

Optical Fiber Sensors
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Lecture 26
Phase modulated sensors - 6

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Overcoming Environmentally-Induced Phase Noise

* Design of a hydrophone

- Phase Generated Carrier
Dandridge et al
IEEE JQE, 1982

Reference $\phi_c \cos \omega_c t$

SPLA Measurement SPLA

Laser Detector

~ 100 Hz Acoustic Signal

Phase vs Time graph showing $\phi_c \cos \omega_c t$ and $\phi(t)$.

$$I_f = I_1 + I_2 + 2\sqrt{I_1 I_2} |\cos(\Delta\phi)|$$

$$\phi(t) = \phi_c \cos \omega_c t + \phi_a(t)$$

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Normal, $I_f = A + B \cos[\phi(t)]$

PGC $I_f = A + B \cos[\phi_c \cos \omega_c t + \phi(t)]$

$I_f = A + B \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos \phi_a$

$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin \phi_a$

$\phi(t) = \phi_c \cos \omega_c t + \phi_a(t)$

$\cos \phi(t) = \left[J_0(\phi_a) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_a) \cos(2k\omega_a t) \right] \cos \phi_c$

$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_a) \cos(2k+1)\omega_a t \right] \sin \phi_c$

$\sin \phi(t) = \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_a) \cos(2k+1)\omega_a t \right] \cos \phi_c$

$+ \left[J_0(\phi_a) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_a) \cos(2k\omega_a t) \right] \sin \phi_c$

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Hello, so far we have been looking at the challenges in phase modulated sensors and overcoming some of those challenges and in the last lecture we looked at overcoming environmentally induced phase noise and that is done through a phase generated carrier technique, which I had mentioned was introduced by Dandridge et al in their IEEE JQE paper in 1982. So, we were just reviewing this technique yesterday and we were saying that one example of where this technique could be useful is the design of a hydrophone.

So, we were looking at the case where you need to pick up acoustic signals, let us say in the order of 100 hertz and so these are essentially pressure variations. So, if you were to pick it up using an optical sensor, we said essentially a phase modulated sensor may be quite appropriate for this application, wherein the pressure waves cause change in the refractive index of an optical fiber through what is called the strain optic coefficient.

And whenever the refractive index changes we know that the phase of the light going through this optical fiber is going to change and so we are essentially converting pressure changes to phase changes in this hydrophone application. And of course, we know that our receiver optical receiver is not able to perceive phase changes, so we need to convert these phase changes to intensity changes and that we do using an interferometer.

And one example of that is what we have shown here is a Mach-Zehnder interferometer and where in one of the arms is the measurement arm which is exposed to these acoustic signals and the other arm is a reference arm. And one thing that we said was if you want to pick this acoustic signal from this noisy background, we could potentially introduce a phase, deliberately introduce a phase variation, so sinusoidal phase variation in the reference, it can be accomplished using a piezoelectric transducer.

And then we went on to look at the math of this and we recognize that what we are essentially doing is any phase variation that we have over here, this part is actually because of this phase carrier that we are introducing, we are converting this phase information into amplitude information through these, when we do a cos of cos, we represent that in terms of Bessel functions.

And which are tagged, the Bessel components are tagged to specific frequency so we get an output spectrum like this, wherein the relative amplitudes of the even and odd frequency components where f_c corresponds to the carrier frequency that we have chosen typically in the order of 10 kilohertz or so, but the magnitude of the ratio of this even and odd components is what is going to give us the phase information that we are looking for.

So, I thought we could go into this in a little more detail today and see specifically how this phase is extracted, so that is what we are gonna do. So, let us continue from where we left off yesterday. So, let us actually look at, it is easy enough to say that the phase information that you need is actually sitting over here in the sin and cos which corresponds to these

frequency terms, but how do you read those values and how do you specifically extract phi of t that is what we want to look at.

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\otimes with f_c , $-B G J_1(\phi_c) \sin[\phi(t)]$
 \otimes with $2f_c$, $-B H J_2(\phi_c) \cos[\phi(t)]$

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Normal, $I_f = A + B \cos[\phi(t)]$
 PGC, $I_f = A + B \cos[\phi_c \cos \omega_c t + \phi(t)]$
 $= A + B \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos \phi(t)$
 $- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin \phi(t)$
 $\phi(t) = \phi_s \cos \omega_s t + \phi_n(t)$
 $\cos \phi(t) = \left[J_0(\phi_n) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_n) \cos(2k\omega_n t) \right] \cos \phi_n$
 $- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_n) \cos(2k+1)\omega_n t \right] \sin \phi_n$
 $\sin \phi(t) = \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_n) \cos(2k+1)\omega_n t \right] \cos \phi_n$
 $+ \left[J_0(\phi_n) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_n) \cos(2k\omega_n t) \right] \sin \phi_n$

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So, let us go back and look, just quickly draw this picture and so we can see where exactly we are at, so this is our laser, the source laser, like I said what type of laser we will come back and look at in a minute, and then we are going into our optical receiver, so this is our 3db coupler or 50-50 coupler, this also 50-50 coupler, and then we are basically saying this hydrophone is wound around a mandrel.

So, you have a mandrel around which you basically wind your fiber coil and similarly on the reference side you have a PZT element, which can allow us and you have the fiber wound

around it, so the PZT can stretch or contract and accordingly impart strain changes on the fiber, which once again changes the refractive index in the phase, in the other arm. So, why do we want to do that?

Well, we said we need to extract these specific frequency components, so what we are interested in is these components, so we are interested in extracting those specific components. So, the way we can do this is we can basically take the receiver output and beat it like we did previously for lock-in detection, we can beat it with essentially the f_c and $2f_c$ signals. So, how do we generate f_c ?

