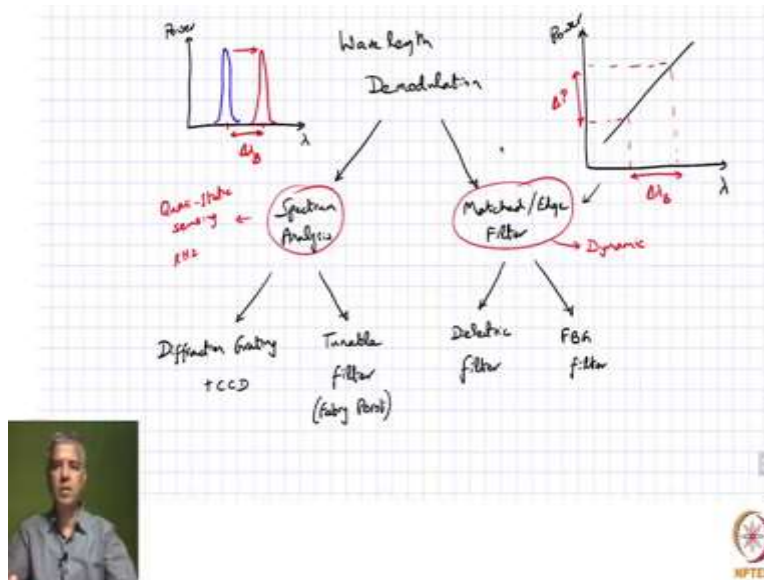


Optical Fiber Sensors
Professor Balaji Srinivasan
Department of Electrical Engineering
Indian Institute of Technology, Madras
Lecture – 7
Wavelength Modulated Sensors

(Refer Slide Time: 0:28)



So, we have been talking about wavelength demodulation techniques for fiber Bragg grating based sensors. And we looked at spectrum analysis type of techniques, which essentially serves the purpose of interrogating in a quasi-static way up to possibly kilo hertz, for a large number of gratings. But on the other hand, if you wanted to do sensing at greater than kilohertz, basically in the ultrasonic range, which is called true dynamic sensing, you would want to use possibly h filter.

(Refer Slide Time: 01:06)

Handwritten diagram illustrating an edge filter system. The diagram shows a Broadband Source connected to an Edge Filter, which is then connected to an Optical Receiver. A graph shows Transmission $T(\lambda)$ versus wavelength λ , with a sharp edge at λ_0 . A grating structure is shown with refractive indices n_1, n_2, n_3 and period d_x . Equations include $n_1 d_x = n_2 d_x = n_3 d_x$ and $d_x = \frac{\lambda}{4 n_x}$. A note states "Detection freq only limited by Receiver bandwidth". A small video inset of a man is in the bottom left, and the NPTEL logo is in the bottom right.

So, we looked at the case of the edge filter, the last lecture and we said it is quite good for high frequencies, but the only issue is that it may not be scalable to multiple gratings. And there is, of course, the issue of the filter itself being a custom component, maybe relatively an expensive solution. Now, that brings up the question, why not we use a fiber Bragg grating itself as an integrator? So, that is what we are going to be talking about in today's lecture.

(Refer Slide Time: 01:54)

Handwritten diagram titled "Matched Filter Interrogation". The diagram shows a Broadband Source connected to a coupler (with coefficients k and $1-k$) leading to an Optical Receiver. A graph shows input signal $s(\lambda)$ with bandwidth $\Delta\lambda_s$. The filter has a transmission $G_e(\lambda)$ with bandwidth $\Delta\lambda_e$. The output signal $G_o(\lambda)$ has bandwidth $\Delta\lambda_o$. A note asks "Why not use another matched FBG for interrogation?" and "Ultrasonic perturbation". The equation $P_o = k^2 (1-k)^2 \int s(\lambda) G_e(\lambda) G_o(\lambda) d\lambda$ is shown. A small video inset of a man is in the bottom left, and the NPTEL logo is in the bottom right.

Handwritten diagram illustrating a fiber Bragg grating (FBG) sensor system. A Broadband Source is connected to a circulator. One output of the circulator goes to an Edge Filter, which is connected to an Optical Receiver. The other output of the circulator goes to a fiber Bragg grating. The diagram includes a graph of transmission $T(\lambda)$ vs wavelength λ , showing a dip at the Bragg wavelength λ_B . Another graph shows the reflection spectrum $R(\lambda)$ vs λ , with a peak at λ_B . The grating is shown with a period Λ and refractive index n_0 . The Bragg wavelength is given as $\lambda_B = 2\Lambda n_0$. A note states "Detection freq only limited by Receiver bandwidth".



Matched Filter Interrogation

- Why not use another matched FBG for interrogation?

Handwritten diagram illustrating Matched Filter Interrogation. A Broadband Source is connected to a circulator. One output of the circulator goes to an Optical Receiver. The other output of the circulator goes to a fiber Bragg grating. The diagram includes three graphs: 1. $S(\lambda)$ vs λ , showing a broad source spectrum. 2. $G_1(\lambda)$ vs λ , showing the transmission spectrum of the interrogating FBG. 3. $G_2(\lambda)$ vs λ , showing the reflection spectrum of the sensor FBG. The overlap of G_1 and G_2 is labeled as "Optimal Interrogation". A note says "Optimal Interrogation". The power P_B is given by the equation:
$$P_B = k^2 (-k)^2 \int_{-\infty}^{\infty} S(\lambda) G_1(\lambda) G_2(\lambda) d\lambda$$



So, in general, this sort of approach is what is called matched filter interrogation. So, can we use a match to filter a filter that is matched to the sensor itself use that for interrogation. So, the key question that we are asking is, why not use another matched FBG for interrogation? And so what exactly are we talking about? Well, we said we will start with the broadband source which is launched into the fiber consisting of this fiber Bragg grating sensor.

