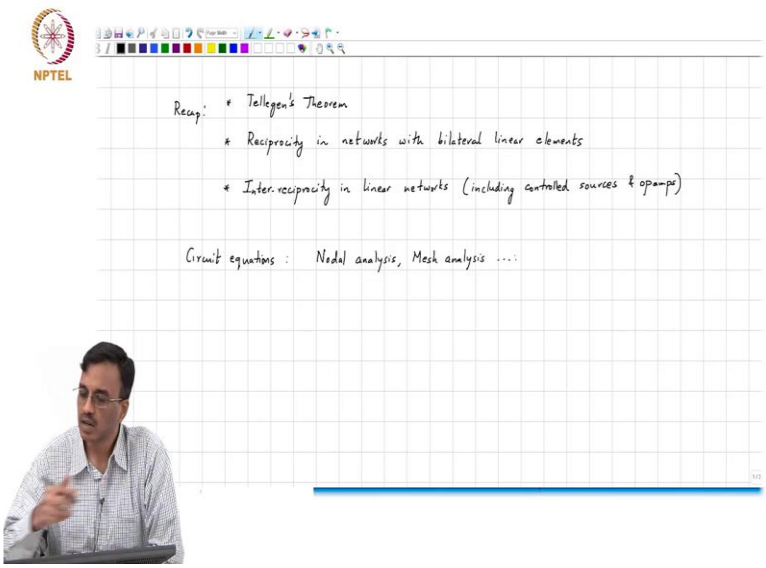
Introduction to Time – Varying Electrical Networks

Professor. Shanthi Pavan
Department of Electrical Engineering
Indian Institute of Technology Madras

Lecture 10

Review of Modified Nodal Analysis (MNA) of Linear Networks

(Refer Slide Time: 00:16)



Lecture five, a quick recap of what we have done in this course so far. So, we learned about Tellegen's theorem. We have learned how Tellegen's theorem was used to prove reciprocity in networks with linear bilateral elements, unfortunately, we found that well, while reciprocity is great and very useful in practice, we found that it is not applicable to a large class of networks that we often deal with namely, those with controlled sources.

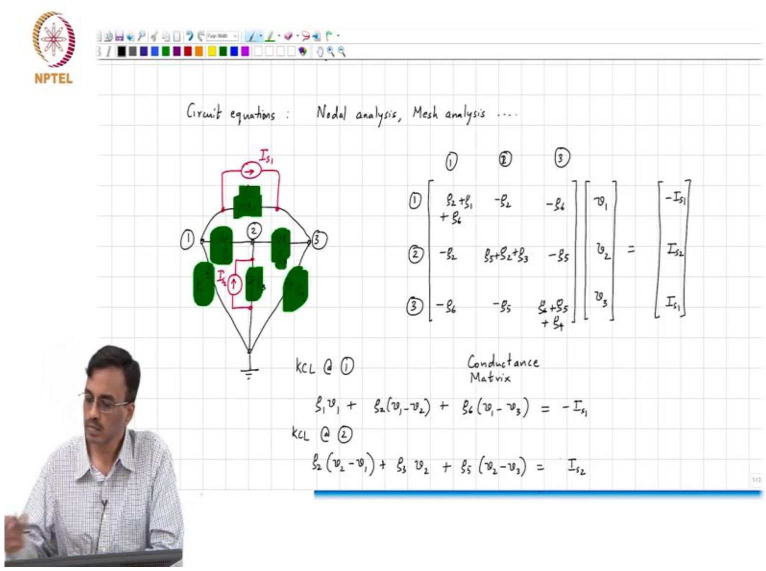
And, then we said well, there is a fix and that is by using the concept of inter reciprocity, where the idea is that while you cannot interchange the location of the excitation and the response in the original network, it is always possible to come up with another network, which is called the inter reciprocal network or the adjoint network, where you can interchange the role of the excitation port and the response port.