Basically you have, let us say a frequency synthesizer that gives us this f_c and this is what we are applying to this PZT so that we can realize this $\phi_c \cos(\omega_c t)$ or ω_c corresponds to $2\pi f_c$ and we use the same source as a reference to beat with the receive signal, so that we can extract the f_c component and similarly you can take a derivative or you can derive, you can do a go through a frequency multiplier, you can derive as harmonic of that frequency to get $2f_c$.

And if you beat with that we can extract the components like we talked about these components. So, when we do this beating what exactly are we getting? Let us examine that a little more closely. So, when you beat with f_c , that is an odd integral multiple, so what does that correspond to? So, you come over here and this corresponds to all the odd integral multiples, so when K equal to 0 that is when you get basically the f_c component.

And so when K equal to 0, this is actually J_1 of ϕ_c and so this goes to 1 and you have a negative sign outside, so and this entire thing is multiplied by B , so we can write this as, so when you beat with f_c , the component that we get is minus B times let us say the strength of this signal generator is such that the strength of the beating signal corresponds to some factor g , and so what we are extracting is J_1 of ϕ_c multiplied by.

You have the $\cos(\omega_c t)$ term, which beats with this $\cos(\omega_c t)$ term here from the reference and so you get something near dc and what we are left with is, sorry, so what we are left with is, so you get a $J_1 \phi_c$ and then you have a $\sin(\phi_c t)$ term. Similarly, if you beat with 2 times f_c you can go back and look at which component we are going to pick up, we are going to be picking up from from this component.

So, 2 times f_c corresponds to K equal to 1, so you have a J_2 of ϕ_c and K equal to 1 there is a minus sign that is coming out and so you have essentially minus B times the, let us say the strength of this $2f_c$ component is given by H , so you have J_2 of ϕ_c and that is associated with \cos of ϕ of t . So, now what we are really interested in is ϕ of t .

So, you can basically say mathematically I take these signals and I can take, basically if you take the ratio of these signals then that will actually give me \tan of ϕ of t and then you can do a \tan inverse to get ϕ of t and all that, so that is nice to say, so for example, if you have some digital signal processing you can possibly you have a computer, you can possibly compute that term. But if you want to suppose extract the signal in an analog form itself, how can we do that?

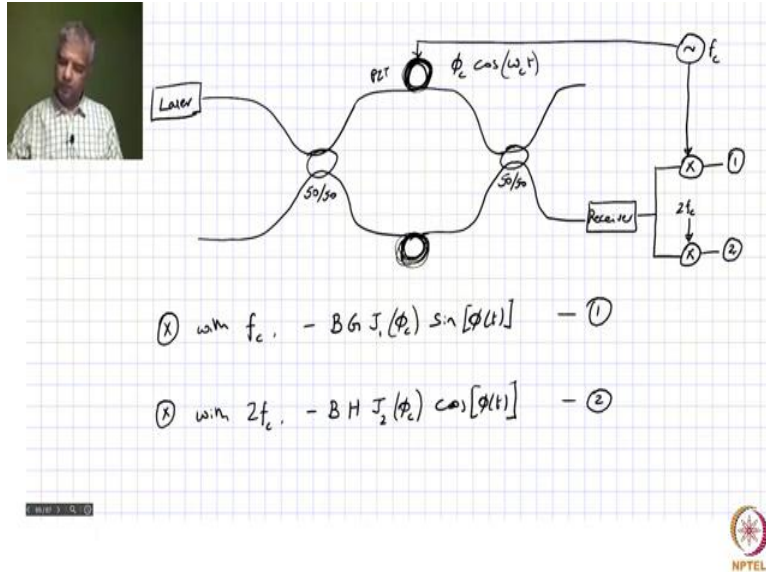
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Differentiate & cross-multiply

③ $-B G J_2(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt}$ ③ \times ② $\rightarrow B^2 G H J_2(\phi_c) \cos^2[\phi(t)] \frac{d\phi}{dt}$ -⑤

④ $+B H J_2(\phi_c) \sin[\phi(t)] \frac{d\phi}{dt}$ ④ \times ① $\rightarrow -B^2 G H J_2(\phi_c) J_2(\phi_c) \sin^2[\phi(t)] \frac{d\phi}{dt}$ -⑥

⑤ - ⑥ $\Rightarrow B^2 G H J_2(\phi_c) J_2(\phi_c) \frac{d\phi}{dt} \Rightarrow$ Integrate to extract $\phi(t)$



Well, we can do that by a principle that we can call as differentiate and cross multiply. So, what does this differentiate and cross multiply scheme is all about? Well, you, what we have coming in is, let us number these things, so let us call this number one and let us call this number two, so you have number one coming here and number two coming here, we are just continuing from, these are essentially what we get over here 1 and 2.

So, let us continue that here. So, if you take that and you put it through a differentiator, differentiate with respect to time, and then you go into a cross multiply term. So, what does this cross multiply do? We will just see that in a minute. So, you essentially take this signal and you are cross multiplying with this and similarly we can take this signal and cross multiply with this term.

So, first of all, and so let us look at what is happening with one. So, let us say the output of this is 3 and this is 4, so let us actually examine what these outputs represent. So, when you differentiate this incoming signal 1 with respect to time, what you get is minus B times G J1 of phi c, which is actually a constant, multiplied by, you have a sin of phi t, so that is just going to give you cos of phi t.

And then you have basically d phi over dt term as well, so when you do this differentiation. So, similarly, when you do this differentiation in the lower arm you have a cos of phi t, which you are differentiating, so you are going to give a minus, you are going to get a minus sign, so what you are going to get is plus times B times H J2 of phi c, which is actually a constant value for a given value of phi c and then you have sine of phi of t d phi over dt.

So, now what you do is you are just going to cross multiply this with 2 because you are taking this and you are cross multiplying with two, so when you do the cross multiply, you get rather, just give you the output, so if you take 3 and you cross multiply with 2 what you get is $B^2 G H J_1(\phi/c) \cos^2(\phi/t) d\phi/dt$ and similarly, if you take 4 and you cross multiply with 1 you get minus $B^2 G H$, sorry, there should be $J_1(\phi/c)$ and then there should also be a $J_2(\phi/c)$.