But in terms of picking up the reflection, you can use a circulator or simply you can use a coupler also say 50 50 coupler where 50 percent of the light goes into the fiber Bragg grating, and whatever is reflected 50 percent of that actually comes back into this other fiber. And what

we are going to now incorporate is actually a filter that is match to the fiber Bragg grating. So, we know that the fiber Bragg grating is going to exhibit a reflection spectrum.

Let us, call this some function, G_s that look something like this. And let us say the Bragg wavelength is λ_B . Now, the question is, can we use a match filter here for the integration. So, we are talking about using another fiber Bragg grating here, which pretty much has the same sort of response. But suppose, if you are looking at it in terms of the transmission of the grating, then so, if you are looking at the transmission as a function of wavelength, how is that going to look?

Well that will be something like this it will be minimum, if you are looking at the transmitted component that is going to be minimum at wavelength, we can say the Bragg wavelength, we just to differentiate with respect to this, let us call it λ_{BR} . Now the idea is, it is going from a transmission of 1 to 0 over here, maybe you do not necessarily go to 0 over here, because there is less than 100 percent reflectivity let us say.

But nevertheless, you have this slope over here on either side of this λ_{BR} wavelength, which essentially constitutes a change in transmission as a function of wavelength. So, remember, what we set out to do, what we need is actually the transmission has to be varying with respect to the wavelength. And here we are actually constructing that we are taking a lot of pains and constructing that with multilayer dielectric coating, which is addressing these different wavelengths.

But instead of that, the question is, if you simply use a fiber Bragg grating another fiber Bragg grating with a match response to this grating, then that constitutes a slope over here and essentially what we are saying is now, your sensor grating is maybe, it is by us somewhere over here. So, when the sensor grating is moving around basically it is constituting change in the Bragg wavelength with respect to this other wavelength.

So, we are talking about the center of this, it starts with λ_B , but it is changing with respect to time. So, that is like moving across this slope over here. So, when you are looking at the transmitted intensity, the corresponding transmitted intensity can be, so as it is moving across

this you have a corresponding transmitted intensity like this. So, and this is of course, with respect to time.

So, this will sort of mimic whatever is your the change in Bragg wavelength as far as the sensor is concerned, which we know is because of some ultrasonic perturbation. So, any ultrasonic perturbation is going to constitute some pressure waves that are incident on this fiber Bragg grating. And so, that will essentially if you are looking at this fiber Bragg grating sitting on a substrate, if it is attached to a substrate, then any pressure changes here is going to constitute a strain in this longitudinal direction.

And because of that strain your Bragg wavelength here λ_B is going to change as a function of time. That is what we are mentioning over here and when that is actually interrogated, using a matched filter, now, you can actually get transmitted intensity whatever we see wavelength change here is going to be converted to a change in the intensity which you can potentially pick up using an optical receiver.

There is one issue in this though, that issue is essentially what, if you concatenate multiple gratings over here, and you try to look at this in this transmitted configuration, it will transmit not only across this spectrum, but if you have another grating, let us say at some other wavelength over here. See, λ_B' if you have one more peak over here, that peak will be somewhere out here it will be unrelated to this peak and because of that, you will see a lot of the light corresponding to the other grating just go into the optical receiver.

So, potentially we want to interrogate in not the transmitted configuration you can interrogate actually in the reflected configuration. So, I can put another coupler over here and whatever is reflected in this direction it is going to be picked up by this coupler. So, in the other arm of the coupler, and that could be sent to the optical receiver. So, instead of picking up the transmitted light, we are actually picking up the reflected light from the interrogator in which case it will reflect only any change in wavelength around λ_B .

If there are other gratings, reflection from other gratings or other background all that will go into the transmitted port of this match filter and they will be lost when whatever is reflected is purely

signal, which is within the bandwidth of λ_{BR} . So, that way, reflected configuration may be better than this transmitted configuration. So, that is a minor level of detail.

But overall, when we are using this coupler, what we are talking about is let us say k percent, or a fraction k of the light is transmitted in the same fiber, $1 - k$ is the fraction that is actually going into, this fiber is terminated over here within the coupler, but $1 - k$ part of the light is going to get into that other, terminated port. So, similarly, you can say, k is the part that goes into the grating, and $1 - k$ constitutes the fraction of light that is picked up by this other fiber, and it is going into this optical receiver.

So, when we look at the power detected, let us call that P_D . So, the power detected now is going to be what it is going to be an integral of the spectrum of this broadband source multiplied by the spectrum of this sensor grating multiplied by the spectrum of this integrated rating. And of course, it is also going to be determined by, these factors K and $1 - k$ and so on.

So, for example, the reflected component is going to have k times if you call that broadband source spectra as we just draw this over here, if you call the broad bands for source spectra of S_A of λ that is a source spectrum that can be modeled as somewhat like a Gaussian shape and that has let us say, a center around λ_{naught} and it is got a RMS width of let us say $\Delta\lambda_{naught}$.

Similarly, this also, the sensor grating response or reflectivity can be modeled as a Gaussian and once again this can have say an RMS with corresponding to $\Delta\lambda_{BS}$. And similarly, when we talk about this grating, that grating also the integrated grating, we can call it $\Delta\lambda_{BR}$, that is the RMS width of that integrated grating that can also be quantified as it can be characterized as a Gaussian function.

