




**Optical Fiber Sensors**  
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**Lecture 31**  
**Phase modulated sensors - 11**

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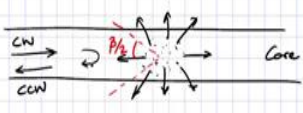


I Fog Limitations

- ① Polarization → Environmentally-induced polarization changes will affect CW/CCW phase ⇒ PM fiber
- ② Coherent Rayleigh Backscattering → ARB along the length of fiber
- ③ Faraday effect → due to external magnetic field, polarization changes (non-reciprocal)
- ④ Shot Noise → due to random arrival of photons @ receiver

Coherent Rayleigh Backscattering (CRB)




Fraction of light collected  $\approx \beta/4$

Ratio of scattered power w.r.t incident power  $= \frac{P_s}{P_i} = \frac{\beta^2}{4} \cdot \alpha_s L$        $\beta \rightarrow 0.1 \text{ rad}$   
 $\alpha_s \rightarrow 0.5 \text{ dB/km}$   
 $L \rightarrow 200 \text{ m}$

Max. phase noise  $\Delta\phi_{\text{max}}^{\text{CRB}} = 2 \sqrt{\frac{P_s}{P_i}} = \beta (\alpha_s L)^{1/2} = 0.035 \text{ rad}$

CRB limited resolution rate  $\sigma_{\text{CRB}} = \frac{V_g \lambda}{4\pi R L} \beta (\alpha_s L)^{1/2} = 341 \text{ deg/hr}$



If we use low coherence source

$$\Omega_{CRB} = \frac{V \lambda}{4\pi R L} \beta(\alpha_s L_c)^{1/2}$$

Low coherence source (LED / Super continuum source)

$L_c \rightarrow 100 \mu\text{m}$  6 orders of magnitude lower  
 $L \rightarrow 200 \text{ m}$

$$\gamma_c \propto \frac{1}{\Delta \nu} \Rightarrow L_c = c \cdot \gamma_c$$

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So, we have been talking about the limitations of an interferometric fiber optic gyroscope and we looked at the various issues with it and one particular issue I said was polarization, which we said we will take care of by going to a PM fiber. And then we also looked at Faraday Effect which is a similar thing in terms of the polarization of the light changing because of an external magnetic field.

And that happens even if you use a PM fiber and that, to reduce that what we essentially have to do is package this in a sort of material with high permeability, so it sort of acts like a shield for the external magnetic field and so there is some special care taken to make sure that the magnetic field is not, external magnetic field is not actually causing, it is not applying on the fiber coil and causing uncertainties with respect to the phase measurements that you are doing. So, those are some practical issues that we take care of.

So, let us say we have taken care of these. Then during our last lecture we looked at the effect of Coherent Rayleigh Backscattering and we said that is because light as it propagates through the core of an optical fiber, it will encounter density fluctuations whose scales are lesser than the wavelength of light that you are using and if that is the case, then you have a Rayleigh Backscattering that is happening, which of course, Rayleigh scattering by itself scatters in all directions.

But what we are really worried about is the scattering that is going back in the same direction because if you have, say clockwise going this way and counterclockwise coming this way, the clockwise propagating wave is now back scattered and it shows up as a counterclockwise propagating wave. And so that actually ends up beating with your regular clockwise propagating wave.

And then you can essentially have phase uncertainties because of that and that phase uncertainty we realize it could be in the order of 0.035 radians, especially when you considered that you have a length of say 200 meters typically and the coherence length of your source is, if you use a high coherence laser, then the coherence length could be in the order of 200 meters and then this will be a huge problem.

Now, of course, coherence length of 200 meters is something that you see only in special single longitudinal mode lasers like a DFP laser, so what this tells you is that you should certainly not use a DFP laser in this sort of a scenario. And then on top of that what we saw was if you use a low coherent source, low coherent source as in the case of an LED or what is called a super continuum source, then you could potentially...