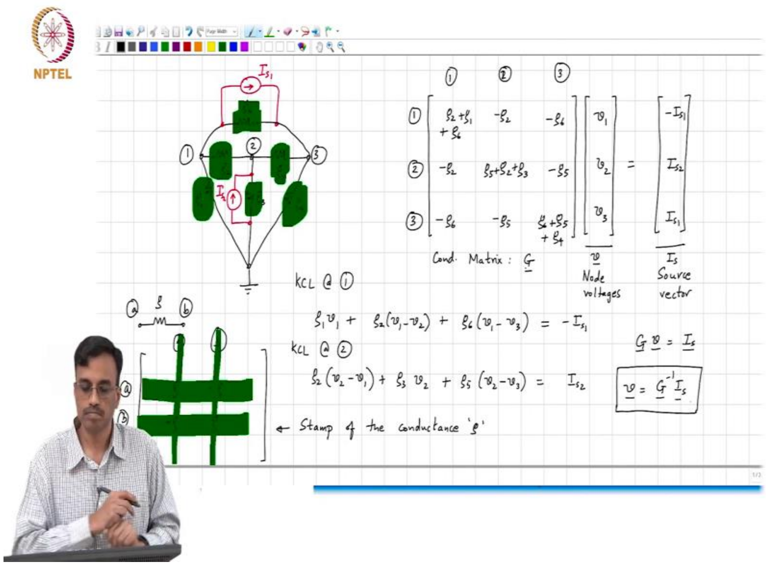
And this is immensely useful in practice, simply because you have, you are now in a position to be able to find multiple transfer functions to a single output, either voltage or current, in one shot without having to rerun the computation again and again.

And, as I was mentioning yesterday, this is the, this is routinely done in all circuits simulators, whether they are doing noise analysis or whether you are doing transfer function analysis. Good. So, having seen that, now let us go a move forward and look at or rather refresh our memories about ways of writing circuit equations. In your earlier classes you have likely seen, you have probably seen many ways of doing this, the nodal analysis is probably the first one that comes to mind.

Then there is mesh analysis, etc etc and for good reason nodal analysis, the basic framework of nodal analysis is very popular and the reason is that it is often involves more work to find measures and then, find the, I mean and then, once you found the mesh currents, then you will do some more operations to get the branch currents.

(Refer Slide Time: 03:00)





On the other hand nodal analysis gives you, in one shot gives you, all the node voltages and since the branch relationships are known, the branch currents can also be found in one shot. So, let us go through a quick refresher. So, let us start with networks which only consist of resistors or conductors and current sources.

Again I am going to take this ice cream cone network as an example. So, I am going to choose some reference, some node as a reference voltage or ground and we will have the conductance are g1, g2, g3, g4, g5 and g6 and, the nodes are numbered 1, 2, 3 and perhaps we have some sources, say, Is1 and we have another current source, exciting the network that is, Is2. How do you write the nodal equations? Well, it is very straightforward as you have seen in the past.

The nodal equations are simply expressing Kirchhoff's current law at a, at a node and once you see that it is not necessary to write equations, one by one carefully staring at the network. We have already seen this in your basic electric circuits and networks class. What are the unknowns that we are trying to solve for? The node voltage vector which is V1, V2 and V3 and if you write Kirchhoff's law at node 1, you can go through it the long form or recognize that you can do it element by element as you have seen in earlier classes.

So, since it is a long time that you saw this, let me just write Kirchhoff's law at node 1 to give you a quick refresher on why we do, what we do. So, if you write KCL at node 1, what do we see? It is g1 times v1 plus g2 times v1 minus v2 plus g6 times v1 minus v3 equals that is the sum total of all the current going out of the node and, that must be equal to the current going into the node and that must be equal to in this case, minus Is1 and likewise, KCL at node 2 gives us g2 times v2 minus v1 plus g3 times v2 plus g5 times v2 minus v3 equals? No, at node 2, what is the sum total of all the current flowing, of the source exciting, that is Is2 indeed. Thank you.

So, now you stare at this and then say well, so g2, this particular conductance appears only in KCL for node 1 and 2, correct and therefore, and how does it appear in this matrix, in the conductance matrix, this is the conductance matrix. You can arrange these equations in matrix form and, the reason it is called the conductance matrix is because every coefficient corresponds to the conductance of some element of the other.

And therefore, as far as g2 is concerned for instance, where do you think which rows will g2 appear in? Well, g2 goes between nodes 1 and 2. So, it must appear in the first two rows and g2 relates, which two node voltages? v1 and v2 and therefore it must also appear in the, it must also appear in the first two columns. So, between one and two basically, how and how does it appear in the first row? How does it appear?

Student: (())(08:26)

Professor: What? Can you fill up the entries of the matrix? Can you tell me which all of those entries in the matrix will correspond to g2, to g2. So, these are the three rows. These are the three columns corresponding to v1, v2 and v3. So, can you help me write the nodal equations? What will happen to g2?