So, P_D now is going to be given by k times $1 - k$ times k times $1 - k$. So, you are going to have k^2 , $1 - k^2$. So, it is going to go through four passes within this two couplers that corresponds to this fraction, multiplied by integral of itself λ , that is the source spectrum, multiplied by this we are calling as G_S of λ . A Gaussian, that is representing the sensor grating.

And let us actually call this GR of lambda that is corresponding to the Gaussian representing the integrated weighting. Mind you this one, we are actually looking at, I have mentioned it as the transmitted component, but in reality, we are, I mean, what we later talked about was, this is actually happening in the reflector component.

So, reflected configuration, so I can just remove that, remove this also maybe, and I can basically replace that in the reflected configuration, it is, going to be something like this, where this now is your delta lambda BR that is the RMS width of the grating, and we are moving lambda BS with respect to lambda BR which means the reflected power is going to get modulated according to this slope over here.

So, coming back to this expression. So, we are saying we are integrating all of this with respect to lambda, and that, we can say, in general, it could be the limits could be minus infinity to plus infinity, but we know that all of this makes sense only within the source spectrum. So, if you define a starting wavelength and stop wavelength for the source spectrum, you can just replace these limits with those values.

So, this is actually the power that we are detecting at the optical receiver. So, let us now go ahead and try to express each one of these by a Gaussian. And let us see how this integral is going to work out.

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$$P_s = k^2 (-i)^2 \int_{-\infty}^{\infty} S(\lambda) G_1(\lambda) G_2(\lambda) d\lambda$$

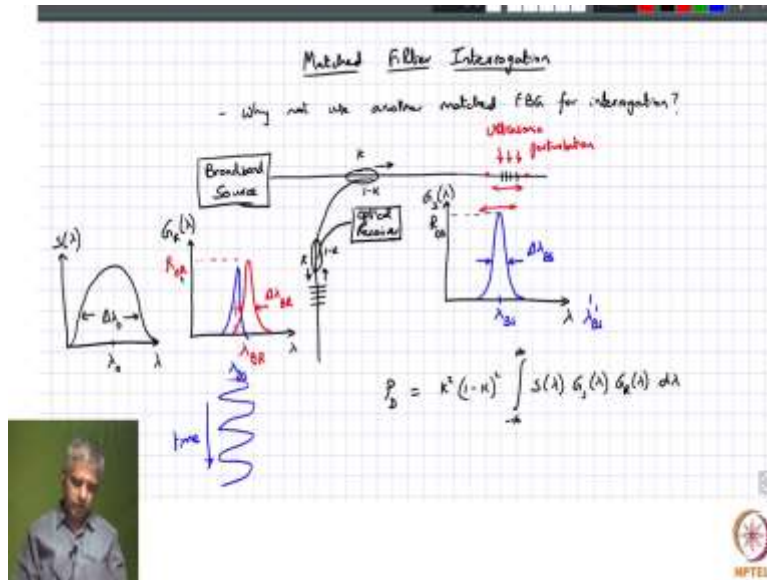
where $S(\lambda) = \frac{P_s}{\Delta\lambda_s} \exp\left[-4.8kz \left(\frac{\lambda - \lambda_s}{\Delta\lambda_s}\right)^2\right]$

$G_1(\lambda) = P_{os} \exp\left[-4.8kz \left(\frac{\lambda - \lambda_{2c}}{\Delta\lambda_B}\right)^2\right]$ → Matched filter

$G_2(\lambda) = P_{or} \exp\left[-4.8kz \left(\frac{\lambda - \lambda_{2c}}{\Delta\lambda_B}\right)^2\right]$ → $\Delta\lambda_{B1} = \Delta\lambda_{2c} = \Delta\lambda_s$

$\Delta\lambda_s \gg \Delta\lambda_B$
 $80-100$
 nm
 $-0.5nm$



So first, let us say, so in maybe I can just copy this here. So, this expression, where SA of lambda is now going to be given by let us say, P naught is the optical power corresponding to this source but that is actually spread over RMS width of delta lambda naught. So, you can say this is normalized. If you are looking at the power spectral density, that is what this is of lambda is, so that has to be normalized by delta lambda naught.

And then you have the Gaussian function, which is given by exponential of minus 4 ln of 2 lambda minus lambda naught divided by delta lambda, naught the whole square. That actually represents the source spectrum where four times ln of 2 is just the normalization constant. And similarly, if you are looking at GS of lambda, that is going to be, modulated around this value, the peak reflectivity Let us call this R naught for the sensor.

So, that value is R naught for the sensor and similarly this value for the interrogator we can call this R naught for R. So, coming back to this so, this is GS of lambda this is corresponding to R naught S exponential of minus 4 times ln 2 lambda minus lambda BS now, it is all centered around lambda BS divided by delta lambda BS. So, that is actually the source the grating response the sensor grading response.

And similarly, you can say GR of lambda equal to the peak value is reflectivity R naught R exponential of minus 4 ln 2 lambda minus lambda BR divided by delta lambda BR the whole square. So, we are doing this integral where we are multiplying all these functions and when we

multiply these functions, we understand that first of all we are using a match filter. So, these two correspond to a matched filter in which case you can just you recognize that delta lambda BS equal to delta lambda BR.