For these sources you could have bandwidths, spectral widths in the order of 100 nanometers, hundreds of nanometers, typically and that means that the coherence for such lasers is very low, the temporal coherence is very low, that is because the coherence time as we saw previously is inversely proportional to, is inversely proportional to  $\Delta \nu$  and this implies that  $L_c$ , which is  $c$  times  $\tau_c$ , so that can be made extremely small.

So, we could probably go to 100 microns, so even 10 microns and even lower, if you choose a source with a much higher coherence, much higher spectral width. So, does that actually ring a bell where have we seen this before, the utility of a low coherent source? If you remember when we started discussing phase modulated sensors, one of the first configurations that we looked at was actually a low coherence.

Sorry, optical coherence tomography and where we said using a low coherent source that actually allows us to look at very fine features, layer by layer features in the eye, that was an example that we looked at several weeks ago. So, it is a similar sort of issue here, so what what we are essentially talking about is if you have a loop like this, so you are not picking up.

You, by going for a low coherent source, if there is any back scattering from this location that actually would not interact with the counter propagating wave that is going in the basically the ccw wave that is coming around here, because the pathline difference between this path and this path is much larger.

So, the only point where backscattering becomes an issue is somewhere in the middle where both this, sorry, this round trip, this backscattering can get confused with the ccw propagated wave. So, this is actually backscattered cw, can get confused with this ccw wave and this can

be like what we talked about is, it can be limited to this  $L_c$ . Now, of course, there is this confusion that if you limit  $L_c$  does that actually limit the phase change that you are getting because of rotation?

Mind you that is not actually getting limited, because of the fact that both your clockwise and as well as your counterclockwise are going through the same propagation distance, propagation time it takes to go around and what we are saying is we are sampling this difference between  $\phi_{cw}$  minus  $\phi_{ccw}$ , the difference we are seeing only within that coherence at time essentially.

So, only those sort of differences we are we are picking up. So, but nevertheless the phase does accumulate over this entire loop, so you are not compromising the sensitivity of your I-FOG because of the low coherent source. So, that is one important aspect as far as OCT, sorry, as far as I-FOG application is concerned.

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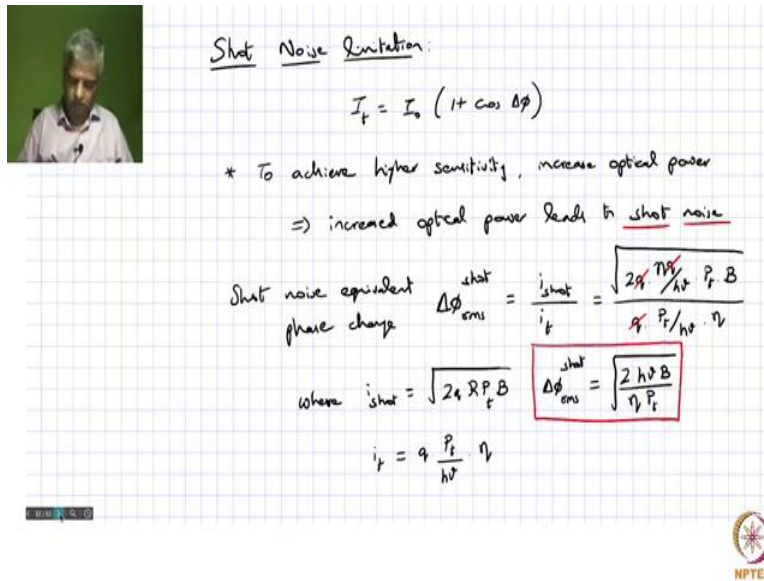
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Now, let us just go back and let us actually now see this last one which is actually a short noise. So, let us see how short noise can affect.