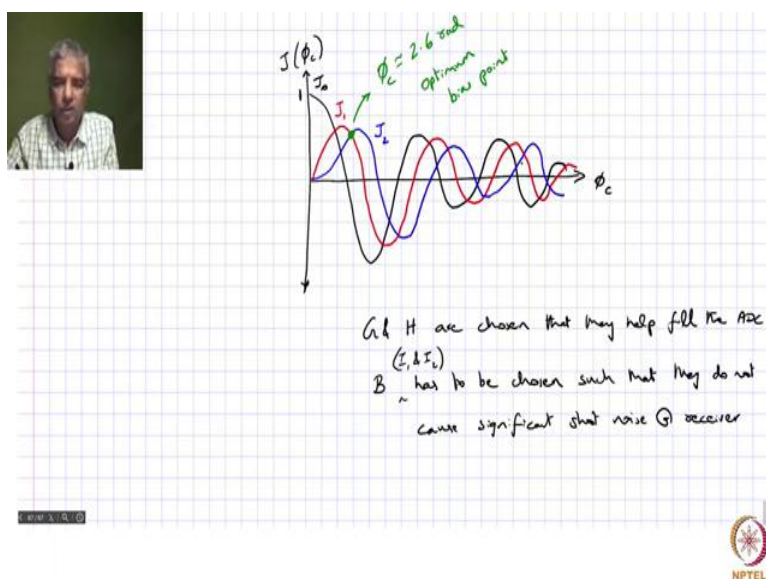
So, you get the same term here $J_1(\phi/c)$, $J_2(\phi/c)$ and here it is going to be $\sin^2(\phi/t)$ and you get a $d\phi/dt$. So, let us call this is 5 and this is 6. So, if you do 5 minus 6, what do you get, you essentially have the negative term becoming positive, so you have a \cos^2 plus \sin^2 term and rest of them are all common, so you get a $B^2 G H J_1(\phi/c)$, $J_2(\phi/c)$ multiplied by \cos^2 plus \sin^2 , which essentially goes to 1.

So, you just have $d\phi/dt$. So, what we are essentially doing is taking this output here and putting it through a subtractor and at the output of this you get this. So, what do we need to do now to get your, to extract your $\phi(t)$? Well, you just have to put it through an integrator. So, this is actually a subtractor circuit and this is an integrator circuit and so what you get out of this.

Let me just put arrows for all this, so it is clear which way the signals are going. So, what you what you get out of this is essentially $\phi(t)$. So, you go through that final step, so from this integrate to extract $\phi(t)$, so that is essentially the way you can go ahead and try to extract this $\phi(t)$. So, let us do a quick recap on what we have been talking about. So, we have been looking at how to extract this phase changes $\phi(t)$.

And of course, we realize that we are going to have to do this differentiate and cross multiply scheme to extract $\phi(t)$, let us just talk about how to optimize this $\phi(t)$. So, to optimize this we need to take care of all these factors J_1 , J_2 , B , G and H and all of that, so let us just look into all of this one by one. Let us first look at the Bessel components. So, what do these Bessel components mean?

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Well, you can draw this Bessel function J of ϕ_c as a function of ϕ_c , so if you draw the different Bessel functions, so let us say you start with J_0 , J_0 is actually going to be an oscillatory component with the exponential decay of the envelope and then you look at J_1 . J_1 is going to start from 0 and then so J_0 actually starts from a value of 1, whereas J_1 starts from a value of 0.

And then once again it is oscillatory, but it does not swing as much as J_0 does. And then J_2 is another oscillatory function, but that does not go up as much as J_1 , so it is relatively smaller in amplitude, but it is, that is also decaying and oscillating. Now the question is what is the optimum value of ϕ_c that you can choose?

(Refer Slide Time: 21:50)

\otimes with f_c , $-B G J_1(\phi_c) \sin[\phi(t)]$ — ①
 \otimes with $2f_c$, $-B H J_2(\phi_c) \cos[\phi(t)]$ — ②

Remember, ϕ_c is something that is up to you because you actually send a voltage from your signal generator to drive your PZT and the voltage should be such a value like it can call this ϕ_c , so such a value that it incorporates a certain ϕ_c through this PZT.

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Differentiate & cross-multiply

$\textcircled{3} -B G J_1(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt}$ $\textcircled{3} \times \textcircled{4} \rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \cos^2[\phi(t)] \frac{d\phi}{dt}$ — ⑤
 $\textcircled{4} +B H J_2(\phi_c) \sin[\phi(t)] \frac{d\phi}{dt}$ $\textcircled{4} \times \textcircled{1} \rightarrow -B^2 G H J_1(\phi_c) J_2(\phi_c) \sin^2[\phi(t)] \frac{d\phi}{dt}$ — ⑥
 $\textcircled{5} - \textcircled{6} \Rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \frac{d\phi}{dt} \Rightarrow \text{Integrate to extract } \phi(t)$



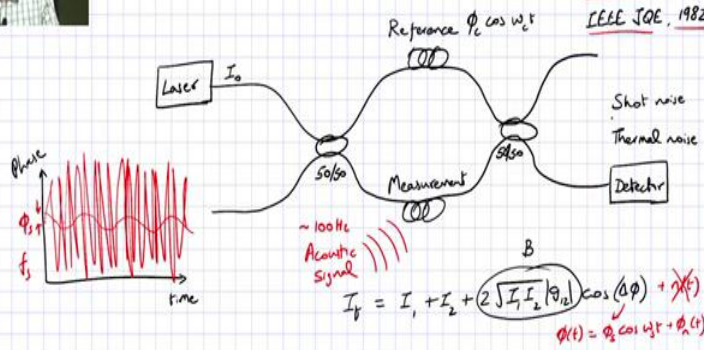
Overcoming Environmentally-Induced Phase Noise