And that can be just in common written as delta lambda B. And what can you say about delta lambda B with respect to delta lambda naught delta lambda naught is typically far greater than delta lambda B that is typically the case you have a broadband source for example, source that has a spectral width of say 80 or 100 nanometers, and then when you are talking about delta lambda B corresponding to a grating, that is actually over a fraction of a nanometer.

So, clearly this is 80 to 100 nanometers and this is typically about 0.5 nanometers. So, this is actually a valid condition. So, in that sort of scenario, the things can be simplified further. So, you can you can simplify it, maybe I can go to the next page and show that.

(Refer Slide Time: 23:27)

$$P_D = A \cdot \alpha(\lambda) \cdot \beta(\lambda)$$

$$\text{where } A = P_0 \cdot \frac{(1-k)^2 R_{31} R_{32}}{\sqrt{2}} \cdot \frac{\Delta \lambda_B}{\Delta \lambda_0}$$

$$\alpha(\lambda) = \exp \left[-2 \ln 2 \left(\frac{\lambda_B - \lambda_0}{\Delta \lambda_B} \right)^2 \right]$$

$$\beta(\lambda) = \exp \left[-4 \ln 2 \left(\frac{\lambda_B - \lambda_0}{\Delta \lambda_B} \right)^2 \right]$$

$$P_D = k^2 (1-k)^2 \int_{-\infty}^{\infty} S(\lambda) G_1(\lambda) G_2(\lambda) d\lambda$$

where $S(\lambda) = \frac{P_0}{\Delta\lambda_0} \exp\left[-4k^2 \left(\frac{\lambda - \lambda_0}{\Delta\lambda_0}\right)^2\right]$ $\Delta\lambda_0 \gg \Delta\lambda_B$
 $B_0 = 100$ nm $\Delta\lambda_0 = \Delta\lambda_{2e} = \Delta\lambda_B$

$$G_1(\lambda) = R_{02} \exp\left[-4R_{02} \left(\frac{\lambda - \lambda_{2e}}{\Delta\lambda_{2e}}\right)^2\right] \rightarrow \text{Matched filter}$$

$$G_2(\lambda) = R_{0R} \exp\left[-4R_{0R} \left(\frac{\lambda - \lambda_{2R}}{\Delta\lambda_{2R}}\right)^2\right]$$


Matched Filter Interrogation

- Why not use another matched filter for interrogation?

$$P_D = k^2 (1-k)^2 \int_{-\infty}^{\infty} S(\lambda) G_1(\lambda) G_2(\lambda) d\lambda$$


So, we can basically say PD now can be expressed in terms of some constant multiplied by alpha function alpha which is actually a function of lambda multiplied by beta which is a function of lambda and these actually where A is actually a constant that accounts for all the constants terms that we discussed here $k^2 (1-k)^2 P_0 \Delta\lambda_0 R_{02}$. So, all of those we can actually represent it with respect to as this constant here and alpha corresponds to essentially lambda the multiplication of these 2 Gaussian functions.

And beta would correspond to multiplication of basically whatever is left which is corresponding to the your source wavelength source wavelength actually falling on this GS. So, essentially what we are talking about is with respect to this is this falling on the grating that is S of lambda on GS

of λ . That is going to be your β and whatever is coming back here the reflected component overlapping with GR of λ , that is what we are calling we are going to call us β .

If we do that, we find that where A is given by all these constants that you have so, P_{naught} multiplied by k^2 $1 - k^2$ the whole square and then you have ROS, ROR and then what we find this when we actually do this integral of the source falling on this on the grating. So, only section of this is going to be carved out. So, when we look at this only small section of this is going to be carved out and picked up as your reflected component.

And that reflected component is going to have something that is given by total power that is given by P_{naught} which is the power with which your sources started multiplied by R_{naught} S multiplied by the effective width of this the effective width of this can be written as $\Delta \lambda_{BS}$ over $\sqrt{2}$. So, you can basically say multiplied by Δ , $\Delta \lambda_{BS}$ divided by $\sqrt{2}$ and then we had another $\Delta \lambda_{naught}$ in the denominator so, I can write this like this.

So, that actually constitutes all the constants corresponding to all the constants that are not actually dependent on the variation with respect to λ . But now we will do the integrals and if we look at α that when you do that integral that is going to result in exponential of minus 2 times \ln of 2 because you are multiplying two exponential corresponding to your sensor as well as your interrogator. So, $\lambda_{BS} - \lambda_{naught}$.

So, this is actually the relative to positions of the sensor grating with respect to the interrogator grating divided by $\Delta \lambda_{BS}$ the whole square that is that corresponds to the overlap of the integral between the sensor grating and the interrogator grating. And similarly if you look at β that is actually the overlap of the source spectrum on the sensor grating. So, that is going to given that will be given by exponential of minus four \ln 2 and you have λ_{BS} with respect to λ_{naught} .

Because as λ_{BS} is away from λ_{naught} as λ_{BS} is away from λ_{naught} it is going to start falling. That is what this is denoting and that is with respect to $\Delta \lambda_{naught}$ it is varying with respect to this overall spectral width of the source. So, you get these so, essentially simplified this integral constituting this multiple Gaussian terms. And now, we can

actually go on to look at the differential of this with respect to λ because what we are interested in is the sensitivity of the minimum.

So, let us go back and look at it. So, what we are interested in the optical receiver is what is the smallest change in λ BS that he can pick up. So, essentially we want to look at what is the smallest change in the sensor grating Bragg wavelength that we can pick up. And of course, that is representing the smallest pressure change corresponding to the ultrasonic wave or the strength of the ultrasonic wave that we can pick up. So, what is that going to be limited by?

