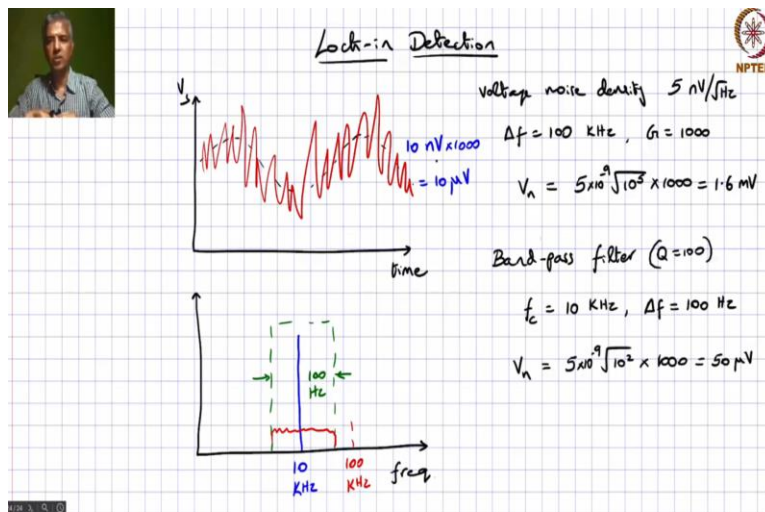


Optical Fiber Sensors
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Lecture No. 13
Lock in detection

So, we have been talking about noise in optical receivers corresponding to an optical sensor. And in the previous lecture we were talking about noise mitigation and we figured that maybe averaging or filtering could be effective techniques to mitigate the noise and hence improve the signal to noise ratio of whatever we are trying to sense.

Now there are conditions under which these techniques that we have considered are not good enough. And we need to look for even better technique in terms of mitigating noise and one of those techniques is a lock-in detection technique or in other words it is also called phase sensitive detection technique. So, that is what we are going to look at today.

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So, let us actually consider a signal. Let us say this is V_s as a function of time and let us say V_s is actually corresponding to RMS voltage of say 10 nano volts. So, you have a sine wave at 10 kilohertz corresponding to, you know with an amplitude of RMS of 10 nano volts. So, we are looking at something like this but of course we know that with the presence of noise you know it is it is going to get corrupted.

If you look at the corresponding say the frequency domain of the signal, what do you expect for a sine wave? Well for a sine wave what we expect is something a delta function right, if it is exactly at 10 kilohertz you expect a delta function at 10 kilohertz. But problem with this is this signal RMS is only corresponding to 10 nano volts and that is not filling your ADC, so you need to boost up the signal.

So, if you try to boost up the signal and you want to use a voltage amplifier, typically a op-amp based amplifier and we looked at the noise characteristics of different op-amps let us say you pick the best op-amp available out there like opa-657 or something like that a fit based amplifier. You have what is called the voltage noise density for the amplifier.

So, let us say we have the voltage noise density of this amplifier which is one of the best amplifiers that you can find op-amps that you can find in the market, you know it is in the order of 5 nano volt per root hertz so we have a signal of 10 kilohertz. So, maybe you need to have a bandwidth in the order of let us say 100 kilohertz. And let us say we are trying to boost up the signal by a factor of 1000. So, the gain that we want to achieve is as a factor of thousands so your signal actually you know gets multiplied by 1000 and becomes 10 microvolt which is a reasonable value. But the question is what happens to the noise?

Well if you make this calculation so you substitute I mean you, you multiply to get the voltage noise, the input referred voltage noise you have to multiply 5 nano volt per root hertz multiplied by root of this bandwidth of the amplifier which is a 100 kilohertz and if you multiply that and then then that is the input referred noise. So, you know if you want to look at what is the output of that amplifier you have to multiply that by 1000 as well. So, your noise voltage is basically 5 multiplied by root of 100 kilohertz is 10 power 5 multiplied by 1000. If you do this multiplication you get to say about 1.6 millivolt RMS of of your noise voltage.