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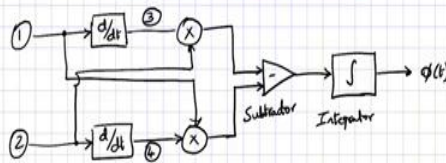


Slide 19.10



Differentiate & cross-multiply

ADC 1V → 1rad



$$\begin{aligned} \textcircled{3} & -B G J_1(\phi_c) \cos[\phi(t)] \frac{d\phi}{dt} & \textcircled{5} & \rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \cos^2[\phi(t)] \frac{d\phi}{dt} \\ \textcircled{4} & +B H J_2(\phi_c) \sin[\phi(t)] \frac{d\phi}{dt} & \textcircled{6} & \rightarrow -B^2 G H J_1(\phi_c) J_2(\phi_c) \sin^2[\phi(t)] \frac{d\phi}{dt} \end{aligned}$$

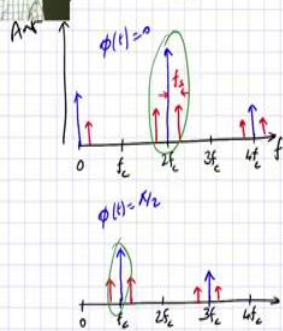
$$\textcircled{5} - \textcircled{6} \Rightarrow B^2 G H J_1(\phi_c) J_2(\phi_c) \frac{d\phi}{dt} \Rightarrow \text{Integrate to extract } \phi(t)$$

Slide 19.11



Normal, $I_f = A + B \cos[\phi(t)]$

PGC $I_f = A + B \cos[\phi_c \cos \omega_c t + \phi(t)]$



$$\begin{aligned} I_f &= A + B \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos[\phi(t)] \\ &\quad - \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin[\phi(t)] \end{aligned}$$

$$\begin{aligned} \cos \phi(t) &= \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \cos \phi_n \\ \sin \phi(t) &= \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \cos(2k+1)\omega_c t \right] \sin \phi_n \\ &\quad + \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k\omega_c t) \right] \sin \phi_n \end{aligned}$$

Slide 19.12



So, then the question is what should be that ϕ_c ? Now, the ϕ_c should be such that when you have small changes in ϕ_c , let us say it is although you give a very specific voltage to get a very specific phase point, the voltage might have some small changes, like it might go from 2.59 to 2.61 it may be fluctuating by about 0.01 radian. If there is any such changes it should not cause a big change in the phase that you are trying to extract.

So, you choose a point such that if it goes less than that value, let us say it goes 2.59, J_1 is increasing J_2 is decreasing, so when you are looking at the product of J_1 and J_2 that is constant and similarly if this ϕ_c value goes up to 0.01 higher, it goes to 2.61, J_2 is increasing, J_1 is decreasing, once again the product is not going to change as much. So, you look for these crossover points between J_1 , J_2 .

So, there is one crossover point here, another crossover point here, another crossover point here and so on. So, you can conveniently choose this crossover point, so that that will be your optimum bias point, so that any small changes in, small fluctuations in ϕ_c should not change the overall ϕ value that you are trying to pick up. And then it comes to optimizing the value of G and H . How do you optimize the value of G and H ?

Now, when you think about the final power value that you are reading, this ϕ of t you are reading, ϕ of t should you typically design for a certain range. There is a maximum ϕ value for which you design this sensor. Let us say that maximum ϕ value is one radian, so it should be such that this final ϕ of t that you are getting is actually a voltage value. So, that voltage should fill your adc.

So, if your adc is say, adc is one volt, then that one volt should correspond to one radian, if one radian is the maximum phase that you are trying to pick up. So, you need to fill your adc, so to fill your adc you can use, you can actually change the value of G and H . What does G and H represent? G represents the strength of the signal that is coming here, H represents the strength of the signal that is coming here.

So, you can actually change those voltages so that you can increase the signal that your, final signal that you are getting, such that it fills the adc. So, that is how you go about choosing G and H . And finally you want to choose B carefully. So, let us actually see what B represents. To see that you need to go back and look at your the original where we all, where it all started, we said we are looking at this beat function, the strength of that beat is what we are calling as B .

So, that is what we used in these expressions, this B is representing this value. And when you look at this closely, it consists of the degree of coherence, which you, of course, try to increase as much as possible, you try to choose a laser, which is relatively narrow line width, so you can get a fairly high degree of coherence close to 1 if possible and then you have the relative values of I_1 and I_2 . So, how do you choose I_1 and I_2 ?

Well, to start with you in an interferometer to get maximum contrast you try to get I_1 to be matching I_2 , so you can adjust the intensities in the two arms by adjusting the losses in the two arms, so you can balance I_1 and I_2 . So that is one thing we do and now overall it is controlled by the intensity from the laser. So, you can actually, this, if you call it I_{naught} , you can adjust I_{naught} , so that you can adjust I_1 and I_2 as well.

But what should those values be? Well, we understand that in a receiver you have what is called shot noise. So, what is shot noise due to? It is because of the random arrival time of photons at the receiver and that actually scales with, the shot noise variance scales with the input power or the input intensity of light that is incident on the detector. So, you need to reduce the intensity I_1 and I_2 such that you can reduce the shot noise component.

But if you reduces to too small a value, let us say you go down to nanowatts of power, in that case you need to actually boost up your signal by having a large gain for your receiver and in that case you have a thermal noise component that comes into the picture. So, you need to essentially adjust I_1 and I_2 , so that you can balance the shot noise and thermal noise, so that you can achieve as high a signal to noise ratio as possible.