Well, we are essentially doing a power measurement at the at the end of this interrogation. So, like all power measurements, when you do when you pick up certain optical power the receiver, that is going to be in a corrupted by noise at the receiver. So, you still have to deal with noise. So, previously I said wavelength modulated sensors are robust to external noise sources. So, whatever I was talking about is that the signal is actually, when it is wavelength modulated the perturbation that wavelength modulation carries all the way through to this point.

But at the receiver, so all this broadband source, this coupler, and everything will actually be kept within a interrogator box. But within that interrogator box, you do the D modulation, and once it is demodulated, it is actually some power variation that you are picking up. So, that is going to be corrupted by the noise, what noise? You could have shot noise you could have thermal noise, so you could have noise corresponding to the ADCs.

And all that, they are all the things that we had previously discussed. So, you if you were trying to find the minimum value of change in the Bragg wavelength corresponding to the sensor, you will have to look at the minimum optical power that you can detect in a reliable manner, and that is going to be limited by your noise. So, that is going to be limited by the noise of the receiver. So, now we will go ahead and try to define the noise in the receiver, and then we will through that, we will define what is the minimum detectable wavelength change.



(Refer Slide Time: 33:10)

Min detectable pow, $J_{\lambda_{21}} = \frac{d\lambda_{21}}{dP_2} \cdot \delta P_2$

$= \frac{1}{\left(\frac{dP_2}{d\lambda_{21}}\right)} \cdot \delta P_2$

slope of interrogator response $\rightarrow \sqrt{\Delta H_{\text{new}}}$ Receiver noise $\frac{1}{2} (2qM^2R_s + \frac{4q^2I}{f})$

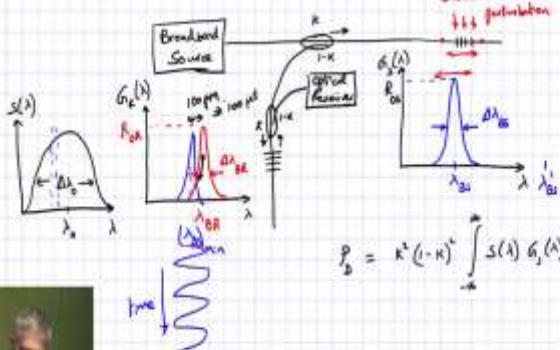
$$\frac{dP_2}{d\lambda_{21}} = A \frac{d}{d\lambda_{21}} (K\beta) = A\beta \frac{d\alpha}{d\lambda_{21}} \text{ since } \frac{dP}{d\lambda_{21}} \ll \frac{d\lambda}{d\lambda_{21}} \left(\frac{2}{\Delta\lambda} \right)$$



$$= A\beta \frac{4\pi n^2 (\lambda_{21} - \lambda_{20})}{\Delta\lambda_0^2} \cdot \alpha$$



Matched Filter Interrogation

- Why not use another matched FBS for interrogation?

Ultrasonic perturbation



$$P_D = K^2 (1-K)^2 \int_{-\infty}^{\infty} S(\lambda) G_1(\lambda) G_2(\lambda) d\lambda$$



The whiteboard contains the following equations:

$$P_D = A \cdot \alpha(\lambda) \cdot \beta(\lambda)$$

$$\text{where } A = P_0 \frac{R^2 (1-R)^2 R_{35} R_{0R}}{\sqrt{2}} \frac{\Delta \lambda_0}{\Delta \lambda_0}$$

$$\alpha(\lambda) = \exp \left[-2 \ln 2 \left(\frac{\lambda_{00} - \lambda_{0R}}{\Delta \lambda_0} \right)^2 \right]$$

$$\beta(\lambda) = \exp \left[-4 \ln 2 \left(\frac{\lambda_{00} - \lambda_{0R}}{\Delta \lambda_0} \right)^2 \right]$$

In the bottom left corner, there is a small video inset of a man with grey hair wearing a light blue shirt. In the bottom right corner, there is a logo for NPTEL.

So, now, we will look at the minimum detectable limit. So, when we look at the minimum detectable limit, you will say this is actually the minimum wavelength change that we can detect, that we can mathematically write as $d\lambda_{BS}$ divided by $D \cdot PD$ multiplied by the noise that you have in terms of the detection. And what is this term represent well, that you can write is 1 over $D \cdot PD$ over $D \lambda_{BS}$.

Well, what does this correspond to? This is actually the slope of the interrogator response. Essentially, if we go back and look at this picture that corresponds to this, this slope over here. That slope actually is what is represented. So, it is inversely proportional. So, larger the slope essentially smaller will be the change in wavelength that you can detect in a reliable manner.

But, of course, 1 limitation as far as this integration technique is concerned is your detection is going to be limited only to the slope and specifically the linear part of this slope because what you want is a linear response. So, essentially this part can be as small as 100 pico meter which means that you may be able to only pick up something in the order of 100 micro strain. So, you will be limited in terms of the strength of the signal that you can pick up.

So, that is one of the limitation as far as this technique is concerned, but 100 micro strains is still a reasonable value reasonable limit to have especially when you are considering perturbations in the order of 100 kilo hertz and above. So, that way, this may be still a viable technique, but

anyway coming back to this, this multiplied by $d\alpha$ PD, which corresponds to the receiver noise. So, that corresponds to the receiver noise.

