

Optical Fiber Sensors
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Lecture 25
Phase modulated sensors – 5

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Challenges in phase modulated sensors

- ✓✓1) Mechanical stability of interferometer → "all-fiber" interferometer
- ✓✓2) Role of source coherence
 - high temporal coherence ($\Delta t \sim \text{kHz}$)
 - phase info from long dist.
 - low temporal coherence ($\Delta t \sim \text{MHz}$)
 - highly precise localized information
- 3) Phase fluctuations
 - Environmental perturbations
 - optical source
- ✓✓4) Polarization → Fresnel-Arago law
 - max visibility when pol. are the same
 - zero visibility for orthogonal polarization

Hello, we have been looking at the challenges in phase modulated sensors and the past few lectures we looked at mechanical stability of interferometer, then the role of source coherence and then the role of polarization and just in the last lecture we started looking at phase fluctuations, phase noise if you may, due to environmentally induced perturbations.

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Effect of phase fluctuations

Environmentally induced phase noise

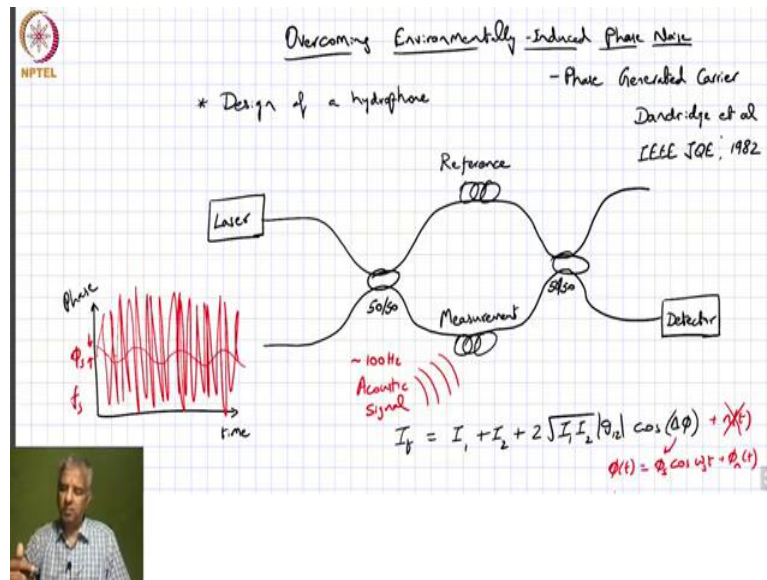
$|E_1|/|E_2|$ vs time

If $\Delta\phi = 0$
 (Max) $\bar{E}_T = \bar{E}_{\text{ref}} + \bar{E}_{S_2}$
 $= 2\bar{E}_{\text{ref}}$
 $|E_{\text{ref}}| = |E_{S_2}|$

If $\Delta\phi = \pi$
 (Min) $\bar{E}_T = \bar{E}_{\text{ref}} - \bar{E}_{S_2}$
 $= 0$

So, we looked at the case where if you, you are trying to extract a signal with say a field of E Sig, but then if that is typically corrupted by noise, now the question is how do you extract that signal from that noise. So that is what we are going to be looking at.

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Overcoming Environmentally Induced Phase Noise and the specific technique that we are going to try to understand in today's lecture is this technique called phase generated carrier. It is quite interesting that this particular technique was actually published way back in 1982, so this is actually a paper by Dandridge et al, I think they published this in IEEE Journal of Quantum Electronics in 1982.

So, it is such a powerful technique that even today a lot of people that use this technique, so let us actually take a deeper look at this. Now, before I go to the details, let us actually set it up as to what sort of scenario we are looking at. Let us say we are trying to design a hydrophone. What is a hydrophone? Well, it is basically a microphone that is kept underwater, so and you can imagine why, what is the use of that.