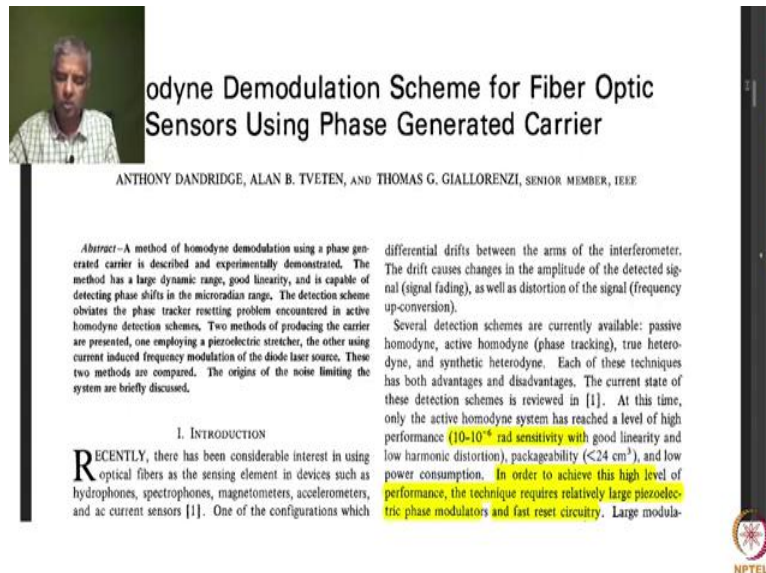
And by adjusting I_1 , I_2 you are adjusting B and through that you are essentially making sure you get a relatively clean signal. So, you can extract this ϕ of t in a reliable manner. Having said that we do have to be careful about this ϕ of t as we defined a little earlier, it corresponds to $\phi_s \cos \omega_s t$, this is the signal that we want to extract but there is still noise incorporated in that.

We are reducing the noise as much as possible through this lock-in detection and low-pass filtering that is something that I did not explicitly mention previously, when you do this beating you also need to incorporate this low pass filter before you extract this one and two, so by low pass filtering to essentially include the essential components like if you are trying to pick up something at 100 hertz, then you do a low pass filter around that, so that you are not actually accumulating a lot of noise from higher frequencies.

So, you need to use this low pass filtering judiciously. On the other hand if you know that your acoustic signal that is incident on this, if the acoustic signal that is incident on this hydrophone, if you know it is going to be at 100 hertz let us say, then instead of a low pass filter you can actually use a band pass filter.

And then just pick up that hundred hertz component or even better you can beat it instead of f_c , you can beat it with f_c plus 100 hertz or something like that and then you can extract the specific component corresponding to, and $2f_c$ will correspond to $2f_c$ plus this 100 hertz or 200hz, those components and then you can extract the specific frequencies of interest for us. So, those are some of the modifications you can do to reduce the noise further, but that is essentially what this phase generated carrier method is all about.

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Active Homodyne Demodulation Scheme for Fiber Optic Sensors Using Phase Generated Carrier

ANTHONY DANDRIDGE, ALAN B. TVETEN, AND THOMAS G. GIALLORENZI, SENIOR MEMBER, IEEE

Abstract—A method of homodyne demodulation using a phase generated carrier is described and experimentally demonstrated. The method has a large dynamic range, good linearity, and is capable of detecting phase shifts in the microradian range. The detection scheme obviates the phase tracker resetting problem encountered in active homodyne detection schemes. Two methods of producing the carrier are presented, one employing a piezoelectric stretcher, the other using current induced frequency modulation of the diode laser source. These two methods are compared. The origins of the noise limiting the system are briefly discussed.

1. INTRODUCTION

RECENTLY, there has been considerable interest in using optical fibers as the sensing element in devices such as hydrophones, spectrophones, magnetometers, accelerometers, and ac current sensors [1]. One of the configurations which differential drifts between the arms of the interferometer. The drift causes changes in the amplitude of the detected signal (signal fading), as well as distortion of the signal (frequency up-conversion).

Several detection schemes are currently available: passive homodyne, active homodyne (phase tracking), true heterodyne, and synthetic heterodyne. Each of these techniques has both advantages and disadvantages. The current state of these detection schemes is reviewed in [1]. At this time, only the active homodyne system has reached a level of high performance (10^{-6} rad sensitivity with good linearity and low harmonic distortion), packageability (<24 cm³), and low power consumption. In order to achieve this high level of performance, the technique requires relatively large piezoelectric phase modulators and fast reset circuitry. Large modula-

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dynamic range, good immunity, and is capable of shifts in the microradian range. The detection scheme uses a tracker resetting problem encountered in active detection schemes. Two methods of producing the carrier are employed: a piezoelectric stretcher, the other using frequency modulation of the diode laser source. These are compared. The origins of the noise limiting the discussed.

1. INTRODUCTION

RECENTLY, there has been considerable interest in using optical fibers as the sensing element in devices such as hydrophones, spectrophones, magnetometers, accelerometers, and ac current sensors [1]. One of the configurations which has shown high sensitivity is that of the Mach-Zehnder all-fiber interferometer. In this configuration, there are many methods of detecting relative optical phase shift between the signal and reference fibers. The design of the detection scheme is made nontrivial by the presence of low frequency random temperature and pressure fluctuations which the arms of the interferometer experience. These fluctuations produce

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nal (signal fading), as well as distortion of the signal (frequency up-conversion).

Several detection schemes are currently available: passive homodyne, active homodyne (phase tracking), true heterodyne, and synthetic heterodyne. Each of these techniques has both advantages and disadvantages. The current state of these detection schemes is reviewed in [1]. At this time, only the active homodyne system has reached a level of high performance (10^{-10} rad sensitivity with good linearity and low harmonic distortion), packageability (<24 cm³), and low power consumption. In order to achieve this high level of performance, the technique requires relatively large piezoelectric phase modulators and fast reset circuitry. Large modulators are undesirable in multi-element sensors since they increase the active sensor's size and decrease its reliability. Additionally, the need for the sensor circuitry to reset itself every time the environmental noise drives it past its dynamic range adds additional noise. In this paper, a passive homodyne technique which obviates the two problems discussed above is presented. Unlike other passive techniques previously reported [2], this technique has been shown to offer a very high level of performance with a linear dynamic range of $\sim 10^7$. This large linear dynamic range allows both small and large amplitude signals commonly encountered in applications to be observed with excellent fidelity. Two methods of utilizing this approach



The present approach outlined below offers improvements in terms of detection accuracy and simplicity of detection.