Now, the we know that the receiver noise will. So, what is that we quantified before the noise variance corresponds to 1 over the responsivity square multiplied by the shot noise component which is given by let us say you are using A PD, because he wants to do a highly sensitive detection. But you do not have a lot of signal power that is incident on the photodiode because, by the time you go through the 2 reflections, you have power levels in the order of nano watts, that are falling on the detector.

So, you are essentially limited by this dark current noise at the receiver plus, you have your shot noise $4 k BT$ over RF if we say that is the gain of your TI stage plus you have the op-amp, let us say you use a fed type op-amp, so you know your current noise density is actually the dominant component. So, you might have something related to that. So, you have all those noise components that are present as far as the receiver is concerned.

And over here we are looking at the slope. So, it is also limited by the slope of the interrogator response. So, if you put all of this together, so, what we are interested in quantifying is, let us say we quantify the slope $d PD$ over $d \lambda_{BS}$. That is essentially differential of this, when you do that, A is a constant it falls off and then when you differentiate alpha and beta, especially when you differentiate beta, we find that $d \beta$ or $d \lambda_{BS}$ is actually a very slow function and that is because this is relatively flat compared to this function over here.

So, you can say that $d \beta$ or $d \lambda_{BS}$ is going to be very small compared to $D \alpha$ over $d \beta S$. So, if you come back here, so, you can basically say, in general you can say this is A constant d over $d \lambda_{BS}$ of alpha times beta, but you can approximate this as A times beta multiplied by $d \alpha$ over $d \lambda_{BS}$. Since, we know that $d \beta$ or $d \lambda_{BS}$ is much smaller compared to $d \alpha$ over $d \lambda_{BS}$.

So, when you do differentiate by parts, you have alpha plus $d \beta$ by $d \lambda_{BS}$ term. but since $d \beta$ or $d \lambda_{BS}$ is far less than that $d \alpha$ and with respect to $d \lambda_{BS}$ we are neglecting that term. So, you just have this and this can be now, when you differentiate a

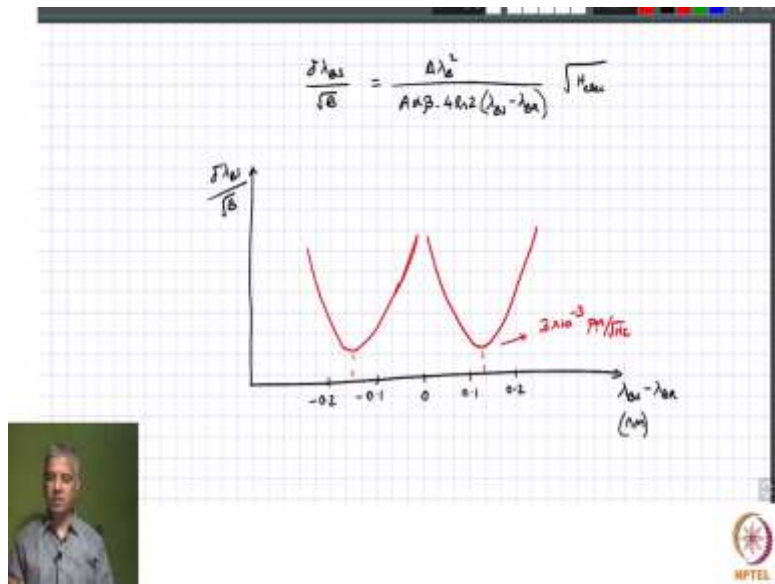
Gaussian function, you will essentially get another Gaussian function but some of those constant terms for out.

So, if you do this differential, you will find that you get A times beta multiplied by 4 ln 2 lambda BS with respect to lambda BR the difference between that divided by delta lambda square. This falls out, and you get alpha as a as the differentiate once again, that corresponds to the Gaussian function itself. So, if you put all of this together, we know that your receiver noise is going to be given by root of B times all the electronic noise terms noise spectral density corresponding to the electronic noise.

Which is what we are quantified over here and where B is corresponding to the bandwidth of your receiver. So, your bandwidth would be corresponding to the frequency range of interest. Let us say you are primarily interested in picking up acoustic waves from 10 kilo hertz to 1 megahertz, your bandwidth will be, would be a passband essentially for the receiver over that frequencies, you will cut off all the 1 over F noise in the lower wavelength side and then anything above 1 megahertz you will cut off when the higher frequencies.

So, lower frequency is 1 over F and higher frequency you have all the op-amp noise and all that you will cut off using bandpass design as far as your receiver responses concern.



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Matched Filter Interrogation

- Why not use another matched filter for interrogation?

$$P_B = k^2 (1-k)^2 \int S(\lambda) G_T(\lambda) G_R(\lambda) d\lambda$$

Anyway if we plug this back into this expression with all of this together, so what you can, what you get is the minimum detectable limit. Let us say we define that with respect to noise spectral density. So, you say if divided by the root of the bandwidth is now given by inverse of that slope function. So, you have $\Delta \lambda B^2$ multiplied by $A \alpha \beta 4 \ln 2$, $\lambda_{BS} - \lambda_{BR}$ multiplied by root of the noise spectral density corresponding to the electronics of the receiver.

So, the key thing to understand is this is inversely proportional to the difference between the two, and you can say that when $\lambda_{BS} = \lambda_{BR}$. This entire function goes to infinity so that means you are not going to be able to do a very sensitive deduction when $\lambda_{BR} = \lambda_{BS}$. So, let us actually look at that in little more closely.