Essentially, let us say you are working in a defense organization and you are being tasked to map out if there are any enemies of submarines that are in the vicinity, you are supposed to have underwater microphone that is capable of picking up enemy submarines. Now, these submarines, whenever they are propelled they cause some pressure variations, essentially acoustics, and by picking up the strength of those acoustics, the strength of the pressure variations as well as the frequency.

You can figure out how far or what type of submarine is approaching you and so on. So, you want to design, let us say you are designing a hydrophone, and a phase modulated sensor is actually very nice for such an application because you could use what is called a Mach-Zehnder interferometer configuration.

If you remember last few lectures we have been looking at a Michelson interferometer, but let us actually in this case consider a Mach-Zehnder interferometer. So, how does the Mach-Zehnder interferometer look like? Well, you have basically two couplers that are cascaded. So, you have one coupler here and another coupler here. So, let us say this is once again a 50-50 coupler, this is also a 50-50 coupler.

So, whatever light intensity at this point is getting split, equal parts to the upper arm and the lower arm and you have a light source, let us say a laser. We will come back and look at what sort of laser we need for such an application and let us say we have a detector over here. So, what are we looking at? Well, light that is coming in here split into two equal halves, let us say I_1 in the upper arm and I_2 in the lower arm.

And then they are going to combine and looking at the relative, based on the relative phase changes between I_1 and I_2 , you can actually get a change in the intensity. So, of course, we have looked at this before we said the total intensity at the receiver is $I_1 + I_2 + 2\sqrt{I_1 I_2}$ and then the degree of coherence that is given by g_{12} and then you have a $\cos \Delta \phi$, that is basically the phase difference between the two arms.

Now, in the case of a hydrophone what you want is you actually have a coil of fiber over here that is exposed to some, let me just use a different color for this, so there is some acoustic signal that is incident on this fiber coil and then that is actually going to give you a change in phase, let us say you just to make sure the fiber length is compensated and you have this reference arm, so this is the reference arm and this is the measurement arm.

So, you protect the reference arm, so that is basically say within the box itself where this laser and detector and all the signal processing is present and you just have this fiber going out into a microphone type of arrangement, a mandrel around which you can actually coil the fiber and this coil is now submerged in the water and then it is exposed to acoustic signals. So, what is the typical sort of challenge in this?

Well, the challenge is that if you are actually trying to pick up this acoustic signal, the acoustic signal by itself, let us say we are plotting the phase as a function of time, the acoustic signal might be something like this, it might have some periodic waveform, periodic changes in pressure, so when that is actually impinging on the fiber coil that is going to actually stretch and compress the coil periodically.

And so that is going to change the refractive index of the fiber accordingly through the, commonly through the strain optic coefficient and that change in the refractive index is going to give rise to a change in the phase in this arm and so if you are measuring with respect to a reference phase, here you are looking at it as $\Delta\phi$, $\Delta\phi$ can be now looked upon as $\phi(t)$, which is ϕ_s , let us say $\cos(\omega_s t)$, where ω_s corresponds to the angular frequency of this acoustic signal.

And, but then the real problem is that you also have a lot of noise that is submerged in, so there is actually a $\phi_n(t)$ as well. There is a lot of phase noise, which essentially is completely sort of burying your actual signal, so this is what we are looking at as the phase of, or if you made the phase difference between the two arms, so you are actually trying to pick up this component, but it is completely buried under a noise.

And what are the sources of noise? Well, a lot of this environmental perturbations, some background changes in temperature, background changes in pressure and strain and so on, and that actually plays a huge role in essentially burying your signal. So, how to extract the signal? Essentially, this phase change, let us say this corresponds to ϕ_s , phase change with amplitude ϕ_s , that is basically the amplitude over here.

And with a periodicity, let us say corresponding to f_s , with frequency f_s , how do you pick up this signal from the noise that is the challenge. Now, of course, this is not something that we, it is not something completely new to us, so when we are looking at amplitude modulated sensors or intensity modulator sensors we were typically dealing with a noise term $n(t)$ over here, and if we had such noise that is, and there remember instead of phase we were picking up amplitude or intensity of the light, and so we were looking at the intensity noise.