The variation in the light intensity detected at the output of the interferometer may be written as

$$I = A + B \cos \theta(t) \quad (1)$$

where $\theta(t)$ is the phase difference between the arms of the interferometer. The constants A and B are proportional to the input optical power, but B also depends on the mixing efficiency of the interferometer. If a sinusoidal modulation with a frequency ω_0 and amplitude C is imposed on the interferometer, then (1) becomes

$$I = A + B \cos(C \cos \omega_0 t + \phi(t)) \quad (2)$$

where $\phi(t)$ includes not only the signal of interest, but environmental effects as well. Expanding (2) in terms of Bessel functions [5] produces

$$I = A + B \left[J_0(C) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(C) \cos 2k\omega_0 t \right] \cos \phi(t) - \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(C) \cos (2k+1)\omega_0 t \right] \sin \phi(t) \quad (3)$$

From this expression it is clear that when $\phi(t) = 0$, only even

multiple of ω_0 and low-pass filtering to remove the terms above the highest frequency of interest.

For the carrier frequencies considered in the experiment, namely 0 , ω_0 , and $2\omega_0$, the output signals after mixing and filtering are

$$\begin{aligned} A + BJ_0(C) \cos \phi(t) \\ BGJ_1(C) \sin \phi(t) \\ -BHJ_2(C) \cos \phi(t), \end{aligned} \quad (5)$$

respectively, and where G and H are the amplitude of the mixing signals for ω_0 and $2\omega_0$.

In order to obtain a signal that does not fade as a function of undesired fluctuations, two signals, one containing the sine $\phi(t)$ and the other cosine $\phi(t)$ are utilized. The time derivative of the sine and cosine terms are cross multiplied with the cosine and sine terms, respectively, to yield the desired sine and cosine squared terms [1]. The process will be illustrated by considering the output signals for ω_0 and $2\omega_0$. The time derivative of these are obtained from (5) and are given by

$$\begin{aligned} BGJ_1(C) \dot{\phi}(t) \cos \phi(t) \\ BHJ_2(C) \dot{\phi}(t) \sin \phi(t). \end{aligned} \quad (6)$$

Multiplying this by the signal for the other frequency produces

$$B^2GHJ_1(C)J_2(C) \dot{\phi}(t) \cos^2 \phi(t)$$



[5] produces

$$\begin{aligned} A + B \left[J_0(C) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(C) \cos 2k\omega_0 t \right] \cos \phi(t) \\ - \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(C) \cos (2k+1)\omega_0 t \right] \sin \phi(t). \end{aligned} \quad (3)$$

From this expression it is clear that when $\phi(t) = 0$, only even multiples of ω_0 are present in the output signal, whereas for $\phi(t) = \pi/2$ rad (quadrature condition), only the odd multiples of ω_0 survive.

In a similar fashion the phase angle $\phi(t)$ can be separated into a signal component of frequency ω and the environmental drifts $\psi(t)$, $\phi(t) = D \cos \omega t + \psi(t)$ and expanded

$$\begin{aligned} \cos \phi(t) &= \left[J_0(D) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(D) \cos 2k\omega t \right] \cos \psi(t) \\ &\quad - \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(D) \cos (2k+1)\omega t \right] \sin \psi(t) \\ \sin \phi(t) &= \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(D) \cos (2k+1)\omega t \right] \cos \psi(t) \\ &\quad + \left[J_0(D) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(D) \cos 2k\omega t \right] \sin \psi(t). \end{aligned}$$

illustrated by considering the output signals for ω_0 and $2\omega_0$. The time derivative of these are obtained from (5) and are given by

$$\begin{aligned} BGJ_1(C) \dot{\phi}(t) \cos \phi(t) \\ BHJ_2(C) \dot{\phi}(t) \sin \phi(t). \end{aligned} \quad (6)$$

Multiplying this by the signal for the other frequency produces

$$\begin{aligned} B^2GHJ_1(C)J_2(C) \dot{\phi}(t) \cos^2 \phi(t) \\ \text{and} \\ -B^2GHJ_1(C)J_2(C) \dot{\phi}(t) \sin^2 \phi(t). \end{aligned} \quad (7)$$

Subtracting gives

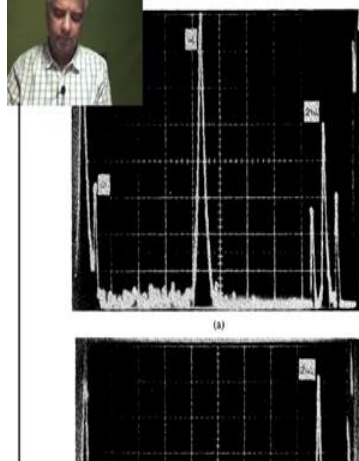
$$\begin{aligned} B^2GHJ_1(C)J_2(C) \dot{\phi}(t) (\sin^2 \phi(t) + \cos^2 \phi(t)) \\ = B^2GHJ_1(C)J_2(C) \dot{\phi}(t). \end{aligned} \quad (8)$$

This output can then be integrated to produce the signal $\phi(t)$ which includes all of the drift information in addition to the actual signal.

A similar set of equations could be written for the 0 and ω_0 . However, in this case a stable offset must be introduced to remove the dc term A from the output. If this term is not removed, the output signal $\phi(t)$ will depend on the value of $\phi(t)$.