Student: (())(08:58)

Professor: It will appear in the first two rows and in the first two columns and as you can see, it must be g2, minus g2, minus g2 and g2 and since g2 does not appear anywhere else, we can now forget about g2 we are done. What about g1?

Student: (())(09:27)

Professor: First row, first column and what about g3? Second row, second column, very good. What about g5?

Student: (())(09:47)

Professor: Between 2 and 3, so that is g5, minus g5, minus g5 and g5. That is done. What about g6? Between 1 and 3. So, that is basically that is done. Then, what about g4? Well, that only appears in third row, third column, correct and this must be equal to where does Is1 appear? It must appear between nodes 1 and 3.

So, in which of these, on the right hand side, which of those rows will it appear in? So, it is minus Is1, because that is the current flowing into node 1 and, this will be plus Is1 and likewise is Is2. So, once we have done this, we have done this, there is no need to go over the same procedure of writing the KCL at every node and then arranging them as a matrix. You can simply look at the network and write the equations and, what is the within quotes algorithm?

If you see a conductance between two nodes a and b and let us call this g, what comment can you make about the, the g matrix? So, we have the g matrix, the conductance matrix. It has the ath row, the bth row, has got the ath column and the bth column and how does this appear?

Student: (())(12:22)

Professor: How does this appear?

Student: (())(12:26)

Professor: It appears as g, minus g, minus g and correct and likewise if you have a conductance only going between a node and ground, it only appears on the, on the diagonal and this is what is called the stamp of conductance. All that it is saying is that, well, by looking at the number of nodes in the network, you already know the size of the conductance matrix. So, you can initiate or initialize a conductance matrix, matrix with that size.

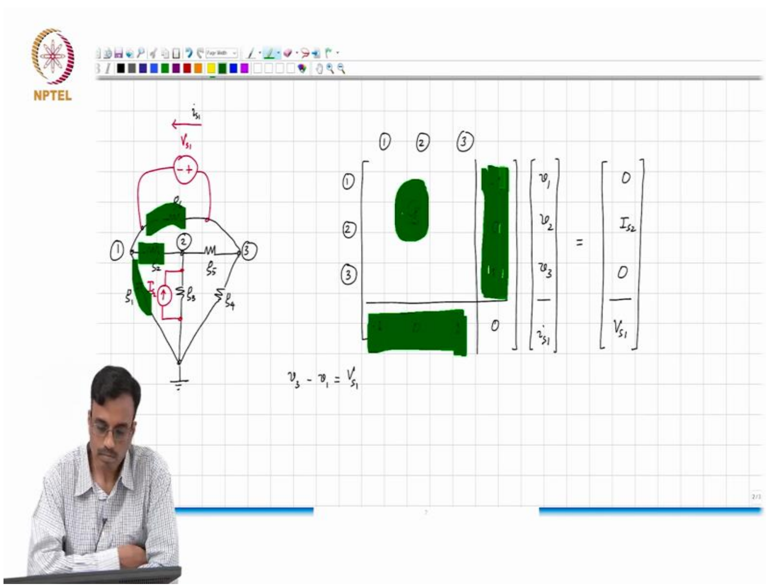
And then you go element by element and replace, I mean, when you go to a particular element in that list of elements you have in your network, you simply plop in that appropriates stamp of that element into the matrix and then you are done with that element you do not have to worry about it anymore and then you have a list of elements. So, you keep going through the list and then you know when you end the list, you are done.

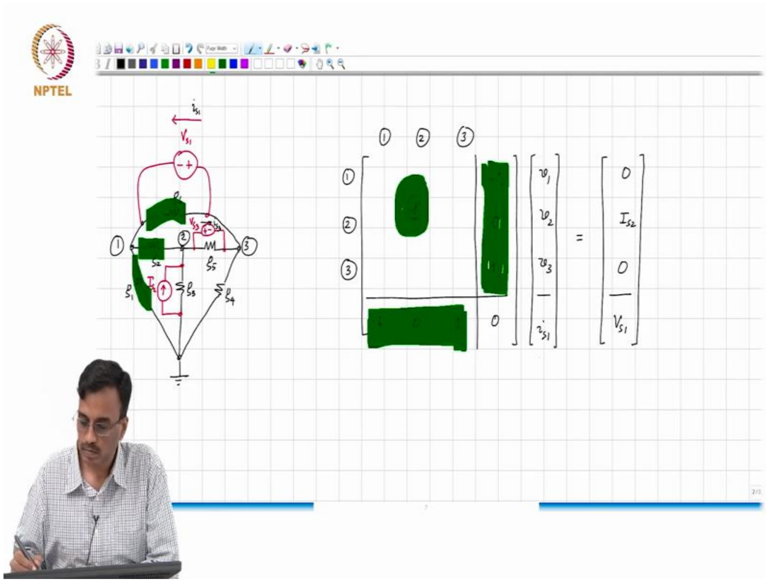
So, that basically, this, I mean, if you can describe it so clearly, it basically means that a computer can do it. There is nothing, deeply philosophical or this thing about it. It is just very straightforward algorithm. So, on the left side you have the conductors matrix, these are the node voltages which are the unknowns.