So, let us plot $\lambda_{BS} - \lambda_{BR}$ as a function of that, this minimum detectable wavelength the noise density corresponding to that, if we do that we say this is 0 this is where $\lambda_{BS} = \lambda_{BR}$ and let us say this is goes as 0.1 minus 0.1 minus 0.2 and so on and this is 0.1 this is 0.2. So, all this is in nano meter. So, if you plot this, what you are likely to find is something like this, it is going to go down to a minimum, and then it is going to be asymptotic with respect to 0.

And that that function goes to infinity mathematically, and similarly this it will be symmetric around the so you will see this is actually going to some minimum at some value on either side.

So, what does that value correspond to? Well, if you look at this that you know, you get the best response or the largest slope corresponding to these points in the middle of this slope on either side. So, that is where you get the largest response.

So, that is what you will that is where you will get the minimum detectable limit also. So, this value for a given grating, we are taken some grading with they have WHM of 0.2 nano meter or something like that. So, that is why we are getting this at 0.12, it really depends on the RMS spectral width of your grating as to where this peak happens. But, this mean where this minimum point happens, but this clearly depends on like I said the middle of that slope.

And this could be as small as for a given example, it could be as small as 3 into 10 power minus 3 pico meter per root hertz. So, what does that correspond to once again, 1 pico meter round about is 1 micro strain. So, we are talking about picking up something the order of nano strain per root hertz.

So, if it is a slow variation, then it is a nano strain. But then if it is something at 1 megahertz, let us say then you have to multiply that by 10 power 3. So, that because you are accumulating noise over that bandwidth, in that case, it is going to be something the order of pico meter. So, that is the minimum detectable Bragg wavelength that you can pick up.

(Refer Slide Time: 46:32)

Analysis of the reflective-matched fiber Bragg grating sensing interrogation scheme

A. B. Lobo Ribeiro, L. A. Ferreira, J. L. Santos, and D. A. Jackson

A technique for the demodulation of fiber Bragg grating (FBG) sensors based on the use of a second wavelength-matched FBG receiver to track wavelength shifts from the FBG sensor is analyzed, particularly regarding its sensitivity as determined by primary noise sources. Numerical and experimental results show that there is an optimum Bragg wavelength difference between the two FBGs that maximizes the sensitivity for this demodulation technique. © 1997 Optical Society of America

Key words: Fiber Bragg gratings, optical fiber sensors, strain sensors.

roduction fiber Fabry-Pérot filters, wavelength-division fiber



some cases an appropriate sensor system may be determined more by the demodulation unit than by the sensor head itself. Most demodulation techniques developed to date rely on optical filtering methods, such as bulk optical edge filters,⁵ scanning

technique is investigated. By consideration of primary noise sources, it is shown that there is an optimum wavelength tuning difference between the two FBG's that maximizes the sensitivity of this demodulation scheme for measured recovery.

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2. System Power Budget

The basic configuration of the sensor-receiver grating pair scheme⁶ is shown in Fig. 1. Light from a broadband source (BBS) is transferred to the sensing grating (FBG₁) by means of a directional coupler with nominal coupling ratio (β), and the light reflected from the FBG₁ then propagates back through the fiber network to the receiving grating (FBG₂), which will perform a matched filter function.

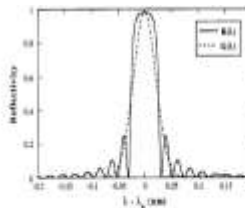
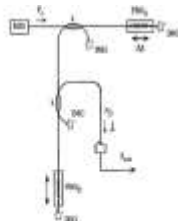
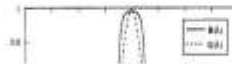


Fig. 1. Diagram of the sensor-receiver grating pair scheme (BBS, broadband source; FBG, fiber Bragg grating).

Fig. 2. Spectral dependence of the FBG reflectivity, considering the exact model, $R(\lambda)$, and the Gaussian approximation, $G(\lambda)$, for $\lambda_d = 100 \text{ nm}$.

Considering a BBS, such as an edge-emitting LED or a superluminescent diode, with a smooth spectral profile, its spectrum can be modeled as a Gaussian distribution of wavelengths with a spectral full width at half-maximum (FWHM) of $\Delta\lambda_0$ and a center wavelength of λ_0 . Hence the Gaussian model gives

$$S(\lambda) = I_{\text{peak}} \exp \left[-4 \ln 2 \left(\frac{\lambda - \lambda_0}{\Delta\lambda_0} \right)^2 \right] \quad (1)$$

λ is the wavelength in vacuum and I_{peak} is the power. Here $I_{\text{peak}} = (P_0 / \Delta\lambda_0) (4 \ln 2 / \pi)^{1/2}$.

the index with the form^{13,14}

$$n(z) = n_d + \Delta n + n_d + \Delta n_0 \cos \left(\frac{2\pi z}{\Lambda} \right) \quad (2)$$

Ω is real and is given by

$$\Omega = \frac{n \Delta n_0}{\lambda} \chi \quad (4)$$

where Δn_0 is the refractive index modulation depth of the grating and χ is the fraction of the integrated



$$P_D = A^2(1 - A^2)N_{\text{dB}} \frac{1}{\sqrt{4 \ln 2}} \times \left[\frac{\Delta\lambda_{\text{FBG}} \Delta\lambda_{\text{BS}}}{(\Delta\lambda_{\text{BS}}^2 + \Delta\lambda_{\text{FBG}}^2)^{3/2}} \right] \times \exp\left[-4 \ln 2 \frac{(\lambda_{\text{BS}} - \lambda_{\text{FBG}})^2}{\Delta\lambda_{\text{BS}}^2 + \Delta\lambda_{\text{FBG}}^2}\right] \quad (8)$$