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Shot Noise Limitation:


$$I_f = I_0 (1 + \cos \Delta\phi)$$

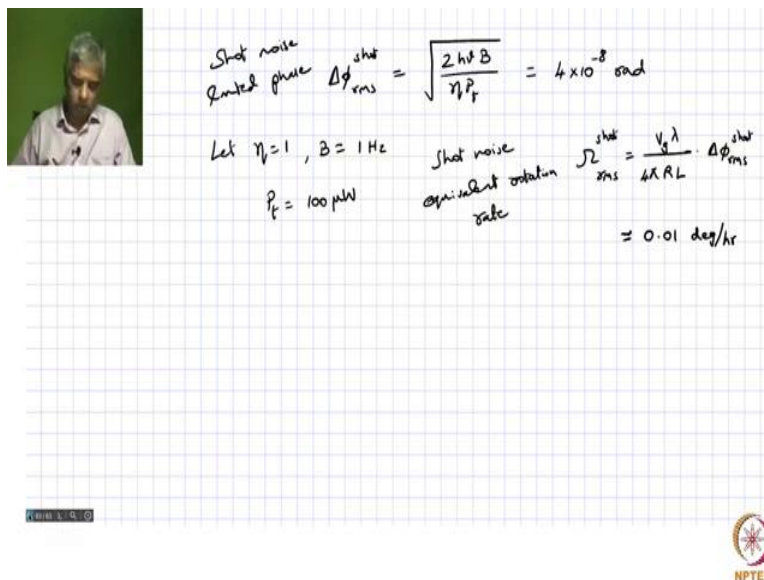
\* To achieve higher sensitivity, increase optical power  
 $\Rightarrow$  increased optical power leads to shot noise

Shot noise equivalent phase change  $\Delta\phi_{rms}^{shot} = \frac{i_{shot}}{i_f} = \frac{\sqrt{2q \eta P_f B}}{q P_f / h\nu}$

where  $i_{shot} = \sqrt{2q R P_f B}$   $\Delta\phi_{rms}^{shot} = \sqrt{\frac{2 h\nu B}{\eta P_f}}$

$$i_f = q \frac{P_f}{h\nu} \eta$$






Shot noise limited phase  $\Delta\phi_{rms}^{shot} = \sqrt{\frac{2 h\nu B}{\eta P_f}} = 4 \times 10^{-8} \text{ rad}$

Let  $\eta = 1$ ,  $B = 1 \text{ Hz}$   
 $P_f = 100 \mu\text{W}$

Shot noise equivalent rotation rate  $\dot{\Delta\phi}_{rms}^{shot} = \frac{\nu \lambda}{4\pi R L} \Delta\phi_{rms}^{shot} = 0.01 \text{ deg/hr}$



So, we are looking at shot noise limitation. So, essentially when we are looking at this gyroscope, we say the output intensity is given by  $I_t$ , which is equal to  $I_0$  plus  $I_0$  multiplied by  $1 + \cos$  of  $\Delta\phi$ , so for a given change in  $\Delta\phi$ , if you want to see a larger intensity at the receiver you need to actually have a larger  $I_0$ , so that is typically the consideration.

But however, we know that, so what we are saying is to achieve higher sensitivity, increase optical power that is reaching the receiver, but this implies that this actually leads to the case where increased optical power leads to more number of photons that are arriving at the photodiode and we saw previously that more number of photons, means there is more uncertainty with respect to their arrival times and that leads to a shot noise.

So, this is actually the primary consideration, so as you scale the optical power. So, the question is can we quantify the limitation in terms of the phase that we can measure in a reliable manner and through that what is the noise limited rotation rate that we can measure? Those are the things that we want to quantify.

So, when we look at basically the short noise equivalent phase change, so let us call this  $\Delta\phi_{\text{RMS}}$  due to short noise. So, we will, we see that the any noise is going to be is, any of this phase noise is going to be inversely proportional to the signal to noise ratio that you have at the receiver, so in this case we, is inversely proportional signal to noise ratio, so that means this is basically given by the short noise current divided by the noise, sorry, divided by the signal photocurrent that you have at the receiver.