And what did we use to actually pick up our signal in the presence of the strong noise? We used lock-in detection, so we essentially modulated the laser intensity at a particular frequency and then at that same frequency we were actually using that as a reference and

beating the signal that you got from your detector and then we were trying to extract the signal in the presence of a strong noise. So, that is what we did previously.

However, the present situation is different. It is not this intensity noise that we are worried about as much, it is actually this noise, this phase noise that we are interested in. So, we need to we need to actually suppress the phase noise and and be able to pick up this signal from that. So, how do we go about doing that? Now, once again we need to use lock-in techniques, one of the common reasons why we need to use lock-in techniques is this signal is in the order of 100 hertz.

And we know that when you are looking at that low frequency signal from your optical receiver you have **one** over f noise because finally you are picking up intensity corresponding to this phase change, so all this **one** over f noise will come into the picture and so that is going to be a huge component.

So, if at all possible you want to actually shift this measurement frequency to some other higher frequency. So, how do you shift this frequency? So, let us actually examine this in a mathematical manner, which is what Dandridge et al had done and then we will come back to how to implement that in the actual system.

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Normal, $I_f = A + B \cos[\phi(t)]$

PGC $I_f = A + B \cos[\phi_c \cos \omega_c t + \phi(t)]$

$$= A + B \left[J_0(\phi_c) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_c) \cos(2k \omega_c t) \right] \cos(\phi(t))$$

$$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_c) \sin(2k+1) \omega_c t \right] \sin(\phi(t))$$

$\phi(t) = \phi_0 \cos \omega_s t + \phi_n(t)$

$$\cos \phi(t) = \left[J_0(\phi_0) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_0) \cos(2k \omega_s t) \right] \cos \phi_n$$

$$- \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_0) \sin(2k+1) \omega_s t \right] \sin \phi_n$$

$$\sin \phi(t) = \left[2 \sum_{k=0}^{\infty} (-1)^k J_{2k+1}(\phi_0) \cos(2k+1) \omega_s t \right] \cos \phi_n$$

$$+ \left[J_0(\phi_0) + 2 \sum_{k=1}^{\infty} (-1)^k J_{2k}(\phi_0) \cos(2k \omega_s t) \right] \sin \phi_n$$

The graphs show Amplitude vs. frequency f . The top graph is for $\phi(t) = 0$ and the bottom graph is for $\phi(t) = \omega_s t$. Both graphs show a central peak at f_c and sidebands at $2f_c, 3f_c, 4f_c$.

Overcoming Environmentally-Induced Phase Noise

- Phase Generated Carrier
Dandridge et al
IEEE JQE, 1982

* Design of a hydrophone

$$I_f = I_1 + I_2 + 2\sqrt{I_1 I_2} |\rho_{12}| \cos(\Delta\phi) + \cancel{\gamma(f)}$$

$$\phi(t) = \phi_c \cos \omega_c t + \phi_m(t)$$

~ 100 Hz Acoustic Signal

So, let us go to the fresh page and then see how to handle this. So, what we are essentially talking about is you want to have a phase modulation at a higher frequency, so in this case if you want to pick up 100 hertz and then we know up to a kilohertz, you have this one, it is dominated by $1/f$ noise, so we want to go outside of that so maybe 10 kilohertz, so what you could possibly do is in your reference, what if you can actually in a controllable manner modulate your reference.

So, if you can modulate your reference with a signal, let us say $\phi_c \cos \omega_c t$, so if you can modulate your reference with this, so the amplitude of this is $5c$ and the frequency corresponds to f_c , where f_c can be in the order of 10 kilohertz, so essentially you have biased all your modulation to a higher frequency.