The actual value of the coefficients in (8) are not important. The value of the argument in the Bessel functions should be





The experimental configuration employed to demonstrate this homodyne scheme was that of a bulk optic Michelson interferometer. This system (described in detail elsewhere) [6] allowed accurate control of the optical path difference and could readily be isolated from environmental noise sources in our setup. Typically, the interferometer noise of this system was 2×10^{-7} rad (at 1 kHz). Both mirrors of the Michelson interferometer were mounted on piezoelectric cylinders so that both "real" and drift signals could be produced. The bulk Michelson interferometer was chosen rather than the fiber system to allow greater flexibility in control of the optical path difference. The fact that a bulk interferometer was used is not essential to determining the usefulness of this technique since all-fiber interferometers (powered by diode lasers) of sub microradian performance have been built and operated routinely [1]. Consequently, the results of this work are directly applicable to these systems. In this experiment a Hitachi HLP 1400 laser was used as the source.

The implementation of the demodulation scheme requires that a high frequency carrier signal be produced in the interferometer. The following two methods were employed: 1) piezoelectric cylinder, and 2) modulating the emission frequency of the laser diode by modulating the laser drive current. The first method has the advantage that a zero optical path difference may be used in the interferometer. The second

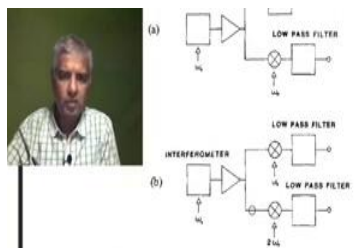
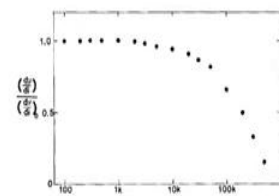


Fig. 3. Circuit used to produce sine and cosine of the phase shift: (a) using the 0 and ω_0 frequency carrier; (b) using the ω_0 and $2\omega_0$ frequency carrier.



the carrier [Fig. 2(a)] or the sideband of the fundamental and first harmonic of the carrier [Fig. 2(b)]. As has been discussed in the theory section, the amplitudes of the sine and cosine terms are determined by the depth of modulation of the carrier. An example of the output after processing using the configuration of Fig. 2(a) and adjustment of the carrier amplitude to 1.4 rad is shown in Fig. 4. Here the Lissajou figure derived from the two outputs (sine and cosine) have been displayed by using them as the x and y inputs of an oscilloscope. With no signal applied, the output appears as a dot which describes a circle as the interferometer drifts into and out of quadrature. To show the circle (indicating that the amplitudes of the sine and cosine terms are equal) more clearly, a low frequency (~ 100 Hz) signal with an amplitude of π rad has been applied to the interferometer.

A similar result to that shown in Fig. 4 was achieved using the configuration shown in Fig. 2(b), but here an amplitude of the carrier close to the predicted 2.2 rad was required for circularity. It should be mentioned that use of the double sideband technique [Fig. 2(b)] conveys the advantage that both outputs from the interferometer may be ac coupled which eliminates a number of problems associated with dc drifts. Obviously, in the configuration used in Fig. 2(a) the signal obtained from the fundamental of the signal frequency needs to be dc coupled.

To obtain the signal from the output of Fig. 2 (i.e., the sine and cosine terms) the differentiate and cross multiply technique described in the theory section was employed. A

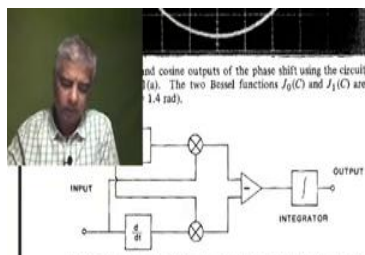


Fig. 5. The schematic of the circuit used to demodulate the sine and cosine of the phase shift.

The frequency response (to the signal frequency f_s) of the system is shown in Fig. 6. The decrease in response at high frequency is due to the low-pass filtering shown in Fig. 2. The frequency response can be extended to higher frequencies if desired by using higher order filters (a simple second order filter was used in this experiment). The rolloff at low frequency was due to the presence of a bleeder resistor across the integrator whose value could be increased if a flat response below ~ 20 Hz is required. The system responded linearly to the amplitude of the input signal between the noise floor of the configuration used (typically 10^{-3} - 10^{-6} rad) and ~ 1 rad. Above this signal level (~ 1 rad), distortions were observed due

to a minimum detectable phase shift was determined for two path length differences, 10 and 2 cm. The result for the 10 cm path difference is indicated on the spectrum analyzer trace shown in Fig. 7, where a 0.01 rad signal has been applied to the interferometer at 470 Hz. The noise floor ($\sim 3 \times 10^{-3}$ rad at 1 kHz) is thought to be due to the phase noise because when the interferometer was operated in a conventional homodyne mode (in quadrature) a noise floor similar to that in Fig. 7 was observed. This experiment was repeated with a 2 cm path difference, and the noise dropped by approximately a factor of 5. To eliminate the effect of the phase noise, the same experiment was repeated at zero path length difference. However, here the modulation for the carrier used the mirror mounted on a piezoelectric cylinder. The results for this configuration are shown in Fig. 8. Again, a 0.01 rad signal was applied to the interferometer. The noise floor corresponds to $\sim 10^{-6}$ rad at 1 kHz. Above 600 Hz, a number of spurious electronic noise peaks related to the line frequency were observed above the true noise floor.