Now, we can define these quantities where we can say that  $i_{\text{short}}$ , what is that given by, its, the variance is  $2q$  into  $i_p$  multiplied by  $B$  where  $i_p$  is the photo current, so you can write it out now in terms of the optical power, so if you, when you look at the RMS noise current that is going to be given by root of  $2q$ .

Now instead of  $i_p$  I would like to write it as a responsivity multiplied by the power that is incident at the receiver so that is corresponding to  $P_t$  multiplied by  $B$  where  $B$  is the bandwidth of the receiver. So, and similarly if you look at it that is the photo currents generated, that is going to depend on the electric charge multiplied by the amount of power that is incident on the photodiode divided by the photon energy.

So, that is the rate at which light is falling on the photodiode that is the optical flux and this multiplied by the conversion efficiency from the optical power to electron hole pairs which is given by what is called this internal quantum efficiency. These are things that we looked at a while ago. So, if you plug these two in here, so in the numerator you have  $2q$ ,  $R$  can be written as  $\eta q$  over  $h\nu$  multiplied by  $P_t$ , multiplied by  $B$ , the whole root divided by  $q$  times  $P_t$  over  $h\nu$  multiplied by  $\eta$ .

So, you have the numerator is  $q$  square under the root, so that has actually  $q$ , so you can basically strike that off, and so  $P_t$  essentially cancels, root of  $P_t$  will cancel, and then you have a root of  $P_t$  in the denominator and then similarly you have root of  $h\nu$  here and so on. So, we can actually simplify this and we can basically write this as, you get  $2$  times  $h\nu B$  divided by  $\eta$  times  $P_t$  the whole root.

So,  $\Delta\phi$  RMS due to the short noise variances is given by this. Now, we can substitute some standard values for this. So, let us actually take this and try to work that out in the next page. So, what we are saying is  $\Delta\phi$  RMS due to short noise is given by  $\sqrt{2}$  times  $h\nu$  multiplied by  $B$  divided by  $\eta$  times  $P_t$ .

So, of course, what it says is in the short noise limit it actually helps to go to a larger value of  $P_t$ , so that you can essentially get a smaller value of the short noise limited  $\Delta\phi$ , the phase change. So, let us say for our application, let us say  $\eta$  equal to 1 for simplicity, anyways if you using something with  $\lambda$  equal to 1.5, use indium gallium arsenide and in that case the internal quantum efficiency typically is about 0.9.

So, we can approximate that to be 1. And then  $B$ , the bandwidth, so what should be the bandwidth of your receiver? Well, we talked about using a phase generated carrier and there we talked about using something in the order of 500 kilohertz for this 200 meter coil that we were having for the gyroscope and so you would say that, yes, the receiver bandwidth has to be at least 500 kilohertz, so that you can pick up those frequencies.

But mind you what we do in this phase generated carrier technique, essentially a lock-in technique, is that we beat with those frequencies and do a low-pass filter to extract the signal. The signal itself, if you look at the information content that we are trying to look at, this is rotation rate, so we are trying to measure in the order of 0.01 degree per hour.

So that is actually corresponding to a very slow change in the, as far as these navigation applications are concerned, there is a very slow change in terms of the rotation rate. And so because of that we can assume the bandwidth to be in the order of 1 hertz, you may not even need 1 hertz actually because the rate at which you are changing is so slow.

But you are happy with getting a phase value or a rotation value every one second basically. So, let us say  $B$  equals to 1 hertz and  $h$  is the Planck's constant,  $6.626 \times 10^{-34}$  and  $\nu$  is optical frequency, so that corresponds to a  $\lambda$  of 1.5 micron. So, basically  $c/\lambda$  we can figure out the  $\nu$ . And let us say we are picking up  $P_t$  equals to 100 microwatt.

Now, of course, we are saying larger the  $P_t$  more will be your or lesser will be your phase noise so, you would be tempted to go to a milliwatt, 10 milliwatt, 100 milliwatt and so on, but mind you, when you go to higher powers you are essentially having higher power density

inside your fiber and at those power densities even when you go to a milliwatt type of power levels, you start having the non-linearities.