It is around that frequency you are actually making your measurement, so you can possibly get away from making a measurement near the base band where $1/f$ noise is dominant. So, let us actually see how this this can be achieved? So, what we are talking about is normal, in a normal case without this phase generated carrier, we are looking at a total intensity which has some dc component let us say A , and then you have some $B \cos \phi(t)$.

That is the kind of signal that you are trying to pick up. But like I said there are problems with picking up the signal because there is, this is once again corrupted by noise and more so near the baseband. So, what we are doing in the phase generated carrier method is you have $A + B \cos \phi(t)$ what you have is $\phi_c \cos \omega_c t + \phi_m(t)$. So, you have this sort of a signal.

Now, if you take a closer look at this what we have is a cosine of a cosine, so when you do a cosine of a cosine what you essentially get is the Bessel components. So, this can be expressed as $A + B$ times so you have a 0th order Bessel function, so that is $J_0(\phi_c)$ depends on the strength of this carrier, how much phase fluctuation you are introducing that is totally up to your control.

And then you have these higher order components, let us say K equal to 1 to infinity of all the odd terms, so you have $\cos(\phi_c)$ to the power of K , the Bessel function of even orders, so then you have basically $J_{2K}(\phi_c)$ and then $\cos(2K\omega_c t)$, all of this now multiplied by your $\cos(\phi_c)$ of, sorry, not ϕ_c , $\cos(\phi_c)$ of t . So, all the even order Bessel terms is getting multiplied by $\cos(\phi_c)$ of t and then you have all the odd terms which correspond to $2 \times K$ equal to 0 to infinity minus 1 to the power of K J_{2K+1} .

So, this corresponds to essentially 1, 3, 5, 7 and so on of ϕ_c and then you have $\cos(2K+1)\omega_c t$ and this component is multiplied by $\sin(\phi_c)$ of t . So that is in the big brackets. So, what are we looking at? So, let us actually look at some examples, let us look at the spectrum. So, essentially what we have done is, whatever changes in phase we have actually converted to these Bessel components and especially it is all tagged to multiple frequency components.

So, essentially if you have, let us say it is a function of frequency you are looking at it, this is let us say and this axis is amplitude, the magnitude of each of these frequency components, so remember originally it was around 0, but now you have shifted everything to some higher frequency. So, now you have essentially f_c , $2f_c$, $3f_c$, $4f_c$ and so on, and so that is what, this is actually, this is a dc term, but then this one corresponds to essentially the even components.

So, this corresponds to $2f_c$, $4f_c$, $6f_c$ and so on, whereas this corresponds to f_c and $3f_c$ and all that, and the key point is this, now the even components, frequency components are associated with $\cos(\phi_c)$ of t and the odd components, odd frequency components are associated with $\sin(\phi_c)$ of t . So, you could potentially have something like, your signal may look like something like this and so on and this would depend on the value of ϕ_c of t .

So, let us take some examples, so if you have let us say ϕ_c of t equal to 0, then all the odd terms are going to vanish, because $\sin(\phi_c)$ of t is equal to 0 that means all the odd frequency components are going to vanish. So, I will basically erase this, so it will look like this. On the other hand if we had the same thing here, let us say 0, f_c , $2f_c$, $3f_c$, $4f_c$ and all

that, then if ϕ of t , sorry, if ϕ of t equal to π by 2, that is it is in quadrature, then what you have is basically it is π by 2 all these $\cos \phi$ of t terms are going to go to 0.

So, that means you will have these terms being the dominant terms, only the odd multiples of f_c , they will, those will show up. And of course, if it is anything in between, then you will have both even and odd components and depending upon whether it is closer to 0 or closer to π by 2, either this will be dominant or this will be dominant. The $2f_c$ component will be dominant or f_c component will be dominant.

So, by looking at the relative strengths of $2f_c$ and f_c components, so how can you find the relative strengths of those components? The same thing that we did previously in lock-in detection, we know what f_c is, so we can generate f_c as well as $2f_c$, and so you can beat with those specific frequencies, then you will, that beat component is going to be dependent on the strength of these signals.