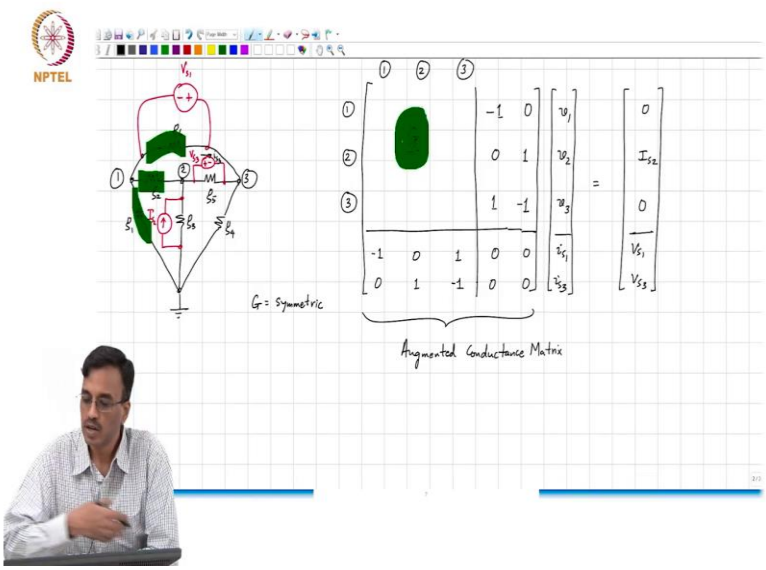
On the right side you have the source matrix, source vector I must say, where you have all the independent sources and once you basically find, so let us call this node voltage, this conductance matrix, let us call that g and the node voltage vector, let us call that v and this is, let us call that I sub s for the source voltages.

So, the nodal equations can be written as g v equals and it is straightforward to find v as simply g inverse times. I mean, this you must have seen. So, there is nothing new about this, I mean you must have seen this before multiple times. It is just a refresher. So, we have covered the four in principle, we have covered networks with conductances and current sources. Unfortunately, these are not the only elements that we encounter in practice.

(Refer Slide Time: 16:05)







Let us see what happens. For instance, if we have, I am going to just simply avoid clutter. I am going to remove one of these things and then replace them, replace it by a voltage source and we like to wonder, how, I mean of course, we know that if you write the equations, you can come up with some set of equations which eventually works and we will be able to solve the set of equations. But the question is, can we come up with a clean way of describing, how you go about writing the equation, so that it can be done on a computer, systematic fashion without having to have intervention every time.

So, the problem with a voltage source is that the current through it is, is unknown. So, well then we say well, let me put this in red, actually and we say well, we do not know the current through the voltage source. So, it is an unknown and we assign an unknown current Is1 through the voltage source and this is the voltage and this unknown has to be determined like, just like any other unknown.

So, earlier we had, earlier we had the g matrix, correct with conductances. I am going to not copy and paste the same thing all over again. So, I am going to call that the g matrix, which is the same as we saw earlier and again, this is node 1, this is node 2 and this is node3 and this is node 1, node 2 and node 3. We now have apart from, what are the unknowns now? What are all the unknowns? v1, v2, v3 are the unknowns. We have, also have an additional unknown, that is?

Student: Is1

Professor: Is1. So, let me just use a small is1 here simply because it is an unknown. is1 and so, if you write KCL for node 1 apart from all the conductances which we have already dealt with and gotten this matrix G, we now also have, if you write KCL at node 1, what do you get, what is the extra time you get? Remember, now there is an extra, I mean, apart from the currents flowing through these guys, which you have already been accounted for in this G matrix.

There is also an additional current which is flowing out, which is minus is1. So, where do you, so what do you think the, the equation will be in the first row? What, what should I do? What should I do? Where will we put the minus 1? Yes.