The medium starts behaving in a non-linear fashion, so specifically you have something called the Kerr effect which depends on the intensity of light that is in the fiber and through the Kerr effect you have what is called self-phase modulation, so those effects will essentially make the measurement process much noisier, so you would want to stay away from those optical powers.

So, that is why we choose a nominal value of 100 microwatt and if you plug all this into this, you get  $\Delta\phi_{\text{short RMS}}$  that corresponds to something in the order of  $4 \times 10^8$  radians. So and of course, what it also tells you is that it is proportional to the square root of the bandwidth and so if you have an application where you need to, you have a short range application, you are navigating over a short range and you need to keep updating say every millisecond, then this would be in the order of kilohertz.

And if it is in the order of kilohertz, then correspondingly you may have to incur higher phase noise and essentially what that tells you is as you go to applications that are basically, these short distance type of applications, control applications, there you may incur higher phase noise, but then on the other hand you may not need the kind of resolution in terms of degree per hour that you need for long distance applications.

So, that is actually the trade-off. But coming back to this, if you have a  $\Delta\phi_{\text{short RMS}}$  in the value around this one, that is actually the, what we call as the short noise limited phase that we can pick up, then you should be also able to figure out the corresponding limitation in terms of the rotation rate, so you can say the short noise equivalent rotation rate.

So, this is the capital  $\omega_{\text{RMS}}$  due to short noise that is basically, like we saw before  $V_g$  over  $\lambda$  divided by  $4\pi R$  times  $L$  multiplied by  $\Delta\phi_{\text{RMS}}$  due to short noise, which is given by this above value. So, if you plug all this back in, previously we assumed  $V_g$  is  $2 \times 10^8$ ,  $\lambda$  is  $1.5 \times 10^{-6}$ .  $L$  is 200 meters,  $R$  is 10 centimeter.

So, if you plug all of those in, you get a value something the order of roughly 0.01 degree per hour, so that is the kind of value that you get as far as the limitation due to short noise. So, those are the typical numbers that you are able to achieve using an I-FOG, and then there is of course, this issue that we talked about, briefly in terms of ensuring quadrature detection.



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How to maintain quadrature point operation in IFOD?

$\Delta\phi = \phi_s + \phi'_{cw} - \phi'_{ccw}$

$= \phi_s + \phi(t - \frac{T}{2}) - \phi(t + \frac{T}{2})$  where  $T = \frac{L}{v}$

Suppose  $\phi(t) = \phi_{m0} \cos(2\pi f_m t)$

$\Delta\phi = \phi_s + 2\phi_{m0} \sin(\frac{\omega_m T}{2}) \sin(\omega_m t)$

$= \phi_s + \phi_m \sin(\omega_m t)$  where  $\phi_m = 2\phi_{m0} \sin(\frac{\omega_m T}{2})$

Max occurs when  $\omega_m T = \pi$

$f_m = \frac{\pi}{2\pi T} = \frac{1}{2T}$

For  $L = 200\text{m}$ ,  $v = 2 \times 10^8 \text{ m/s} \Rightarrow f_m = \frac{v}{2L} = 500 \text{ KHz}$

NPTEL

$I_T = I_0 [1 + \cos(\Delta\phi)]$

$= I_0 [1 + \cos(\phi_s + \phi_m \sin \omega_m t)]$

$I_T/I_0 = 1 + \cos \phi_s \cos(\phi_m \sin \omega_m t) + \sin \phi_s \sin(\phi_m \sin \omega_m t)$

$= 1 + \cos \phi_s \left[ J_0(\phi_m) + \sum_{k=1}^{\infty} 2J_{2k}(\phi_m) \cos(2k\omega_m t) \right]$

$+ \sin \phi_s \left[ \sum_{k=0}^{\infty} 2J_{2k+1}(\phi_m) \cos((2k+1)\omega_m t) \right]$

↓

NPTEL

So, for quadrature detection we said we are driving this PZT at a frequency of 1 over 2T where T corresponds to the round trip time and then we went through the math to figure out what sort of signals you are getting. Essentially, we are saying that we would either use this component, this, basically the dc component and your odd frequency components. So you take the ratio of that to figure out your phi s.