Now your signal voltage is 10 micro volts and your noise voltage is 1.6 millivolts. So, that is much much larger than the signal. But of course you realize that your noise is actually something like this. It is it is basically it is it is broadband noise and you do not necessarily need to integrate of course we are talking about integrating up to 100 kilohertz here, that is the bandwidth that we are considering. So, we are integrating only up to 100 kilohertz but nevertheless, you do not necessarily have to I mean for a for a case like this where you are you are trying to pick up a

particular tone at 10 kilohertz you do not necessarily need all that bandwidth from your amplifier.

So, you might argue that hey, why cannot I you know throw in a band pass amplifier right and limit my noise. So, I would say, I use a band pass, so I use a band pass filter right I use a band pass filter with say a Q factor of 100. So, that is the Q factor for the filter. Well which actually says ok the ratio of the center frequency over the the bandwidth of the filter that is a factor of 100. So, in our case our center frequency is 10 kilohertz. So, you would say that in if you use something like this your Δf , the filter bandwidth is going to be still you know 10 10000 divided by 100 so that is going to be 100 hertz.

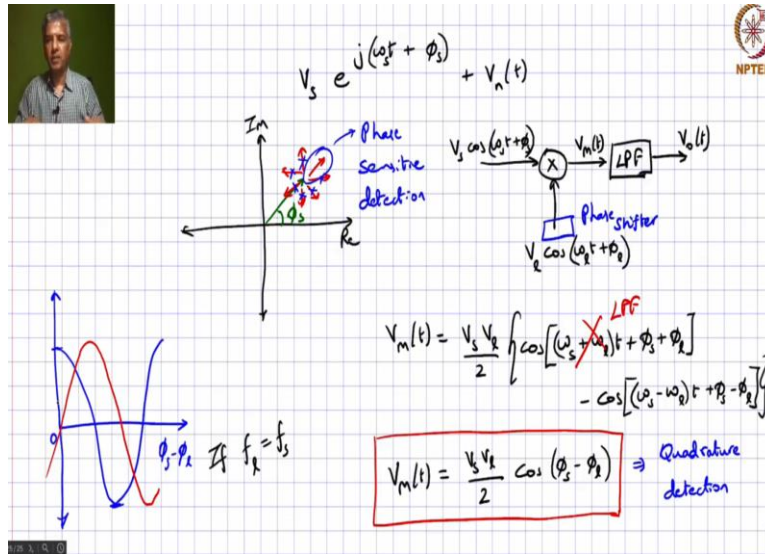
So, effectively what we are saying is I am going to now filter my I am going to apply a filter whose width is 100 hertz and try to pick up the signal. And that way what I am doing is I am actually erasing all of this and instead I have only you know noise within that filter bandwidth. So, I am improving my signal to noise ratio. Now let us go back and compute what is the signal to noise ratio expected here. So, in this case if you do v_n , v_n is 5 multiplied by root of 10 power 2 multiplied by 1000 by the way so this is actually 5 nano volts.

So, this is there is actually 5 into 10 power minus 9 coming to the picture that is that is why you get this value here. So, this is 5 into 10 power minus 9 multiplied by this and if you do this you get something like 50 microvolts. So, better than before but you still have a signal to noise ratio that is much less than 1. So, we are not really doing the job we are not we are not actually able to pick up the signal very well from from the noise. So, so we are essentially still considering a case where this is looking like this. So, we are not we are not able to pick up the signal properly from this.

So, what do we do in in this case? Well 1 thing that you want to realize here is when we are doing this detection we are actually integrating over this noise and the noise essentially has a random face, the signal has got a very well defined face because it is got a nice sinusoid it has got a very well defined face. But the noise is is all over the place the the face of the noise is all over the place. So, if you are able to limit your detection to a face corresponding to only the signal phase and you are eliminating all other phase components of the noise. If you are able to

do that, that will constitute a method where you can get the highest signal to noise ratio. So, what do I mean by that?

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Well let me try to explain. So, let us actually represent our signal as V_s . You can just say it is it is it corresponds to a particular frequency we picked up 10 kilohertz as a frequency and some arbitrary phase is caught. So, you can represent it in terms of a phasor $e^{j\omega_s t + \phi_s}$. This is a signal that we are trying to pick up but of course it is you know it is corrupted by some noise.