So, possibly by taking the ratio of those two components you get basically, let us say you take even, sorry, odd component divided by even component, it is corresponding to a $\tan \phi$ of t , so we can do an inverse \tan of that and then you can potentially retrieve ϕ of t from there, so that is what we did previously in quadrature detection, so you are quite familiar with that, from that perspective.

However, the problem does not end there, because this ϕ of t , if you go back and look at it, this ϕ of t has both ϕ s as well as this noise. So, we need to essentially find a way to extract this ϕ s from the noise, so ϕ of t can be now expressed as ϕ s \cos of ω s t plus ϕ n of t . So, you have the background, the environmental perturbations also present along with this signal that you want to extract.

So, now the question is how are you going to extract your signal? Well, the same way, if you look at \cos of ϕ of t , ϕ of t is this, \cos of ϕ of t is once again like what we did \cos of \cos , we looked at it and then we wrote down all of this, so the same thing we are going to do, \cos ϕ of t is going to become, is going to be J n $aught$, that is the Bessel function of order 0 of ϕ of s plus, by the way so this J n $aught$ over here I should mention is corresponding to a term over there as well, so that is exactly a dc term that you get.

So, similarly you will get a dc term and then you will have all the even components, Bessel components, so that is going to be minus 1 to the power of k , J **two** k of ϕ s now, because

that is the strength of your signal that you want to pick up and then \cos of $2K \omega s t$ and this whole multiplied by \cos of ϕ_n now, and then minus $2K$ equal to 0 to infinity, minus 1 to the power of K , $J 2K$ plus 1 $\phi_s \cos$ of $2K$ plus 1 $\omega s t$, so the whole thing multiplied by \sin of ϕ_n .

So, does not matter what ϕ_n is, it is going to be, either these terms are going to be dominant or these terms are going to be dominant so if you are looking at the ratio of those terms, now you can actually pick this up and essentially this is around $\cos \phi_t$, so that actually means that it is a component around this. So, what are we looking at this component is actually something around these even term components.

So, there will be a component here also and then component here and so on. So essentially, this is basically $2f_c$ plus ωs , so in this case if this is dominant then this corresponds to f_s and of course, f_s^2 , f_s and so on, there could be components around here depending upon which of these terms are dominant, so essentially by picking up these components here, this $2f_c$ component for example, then you can figure out what is f_s , what is the signal frequency.

Just to complete this picture we can also look at \sin of ϕ of t in which case that is going to correspond to 2 times K equal to 0 to infinity minus 1 to the power of K , $J 2K$ plus 1 $\phi_s \cos$ of $2K$ plus 1 $\omega s t$, the whole thing multiplied by \cos of ϕ_n and then you are going to have plus J naught of ϕ_s plus $2K$ equal to 1 to infinity minus 1 to the power of K , $J 2K$ of $\phi_s \cos$ of $2K \omega s t$, the whole thing multiplied by \sin of ϕ_n .

So and these are terms that will be around these frequencies. So, if you do not find them here, you will find them in, if you do not find them here you will find them here and that corresponds to content around these frequencies. So, you can actually figure out the strength of the phase ϕ_s , which actually corresponds to the strength of the pressure signals that you are detecting, that will determine the amplitude of these components.

And the frequency at which these acoustic wave is incident on this hydrophone that will determine the exact position of these peaks around these main peaks, the main peaks mind you are what you have chosen, so you have actually chosen say 10 kilohertz, so then this corresponds to 10 kilohertz, this is 20, this is 30, this is 40 kilohertz, so you know where those frequencies are, so you basically lock into those frequencies and then look for some components around it.

And those components are essentially going to tell you what is the actual signal corresponding to the acoustic signal that is incident on the hydrophone. So that is actually the basis of phase generated carrier, but let us actually go into little more details now and let us see how to specifically extract those phase components based on this $2f_c$, the strength of the $2f_c$ and the f_c signals. So, go to that.