Or you could also take what we did previously which is 2 omega m signals, you look at the 2 omega m component and then with respect to the omega m component that can also give you your phi s. So, let us say we are taking the, in this case let us say we are taking the dc component.

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Shot noise limited phase  $\Delta\phi_{rms}^{shot} = \sqrt{\frac{2h\nu B}{\eta P_t}} = 4 \times 10^{-8} \text{ rad}$

Let  $\eta = 1$ ,  $B = 1 \text{ Hz}$   
 $P_t = 100 \mu\text{W}$

Shot noise equivalent rotation rate  $\Omega_{rms}^{shot} = \frac{V_g \lambda}{4\pi RL} \Delta\phi_{rms}^{shot} = 0.01 \text{ deg/hr}$

Total noise equivalent rotation rate (incl. PGC scheme)

$$\Omega_{rms} = \frac{V_g \lambda}{4\pi RL} \sqrt{\frac{2h\nu B}{\eta P_t} \frac{1 + J_0(\phi_m)}{2J_1(\phi_m)}}$$

At  $\phi_m = 1.8$   
 $J_1(\phi_m) = 0.58$   
 $J_0(\phi_m) = 0.34$

$\approx 0.012 \text{ deg/hr}$

How to maintain quadrature point operation in IFOD?

$\Delta\phi = \phi_s + \phi'_{cw} - \phi'_{ccw}$

$= \phi_s + \phi(t - \frac{T}{2}) - \phi(t + \frac{T}{2})$  where  $T = \frac{L}{v}$

Suppose  $\phi(t) = \phi_m \cos(2\pi f_m t)$

$\Delta\phi = \phi_s + 2\phi_m \sin(\frac{\omega_m T}{2}) \sin(\omega_m t)$

$= \phi_s + \phi_m \sin(\omega_m t)$  where  $\phi_m = 2\phi_m \sin(\frac{\omega_m T}{2})$

Max occurs when  $\omega_m T = \pi$

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for  $L = 200 \text{ m}$ ,  $v = 2 \times 10^8 \text{ m/s}$   $\Rightarrow f_m = \frac{v}{2L} = 500 \text{ KHz}$

If you take the dc component then that actually needs to be worked into this, so when we look at the total noise equivalent rotation rate, so when you say total noise equivalent rotation rate, you are saying we are including this PGC - Phase Generated Carrier scheme, and let us assume that we are going to use the dc term for the detection. Then I can write the overall noise equivalent rotation rate  $\Omega_{RMS}$  as all these quantities  $V_g \lambda$  divided by  $4\pi RL$ , multiplied by root of  $2h\nu B$  over  $\eta P_t$ .

And now that is scaled according to, so now we have  $1 + J_0(\phi_m)$  divided by  $2J_1(\phi_m)$ , so this is actually the total noise equivalent the rotation rate that you would have. So, you want to actually go to as small a rotation rate as possible for a high

performance I-FOG, and that essentially means that you need to make this as large as possible.

And to do that you have to look at your Bessel functions, so your Bessel functions as a function of  $\phi_m$ , if you look at it, your 0 order Bessel function is something like this and your first order Bessel function is something like this. So, it actually goes to a, so you want to pick up a point where  $J_1$  of  $\phi_m$  goes to a maximum, you want to pick  $\phi_m$ ,  $\phi_m$ , mind you is your free parameter.