The results of the three experiments performed above are summarized in Fig. 9, where the solid line in the upper two curves represent the average level of the phase noise. The lowest curve was obtained for the interferometer held manually in quadrature using the conventional homodyne detection scheme. As can be seen, the passive homodyne result is approximately a factor of 2 worse than the result obtained with the conventional homodyne. The noise in the conventional homodyne mode was dominated by the limited detection



... higher order filters (a simple second order in this experiment). The rolloff at low frequency to the presence of a bleeder resistor across the input value could be increased if a flat response is required. The system responded linearly to the input signal between the noise floor of the input used (typically 10^{-3} - 10^{-6} rad) and ~ 1 rad. At higher signal levels (~ 1 rad), distortions were observed due to the differentiators overloading from the presence of the residual carrier signal, thus the dynamic range of the system at high signal levels may be increased by 1) lowering the gain prior to the differentiations or preferably, 2) having a sharper cutoff of the carrier modulation (i.e., a higher order filter). Below this region of obvious distortion, the first harmonic of the signal was typically buried in the noise floor and hence was unobservable, but the ratio of the fundamental to the first harmonic was typically greater than 70 dB (i.e., 3×10^3).

2) As discussed earlier, the operation of the current induced modulated carrier requires the interferometer to use a nonzero path length difference such that the frequency modulator is converted to the required phase modulation. Consequently, this nonzero path difference allows the phase noise to contribute to the interferometer noise. Using the diode laser modulation scheme, the frequency dependence of the minimum detectable phase shift in the interferometer is given by

$$\psi_m = 2\pi D \left(\frac{b\nu_n}{c} \right)$$

The results of the three experiments performed above are summarized in Fig. 9, where the solid line in the upper two curves represent the average level of the phase noise. The lowest curve was obtained for the interferometer held manually in quadrature using the conventional homodyne detection scheme. As can be seen, the passive homodyne result is approximately a factor of 2 worse than the result obtained with the conventional homodyne. The noise in the conventional homodyne system was determined by the intrinsic intensity noise of the diode laser [8]. Owing to the carrier modulation, the average level of the intensity noise remains the same as with the conventional homodyne interferometer field in quadrature, however, the amplitude of the phase signal is reduced by the magnitude of the relevant Bessel function (5). Consequently, in an interferometer dominated by the laser intensity noise, a factor of ~ 2 difference between the two detection methods is expected for this modulation amplitude.

In an unbalanced interferometer powered by a laser diode, the primary noise source becomes the phase noise due to the fluctuation of the emission frequency of the laser. For a signal-to-noise ratio of 1, the minimum detectable phase shift ψ_m in the interferometer is given by

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Of course, I want to acknowledge that most of this material is coming from this 1982 paper, that is in Journal of Quantum Electronics in 1982, is when this scheme was first proposed and by Dandridge et al and they were looking at these picking up phase values in the presence of low frequency random the temperature and pressure fluctuations, so that is what they want to deal with and they are targeting 10 radians to 10 power minus 6 radian type of range.

And to achieve that they are essentially proposing this phase generated carrier technique, which involves using this piezoelectric phase modulators and this is all the math that we went through previously, so all that comes from this paper here and once we have this $\phi(t)$, $\sin \phi(t)$ terms, then you do this cross differentiate and cross multiply scheme first, you do the beating and you get the f_c and $2f_c$ components.

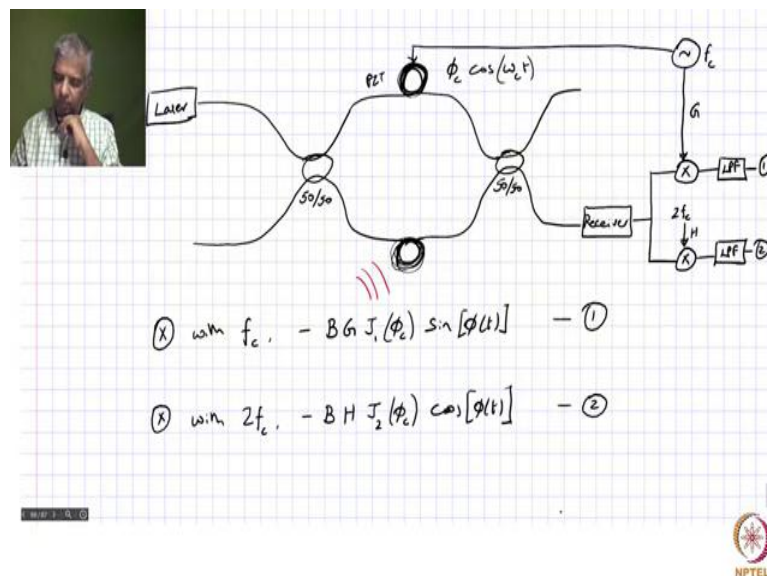
And then you do the cross differentiate step and then the cross multiply step and then your subtraction and then finally you integrate to get the $\phi(t)$ term. And here are some sample waveforms that they acquired experimentally at that time. This is actually for a condition where your f_c component is dominant, compared to this $2f_c$ components, so this is closer to quadrature and this is actually a case where your $2f_c$ and 0, the dc components are dominant compared to the f_c component.

So, this corresponds to a case where ϕ is closer to 0. So, of course, they talk about this beating at the f_c and $2f_c$ to get those beat components, and then they talk about how to do this, differentiate and cross multiply scheme to extract $\phi(t)$. And what they mentioned there is they are able to get to about 10 power minus 6 radians to about 1 radian in their work, which is, of course, has been, this was almost 40 years ago, so it is been improved since then.

But they also make another important point, which is - what is the minimum detectable limit as far as this phase generated carrier technique is concerned? What is it really limited by? What they observe is limited by the fluctuation of the emission frequency of the source. If you have a single frequency source, then you have a single phase, very definite phase associated with it, but if your source is actually having multiple frequencies then the phase actually does not become deterministic, there is some uncertainty in the phase itself.

And that uncertainty is going to constitute the minimum detectable limit as far as the phase detection is concerned. So, not so surprisingly that detectable limit in terms of phase is proportional to delta nu, where delta nu correspond to the spread in the frequencies of your source. So, we will look into this aspect a little more detail possibly in the next lecture.

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So, that is what we have a phase generated carrier technique to to pick up phase changes in the presence of environmentally induced phase noise.