For simplicity, it is assumed that both FBG's have the same spectral width, i.e., $\Delta\lambda_{\text{BS}} = \Delta\lambda_{\text{FBG}} = \Delta\lambda$. Therefore, also considering relation (1), the output power from the system under study is

$$P_D = \frac{k^2(1 - k^2)^2}{2} P_0 R_{\text{FBG}} R_{\text{BS}} \frac{\Delta\lambda}{\Delta\lambda_0} \beta(\lambda_{\text{BS}}) \alpha(\lambda_{\text{BS}}) \quad (9)$$

with

$$\beta(\lambda_{\text{BS}}) = \exp\left[-4 \ln 2 \frac{(\lambda_{\text{BS}} - \lambda_0)^2}{\Delta\lambda_0^2}\right] \quad (10)$$

$$\alpha(\lambda_{\text{BS}}) = \exp\left[-2 \ln 2 \frac{(\lambda_{\text{BS}} - \lambda_{\text{FBG}})^2}{\Delta\lambda_0^2}\right] \quad (11)$$

where P_0 is the total power injected into the fiber by the laser source.

As the FBG scans in wavelength the light reflected by the FBG, the optical power at the output system (P_D) will change following an exponen-

Gaussian approximation for the fiber Bragg grating reflectivity profile.

3. Sensitivity Analysis

In this section, and considering the primary noise sources (shot noise and electronic noise), the minimum Bragg wavelength shift in the sensing grating detected by the system is evaluated.

A variation of $\Delta\lambda_{\text{BS}}$ in the Bragg wavelength of the FBG sensor will produce a variation in the optical power arriving at the detector (P_D) such that



$$\Delta P_D = \frac{1}{(dP_D/d\lambda_{\text{BS}})} \Delta P_D \quad (12)$$

where $dP_D/d\lambda_{\text{BS}}$ is the derivative of Eq. (9) with relation to λ_{BS} and is given by

$$\frac{dP_D}{d\lambda_{\text{BS}}} = \left(\frac{k^2(1 - k^2)^2 P_0 R_{\text{FBG}} R_{\text{BS}} \Delta\lambda}{2 \Delta\lambda_0} \right) \frac{d}{d\lambda_{\text{BS}}} \left(\beta \alpha \right) \quad (13)$$

When $\Delta\lambda_{\text{BS}} \gg \Delta\lambda_0$ and $(d\alpha/d\lambda_{\text{BS}}) \gg (d\beta/d\lambda_{\text{BS}})$, which is normally the case, it follows that

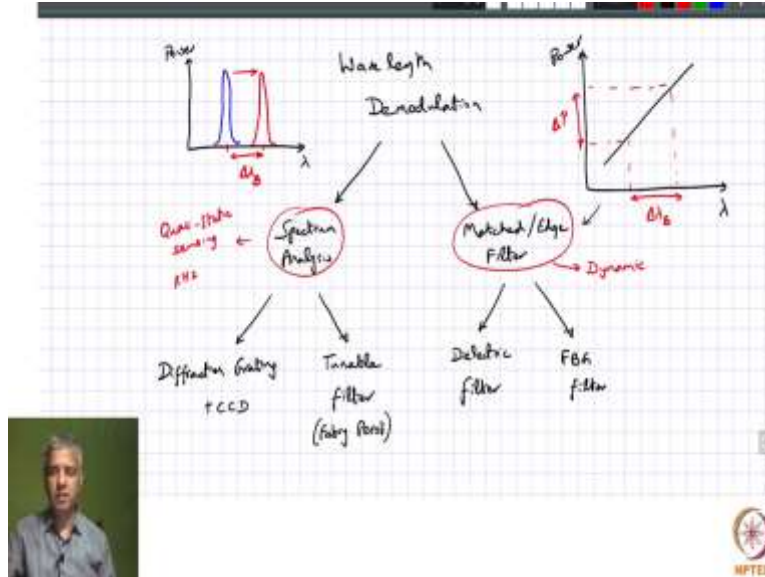
$$\frac{dP_D}{d\lambda_{\text{BS}}} \approx A \frac{d\alpha}{d\lambda_{\text{BS}}} \quad (14)$$

And just to acknowledge, all this is coming from this paper from Ribeiro at all, so, this is actually a paper in applied optics in 1997, where they talk about this reflected configuration and, represent these functions as a Gaussian.

And based on that, they look at the sensitivity analysis. And that is what we presented and this is the final analysis that we have picked up where they talk about the complete noise versus just the short noise limit and so on as a function of lambda BS minus lambda BR. But like I talked about it, we get a similar type of response, becomes the minimum detectable limit is determined by the slope of that response and the noise in the receiver. So, that actually completes our discussion related to fiber Bragg grating sensors.

(Refer Slide Time: 47:43)



So, if we go back here, just to summarize, when we were discussing all these things, we already talked about, when you have a large number of sensor a large number of locations where you need to send strain or temperature, which is actually varying at kilo hertz rates or less, then you go to one of these spectrum analysis techniques.

But if you want to really go to dynamic sensing, you want to either go for a dielectric filter or an FBG filter. FBG filter may be a cheaper solution compared to a dielectric filter. So, that may be attractive for certain dynamic applications. But, of course, the limitation is that this is not highly scalable, so, you are not going to be able to go to dozens of sensors as you did in the other case that for quasi static sensing. So, that completes our discussion related to fiber Bragg grating based sensors.