$\phi_m$  actually depends on like we talked about, it is corresponding to  $\phi_m$ , I mean  $\phi_m$  naught multiplied by  $\sin \omega_m t$  over 2, this we are taking care of by choosing an appropriate frequency, so this becomes 1, so then this is equal to  $\phi_m$  naught and  $\phi_m$  naught is the modulation depth of the signal that we are providing.

So, now essentially we are seeing what can we choose for  $\phi_m$  naught. So, we choose  $\phi_m$  naught such that when we look at  $\phi_m$ , it corresponds to the maximum of this and that actually corresponds to something over here and at that point if you look at  $\phi_m$  that corresponds to a value of 1.8 that is the first maximum of  $J_1$ . So, this is  $J_1$  and this is  $J$  naught. So, the  $J_1$  becomes maximum at  $\phi_m$  equals to 1.8.

And so and that value  $J_1$  of  $\phi_m$  at 1.8 it is given by 0.58 and  $J$  naught of  $\phi_m$  would correspond to a value of 0.34. So, that is a slightly smaller value over here. So, if you plug that in, so this 1 plus  $J$  naught is 1.34 divided by 2 times 0.58, which is 1.16, so that is slightly over this value.

So, this will correspond to about 0.012 degree per hour, so that does not change things too much fortunately, so you still around about 0.01 degree per hour and of course, commercial gyros, you do have an issue with  $R$  being 0.1 meters, 10 centimeters, you want a more compact gyro so  $R$  is typically chosen as 5 centimeters in which case this entire thing gets scaled by a factor of 2, so in a round about that number is where you find the commercial I-FOG performing something around 0.03 degree per hour.

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Fiber Limitations

- 1) Polarization → Environmentally-induced polarization changes will affect CW/CCW phase ⇒ PM fiber
- 2) Coherent Rayleigh Backscattering → AR along the length of fiber
- 3) Faraday effect → due to external magnetic field, polarization changes (non-reciprocal)
- 4) Shot Noise → due to random arrival of photons @ receiver
- 5) Thermal issues → Quad winding  
?cf

Diagram: A cross-section of a fiber coil with four layers. The innermost layer is labeled 'CW' and the outermost 'CCW'. A temperature differential  $\Delta T/H$  is indicated between the layers. The winding is labeled 'Quadrupolar winding' and 'receiver'.

Of course, something else that that we did not talk about but it is very important is, maybe I can just add it over here, thermal issues, so mind you, you are actually coiling the fiber into a compact form factor and when you are coiling the fiber, if you start coiling, say from the cw path and then, so you start coiling cw and then you end up at the top, after several layers of coiling you end up with the ccw port, we clearly have a temperature differential between the innermost coiling diameter.

So, what I am talking about is if you start with the coiling around or maybe I should just show you the around that mandrel how that coiling, the cross section of the mandrel, so if you start with the coiling over here and then you stack other layers on top, so you may start with your cw here and you end up with ccw.

So, if you have a winding like this, then you might have a temperature differential between the different layers and that might actually cause issues for you as far as, and this is actually the temperature differential is actually a function of time and so because of that, that will introduce some errors. So, what you actually want to do is something like this.

So, you want to start with say the cw coil and then what you want to do is to wind from the other end, so you wind one more layer corresponding to the ccw and then one more layer corresponding to the cw followed by a layer at cw and so on. So, you can actually take both ends of the fiber coil, one first and then coil the other end and then coil the first one again.

So, if that is what you call as quadrupolar coiling or quadrupolar winding, so if you use a quadrupolar winding you can reduce some of those thermal issues. So, you take care of

thermal issues with quad winding and recently there have been also reports of using what is called a photonic crystal fiber instead of using a regular panda fiber or an elliptical core fiber.

People have actually looked at photonic crystal fiber, which is actually proven to be less thermally sensitive, so when you have the coil made of photonic crystal fiber. But of course, the fiber is more expensive than your regular PM fiber, so you pay an extra cost, but that is the, what you get in return is actually a lower rotation rate that you can detect. So, those are some of the typical issues as far as an I-FOG is concerned. Let us stop there.