Student: (())(19:42)

Professor: First row, last column. Very good. So, we put minus 1 and likewise where else do you think you will get I mean, so the voltage Vs1 is connected between nodes 1 and 3. So, where do you think, what do you think will happen to, how will I account for is1 flowing out of node 3?

Student: (())(20:09)

Professor: Very good. So basically you can see we have a minus 1 there, we have plus 1 there and what comment can I make about the element there? That is 0 because is does not touch node 2. So, we have one more equation, we have one more unknown. So, we better have one more equation and what is that equation?

Student: (())(20:39)

Professor: Well, the extra equation is that v3 minus v1 equals Vs1. So, what should I do? This is the, how should I write that in this matrix? Fourth row, minus 1, 0, 1, 0, must be equal to. What should the right hand side look like? Right hand side remember must have only the independent sources. So, where are the, what are the independent sources? The first three rows will have only, will have only the independent current sources. So what is, what are the independent current sources? Flowing into node 1 is 0, Is2, 0 and what should be there?

Student: Vs2

Professor: Vs2. Vs1 and so again, this is the original conductance matrix augmented with something else to account for the voltage source and please note, I would like to draw your attention to the following. This is always going to be, what comment can you make about this part and this part?

If take that part this is going to be simply the transpose of that and I think it is a pretty apparent how that happens. So, the current flows between the current, the voltage source flows between the two nodes and the voltage force, source forces the difference between those two notes to be something. So, it stands to reason that this row is simply the transpose of this column.

Now, if you had any, if you had another source, what do you think you will do? Let me just make life a little more complicated by say for instance, Vs3 for instance. What do I do now? I mean, do I need to start all over again or can I just simply, kind of…

Student: (())(24:02)

Professor: Well, as straightforward as you all point out, very good. So, this current is, I am going to call that Is3 and what do I do? Well, I add another, there is an extra unknown. There is an extra unknown and that is, what is that saying is, that is between nodes 2 and 3 and how do I will, so what should I do? The new unknowns are basically, basically v1, v2, v3, Is1 and Is3 are the unknowns. Yes, so should I do, please?

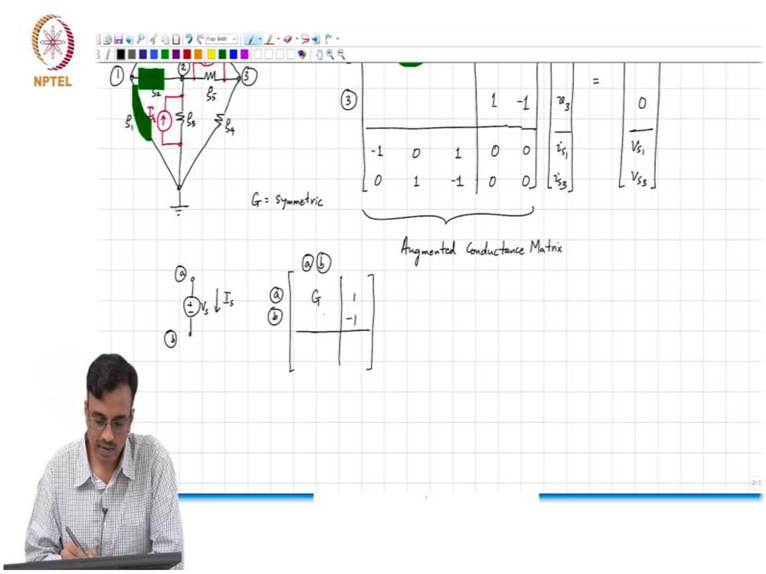
So, for every extra voltage source you argument this original conductance matrix by one extra row and one extra column and what happens to this the new row and new column that we need to add now, what is that?

Student: (())(25:56)

Professor: So, Vs3 goes between nodes 2 and 3. So, it should, so basically, it must be 1 and minus 1 and likewise what it should, this new row b? 0, 1, minus 1 and this is 0 and as usual on the right hand side, the first three rows are basically well 0, Is2 0, like we had before and you have the independent sources Vs1 and Vs2. So, therefore we are now in a position to cleanly, so this is basically the augmented conductance matrix. What comment can we make about G?

Can we make any comment about the structure of G? Well, it is symmetric. Because if you remove all the current sources and I mean, you remove all the current sources and remove, physically remove all the voltage sources, then all that you have is conductances which are obviously, if a conductance is going between two nodes it appears symmetrically across and you can therefore, solve this set of equations and then in one shot get all the voltages and node voltages and currents through the independent sources. So, this is telling us what to do, this analysis is basically telling us what to do when you have a voltage source.