Now if I want to represent this signal you understand this is actually a complex signal. It is got a particular face associated with it. So, if you want to represent the signal you go for representation in the complex plane, you actually try to represent it as a phasor. So, you go for this complex plane representation where this is the real axis this is the imaginary axis. And the signal that we are trying to get is basically let us say a particular phasor here in this which has a particular phase we are calling it ϕ_s . And of course this is a periodic signal.

So, it just means that as a function of time this phasor is actually you know just rotating around this plane 360 degrees for every 1 cycle of the frequency you basically complete 1 rotation that is what happens in a phasor. So, phasor does not really represent the frequency. So, it just it just represents the face because that is what we are interested in as far as what we want to discuss is concern.

Now that is actually the signal but what about the noise? Even if you consider noise at that particular frequency you will find that the noise has got arbitrary phase. Of course it can have 1 component along this signal, but it also has other components. So, effectively it either adds to the signal or it acts against the signal or just add in some arbitrary manner and that is what we see as variations in your signal. So, that is that is what we get to see as these variations around the signal because that is because of this noise phasor is actually uniformly distributed over 2π phase.

Now is there a way we can actually pick up one particular phase vector of the phasor of the of the noise from all of this and reject all other phasors. Is there is there a way we can do that that is the key question we are asking. So, essentially what we are talking about is if we can do a phase sensitive detection, we can do a phase sensitive detection then you can eliminate all these other noise components.

And then of course, you still have to deal with this particular noise component which may not only be even though it is of the same phase it might still have some fluctuation in terms of the actual amplitude but that is the that is the limit whatever fluctuations is there you have got to take that and and move on. But you are still able to eliminate all the other phasors. So, that is the key part. So, how do you do that? Well we can do that provided you take this signal here this is basically let us just consider instantaneous representation of the signal. So, this is $V_s \cos(\omega_s t + \phi_s)$ that is the incoming signal.

Now I beat it or mix it with another signal ok where we say this is $V_l \cos(\omega_l t + \phi_l)$. So, what is the resultant of that mixing? Let just call that say V_m of p . So, v_m of t if you write it out that is going to correspond to V_s multiplied by V_l plus, multiplied by $\cos(\omega_s t + \phi_s)$ and $\cos(\omega_l t + \phi_l)$. So, let us say this is $\cos a \cos b$. So, you have a $\cos a$ multiplied by $\cos b$. So, if you are taking if you are taking if you are looking at $\cos a \cos b$, then you say this is corresponding to $\cos(a+b)$ minus $\cos(a-b)$ divided by 2.

So, I would say this is divided by 2 and then you have a \cos of $a+b$ which is corresponding to $\omega_s + \omega_l$ multiplied by $t + \phi_s + \phi_l$. That is a \cos of $a+b$ term minus \cos of $\omega_s - \omega_l$ multiplied by $t + \phi_s - \phi_l$, the whole thing in this bracket. So, you that is actually the mixing term that we have at the end of this at the output

of this mixer. Now you can easily eliminate this part. So, you say this is actually ω_s plus ω_l . So, that is actually fairly high frequency.

Let us consider a case where ω_l is almost you know equal to ω_s . So, you are talking about 2 times ω_s type of frequency. So, you can you know deploy a low pass filter here and you get $V_m \cos(\omega_s t)$. And on top of that let us say we have a way of tuning. So, so essentially we are we are getting this term. So, we so we get rid of this term because of this LPF and we are left with just this term.

Now what we can do is if we have a way of matching ω_s and ω_l , f_s and f_l let us say. If rather f_l is made equal to f_s then this component goes away then you have $V_m \cos(\omega_s t)$ corresponding to V_s , V_l divided by $2 \cos(\phi_s - \phi_l)$. Of course there is a minus sign I would not bother about that because that is just a π phase shift and that is not really matter I mean we can we are just looking at the magnitude of this.

Now if we look at this, what does that tell you? It says that I am able to pick up the signal and I can I can basically my signal can be maximized by tuning this ϕ_l and making it equal to ϕ_s . Then this becomes 0 then \cos becomes 1 so that is when you get the maximum signal. So, all we are talking about here is if I incorporate a phase shifter, a tunable phase shifter, I can tweak that phase until I see maximum voltage there. And at that point now you have maximized your signal and effectively what you have done in this case is you are picking up only this component only a very particular phase of your incoming signal. So, how does this differ from a filtering?

