18:07)

Professor: It will enter here and therefore, if by flipping the ports all that you do is, if you flip the ports, what do you basically have is the transpose of the MNA stamp of this guy. So which control, which of the control sources does flipping the ports result in the transposed MNA stamp?

Student: (18:30)

Professor: It is just an observation, flip the controlled and controlling ports. In, which control sources?

Student: VCCS and CCVS.

Professor: VC, very good and?

Student: CCVS.

Professor: CCVS. And, so if you transpose the MNA stamp of the voltage-controlled voltage source. If you transpose, it is MNA stamp, what will you get? That two things that you need to observe here.

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The slide displays the following MNA stamps:

- CCCS: 
$$\begin{bmatrix} a & b \\ b & -g & -g \\ -g & g & g \end{bmatrix}$$
- CCVS: 
$$\begin{bmatrix} a & b \\ c & g & -g \\ d & -g & g \end{bmatrix}$$
- VCCS: 
$$\begin{bmatrix} a & b & c & d \\ c & & 1 & -1 \\ d & & -\mu & \mu & 1 & -1 \end{bmatrix}$$
- VCVS: 
$$\begin{bmatrix} a & b & c & d \\ a & b & c & d \\ c & d & \mu & -\mu \\ 1 & -1 & & \end{bmatrix}$$

Note: Interchange the controlling and controlled parts

Professor: If we transpose this guy here, what do we get? We will get, please see this clearly. What will we get?

Student: (20:17)



Professor: So, you will get minus mu, mu, 1, minus 1. And which rows are they in, a, b, c, and d. And what will happen on the last row?

Student: ( ) (20:38)

Professor: 1 and minus 1 and they will be in the?

Student: ( ) (20:42)

Professor: c and dth column. So, that looks suspiciously similar to this, but there is a small twist, what is that twist?

Student: ( ) (20:57)

Professor: I mean is this the same as that?

Student: No, sir.

Professor: It is not that. So, if you take the transpose of the MNA stamp of the voltage-controlled voltage source, you get something which seems like a current-controlled current source, but it is not, there is a twist and that is?

Student: ( ) (21:15)

Professor: The controlling and controlled ports are, interchange the controlling and control ports. And then you will get the current-controlled current source. Does it make sense people?

At this point this is just an observation, it is clear so far? And, if you take the transpose of a conductance, if you take the transpose of the MNA stamp of a conductance, what comment can we make? Well, that is a symmetric matrix. So, if you take the transpose, it remains the same conductance. Is that clear?

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The image shows a digital whiteboard with a grid background. At the top left, there is a logo for NPTEL (National Programme on Technology Enhanced Learning) and a toolbar with various drawing tools. The whiteboard contains two lines of handwritten text in black ink:

MNA stamp transpose  $\rightarrow$  Flip controlled & controlling ports in VCCS and CCVS

VCVS : MNA stamp transpose  $\rightarrow$  CCCS with ports interchanged

A professor is visible in the bottom left corner of the frame, sitting at a desk and gesturing with his right hand.

Professor: With this background. So, this basically you get a CCCS with ports interchanged.