(Refer Slide Time: 28:17)

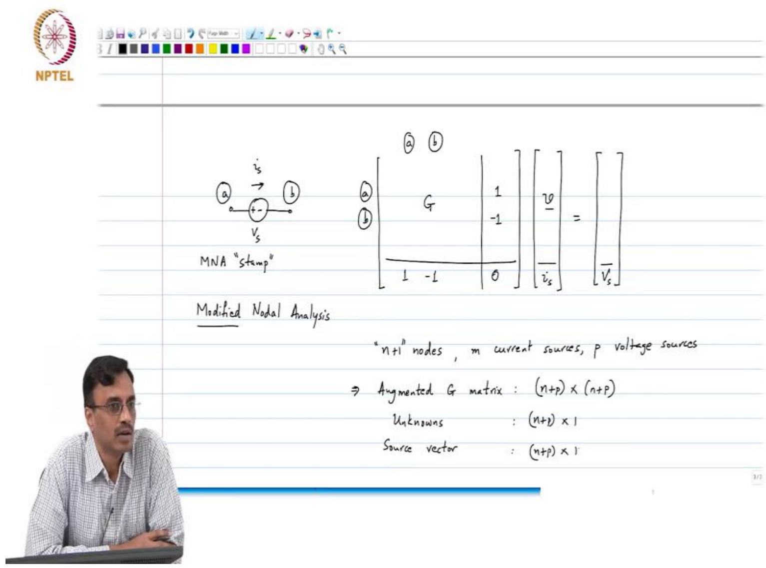


So, when you have a voltage source between nodes a and b and Vs, you add a new unknown is and what happens? You had the original G matrix and then what do you do? You augment it with, so you have the ath row, bth row, ath column, bth column. So, what should you do? What did we do when and remember you should be familiar with this stuff, so that if you look at a network and write down the equations without the matrix, without having to go through the whole…

Student: (())(29:27)

Professor: Very good. So basically, you must have 1 here and minus 1 here and in the ath row you put a 1, and the bth row you put a minus 1 and likewise.

(Refer Slide Time: 29:39)



So, that is the stamp of a voltage source, if you had nodes a and b is the following and that is, you had the ath row and the bth row as well as the ath row and the bth column and you add an extra. So, this is the Vs, this is the current extra variable Is and with the augmented conductance matrix, what you do is you put in a 1 and a minus 1, between the nodes a and b, in the ath and bth rows that corresponds to the accounting for the current that is flowing through the voltage source.

And, you put in 1 and minus 1 for in the ath and bth columns in the last row and this matrix is going to that part that entry is going to be 0. So, this is the node voltage vector. This is the original conductance matrix of the network stripped of all current sources, independent current sources and voltage sources and, then you have this new variable Is and this is equal to, what do you have? What should you have here? Here you have all the independent current sources and then now you have Vs.

So, the way this method of accounting for voltage sources or in other words, accounting for branches where the branch current cannot be expressed as a linear function of the branch voltage, is basically, this is an extension of the nodal analysis that we know already and this is what is called the modified nodal analysis and, as we have seen so far, if we have networks with conductances, current sources and voltage sources, I mean, the algorithm for writing these matrices is very clear.

You just make a, you describe the netlist and in a computer, you have a parser which takes a circuit diagram that your draw and generates a spice netlist and you keep going line by line in that netlist and you look at the element and you can simply plop its respective MNA stamp is what is called, into the matrix. So, if you have n nodes in the network or let us say, I deliberately choose n plus 1 nodes in the network and M current sources and p voltage sources. What comment can you make about the size of the augmented G matrix?

Student: (())(33:31)

Professor: Well, you have n plus 1 nodes in the network, one node is ground you have n other nodes, whose voltages you need to find. So, you will have n unknowns as far as the node voltages are concerned. You have p voltage sources. So, out of p voltage sources, the currents through the p voltages are all unknowns. So, the total number of unknowns is n plus p. So, the augmented G matrix is, what is the size of the matrix? n plus p cross n plus p. The unknowns form a, what vector?

How many unknowns do we have? n plus p. So, this must be n, n plus p cross 1 vector and the source vector, obviously better be also n plus p cross and where is the, the, the source vector? The lower three, I mean the lower p entries will be, will correspond to the strengths of the voltage sources.