Well in filtering we are of course reducing the frequencies and we are limiting the frequencies to a to a very small part around the frequency of interest. But you do accumulate noise and the noise has some random phase. But in this case what we are doing is by actually doing lock-in detection you are doing phase sensitive detection. So, you are limiting the noise to essentially this this phase only. So, that is essentially the concept of lock-in detection. So, that seems fairly powerful but is that good enough?

Well the answer is it is probably not the best because when $\phi_s - \phi_l$ so this is a cosine function so if you are plotting this $\phi_s - \phi_l$ so it is it is it is a maximum and a minimum and so on. But over here it is actually not very sensitive. It flattens out over here at a phase of 0. And similarly, you can say at this point it once again flattens out and at those places it is very

hard to get your signal maximize the signal because the phase resolution that you get is is very small.

So, if you want to get around that what you need to do is not just this but you need to do quadrature detection. What is quadrature detection? What if along with your cos you are able to get a sine signal of the same phase difference as well. What if you are able to simultaneously get cosine as well as the sine. So, whenever the cosine it is relatively insensitive. If you are picking up the sign you can say that the if you if you have another channel which provides you this with this sort of a function, then wherever the sensitivity is very low. Here the sensitivity is very high. And just by looking at both of these components you can actually pick up the phase accurately. So, that is the advantage of doing quadrature detection. But I I do want to mention that a lot of these examples are being considered.

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Lock-in amplifiers are used to detect and measure very small AC signals of the type shown in the figure. Accurate measurements may be made even when the signal is obscured by noise because many thousands of times larger. Lock-in amplifiers use a technique known as phase-sensitive detection to extract the component of the signal at a specific reference frequency and phase. Most signals of frequencies other than the reference frequency are rejected and do not affect the measurement.

Why use a lock-in?

Let's consider an example. Suppose the signal is a 10 mV sine wave at 10 kHz. Cheap noise amplifiers are required to bring the signal above the noise. A good low noise amplifier will have about 1 μ V of rms noise. If the amplifier bandwidth is 100 kHz and the gain is 1000, then the rms signal can be up to 10 μ V of signal (10 mV \times 1000) and 1.6 mV of broadband noise (1 μ V rms \times 100 kHz \times 1000). We won't have much luck measuring the output signal unless we single out the frequency of interest.

If we follow the amplifier with a bandpass filter with a Q-factor of 1000, good filter centered at 10 kHz, any signal in a 100 Hz bandwidth will be detected (10 mV \times 1000) \times 1000 Hz \times 1000 and the signal will still be 10 μ V. The noise in the filter pass band will be 1.6 μ V (1.6 mV \times 100 Hz \times 1000) while the signal is still 10 μ V. The signal to noise ratio is now 10 and an accurate measurement can now be made. Further gain will help the signal to noise problem.

How to follow the amplifier with a phase-sensitive detector (PSD). The PSD can detect the signal at 10 kHz with a bandwidth as narrow as 0.1 Hz. In this case, the noise in the detection bandwidth will be only 0.1 μ V (1.6 mV \times 0.1 Hz \times 1000) while the signal is still 10 μ V. The signal to noise ratio is now 100 and an accurate measurement of the signal is possible.

What is phase-sensitive detection?

Lock-in measurements require a frequency reference. Usually an oscillator is locked at the lock frequency (from an oscillator or function generator) and the lock-in detects the response from the experiment at the reference frequency. In the diagram below, the reference signal is a square wave at frequency ω_0 . This might be the sine output from a function generator. If the sine output from the function generator is used to excite the experiment, the response might be the signal waveform shown below. The signal is $v_{sig} \sin(\omega_0 t + \phi)$ where v_{sig} is the signal amplitude, ω_0 is the signal frequency, and ϕ is the signal's phase.

Lock-in amplifiers generate their own internal reference signal usually from a phase-locked loop locked to the external reference. In the diagram below the external reference, the lock-in's reference and the signal are all shown. The internal reference is $v_{ref} \sin(\omega_0 t + \phi_{ref})$.

Reference

Signal

Lock-in

The PSD amplifies the signal and then multiplies it by the lock-in reference using a phase-sensitive detector or multiplier. The output of the PSD is simply the product of the two waves.

$$v_{sig} = v_{sig} \sin(\omega_0 t + \phi) + v_{sig} \sin(\omega_0 t + \phi + \pi)$$

$$v_{sig} = 10 \mu\text{V} \sin(2\pi \times 10^4 t + \phi) + 10 \mu\text{V} \sin(2\pi \times 10^4 t + \phi + \pi)$$

The PSD output is two AC signals, one at the difference frequency ($\omega_0 - \omega_0$) and the other at the sum frequency ($\omega_0 + \omega_0$).

If the PSD output is passed through a low pass filter, the AC signals are removed. What will be left in the general case setting. However, if $\phi = \phi_{ref}$, the difference frequency component will be a DC signal. In this case, the filtered PSD output will be

$$v_{sig} = 10 \mu\text{V} \cos(2\pi \times 10^4 t - \phi_{ref})$$

This is a very nice signal - it is a DC signal proportional to the signal amplitude.

It's important to consider the physical nature of this

Lock-in Amplifiers

This is actually a very good reference Stanford research systems is a pioneer as far as lock-in amplifiers goes. And they have a very nice application note where they consider some of these examples which I have used here.

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Quadrature Lock-in Detection

Source f_m → Receiver

Mixer 1: $V_s(t) \times V_L \cos(\omega_L t + \phi_L) \xrightarrow{\text{LPF}} V_{ox}$

Mixer 2: $V_s(t) \times V_L \sin(\omega_L t + \phi_L) \xrightarrow{\text{LPF}} V_{oy}$

$$V_{ox} = \frac{V_s V_L}{2} \cos(\phi_s - \phi_L)$$

$$V_{oy} = \frac{V_s V_L}{2} \left[\sin[(\omega_s + \omega_L)t + \phi_s + \phi_L] + \sin[(\omega_s - \omega_L)t + \phi_s - \phi_L] \right]$$

$$V_{oy} = \frac{V_s V_L}{2} \sin(\phi_s - \phi_L)$$

Graph: $\phi_s - \phi_L = \tan^{-1} \left(\frac{V_{oy}}{V_{ox}} \right)$ & $|V_0| = \sqrt{V_{ox}^2 + V_{oy}^2}$

$V_s e^{j(\omega_s t + \phi_s)} + V_n(t)$

Phase Sensitive detection

Mixer: $V_s \cos(\omega_s t + \phi_s) \times V_L \cos(\omega_L t + \phi_L) \xrightarrow{\text{LPF}} V_M(t)$

Phase shifter: $V_L \cos(\omega_L t + \phi_L)$

$$V_M(t) = \frac{V_s V_L}{2} \left[\cos[(\omega_s + \omega_L)t + \phi_s + \phi_L] - \cos[(\omega_s - \omega_L)t + \phi_s - \phi_L] \right]$$

$V_M(t) = \frac{V_s V_L}{2} \cos(\phi_s - \phi_L)$ ⇒ Quadrature detection

So, let me just get back here and let us actually explain what is quadrature lock-in detection. So, what if we are able to take this incoming signal and split it into 2 parts. So, let us say they are both equal parts V_s of t and V_s of t here and they go through their respective mixers. And the signal that you get from what that V_L of t that is actually called the local oscillator. So, the local oscillator let us say is this source which is providing V_L of t . In this case this will be like a \cos of $\omega_L t$ plus ϕ_L .

But in the other side you actually have 90 degree phase shifter and then you do the mixing and then of course just like previously you take the mix output. Let us start let us say this channel is

called V_{mx} of t and this channel is called V_{my} of t and both are actually going through the respective low pass filter. And the output we call this V_{ox} and this is V_{oy} . So, in this case what happens?

Now of course V_{ox} we can directly write right. So, V_{ox} is gonna correspond to V_s multiplied by V_l divided by $2 \cos$ of ϕ_s minus ϕ_l . So, that part we already know from our previous discussion but what about V_{oy} ? Here in this case this V_l of t for for this mixer corresponds to V_l multiplied by \sin of $\omega_l t$ plus ϕ_l . In this case it is it is basically what is what is going in is $V_l \cos$ of $\omega_l t$ plus ϕ_l . That is what is going in here but what is what is actually going into this mixer is $V_l \sin$ because we have incorporated a 90 degree phase shift. So, then when you mix this so V_s of t is $V_s \cos$ of $\omega_s t$ plus ϕ_s .

So, when you mix with this you have a sine and cos multiplication. So, that you know it corresponds to \sin of a plus b plus \sin of a minus b . So, in in in that case and then of course you do your low pass filtering. So, what you get out of that is maybe I, I will just write it out. So, you have \sin of $\omega_s t$ plus $\omega_l t$ plus ϕ_s plus ϕ_l plus \sin of $\omega_s t$ minus $\omega_l t$ plus ϕ_s minus ϕ_l . So, this is actually what you have for V_{my} of t .

So, if you send it through the low pass filter and also make sure that I mean as we did before we are already making sure that the frequency of the local oscillator corresponds to the frequency of the incoming signal. Then what you get is V_{oy} corresponds to V_s multiplied by V_l divided by $2 \sin$ of ϕ_s minus ϕ_l .

So, you can see that V_{ox} and V_{oy} are in quadrature with respect to each other and that actually describes this situation that we have here where you know where you can say that this corresponds to V_{ox} and this corresponds to V_{oy} . So, so you you that is that is how we are able to implement this quadrature detection technique. And of course once you have this then you can you can say based on this you can say ϕ_s minus ϕ_l is going to be given by \tan inverse of V_{oy} divided by V_{ox} .

So, you can get the phase of the incoming signal if ϕ_l you you you are able to get the phase of the incoming signal and and if you want to get the magnitude of this so the magnitude is going to be root of V_{ox}^2 plus V_{oy}^2 . So, you can you can get both the magnitude and the phase of the you can retrieve the magnitude in the phase. And of course we are multiplying with

V_L here. So, effectively you know if you if you write it out that corresponds to $V_s V_L V_s$ multiplied by V_L . So, that is essentially telling you that you can also boost up your signal by boosting up V_L to some extent you can you can you can do that as well. So, you get an amplification of your signal as well.

So, but the but the key point is this is actually phase sensitive detection. So, you are able to keep the noise level to a very low value. Now what is the downside of this? Well we are characterizing V_s as as a single frequency signal. So, if you have a single frequency signal then this this works out very well. In so how do you ensure that and also we are saying f_s has got to be equal to f_L . So, how do we how do we ensure that? What you can possibly do is and we will see examples of this.

We can modulate our light source we are talking about implementing this in a optical sensor. So, we have a light source and a light detector. This is actually you know what what we are seeing here is the input to this. This is beyond that initial photodiode and and then you have a certain voltage and that is after that is where you are actually implementing this lock-in detection. So, what if you can actually modulate I mean let us say in a sensing application you are picking up strain or temperature which is varying very slow with respect to time. So, the most of my information is actually in in dc around dc right, it is it is it is quasi-static type of information.

So, let me just explain this. So, what what I am considering is if I am looking at the spectrum of my information my information may be the information that I am sensing may be only around this. But if I use a modulated light if I am if I am actually modulating at say a frequency of V_m , F_m , then I am looking at in a perturbations to that particular carrier the carrier frequency is F_m and I am looking at perturbations to that carrier.

So I have basically something like this. That is my information ok and since you know since you are the one that is providing this modulation frequency. So, you you say basically take the same so this is my let us say my, I should not draw there. So, this is my source I am modulating with this F_m . And that is actually going through perturbation and then it is going coming to the receiver. And within this receiver you basically need this reference.

So, this reference essentially could be the same reference as as this. So, that actually you do not need an independent source you basically use that same reference and think so that way you

make sure that F_s and F_l are the same. And then you can do your entire detection to to get these signals. So, that is how you would implement in in a in a real sense as far as the optical sensor is concerned.