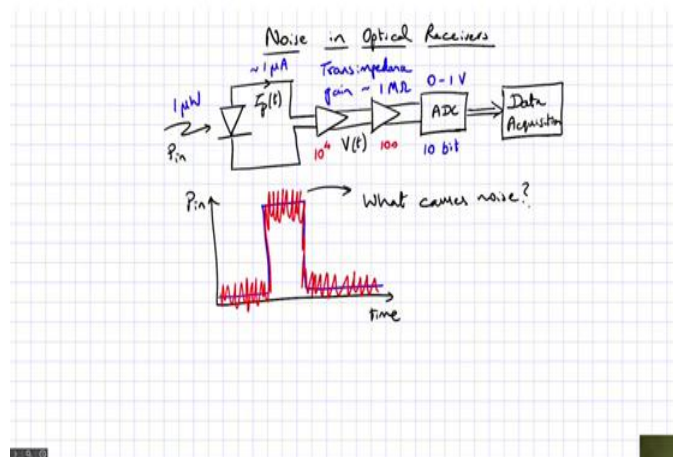


**Optical Fiber Sensors**  
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**Lecture : 10**  
**Noise Analysis**

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We have been looking at the design of optical receivers from a sensor perspective. And we have so far been looking at how light is actually detected using a photodiode a PIN or an APD. And then we looked at how it is, that the photocurrent is converted into voltage using a trans-impedance amplifier, which might, of course, also need further amplification using an voltage amplifier.


And then it goes on to the analog to digital converter and data acquisition system and all that, but so far, we have been looking at the signals. Let us say, for example, you have with respect to time, you have the incident power at the receiver, let us say it is corresponding to a signal like this.

So, this is what we should be detecting, but in reality, when we convert this to photocurrent and then try to amplify that we end up actually generating, adding noise to this optical signal. So, eventually, when we look at the signal, it may not be exactly like this, it may be something like this. So, it is nothing but periodic although I have drawn it as if it is periodic it is nothing but periodic. It is actually typically wideband noise, typically white noise.

So, we will look at what is, in today's lecture, we will look at what causes noise in this optical receiver. And specifically, how can we try to keep that noise that is added to a minimum, and

then we will also look at potentially noise mitigation techniques to improve the signal to noise ratio. So, let us move on, let us actually try to see what are the possible sources of noise.

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Shot noise

Intensity vs time graph: Shows a series of discrete pulses representing photon arrivals. A horizontal line indicates a constant intensity level.

Optical Receiver  $\Delta f$

$I_p(t)$  vs time graph: Shows a fluctuating signal intensity over time, representing the signal after passing through the receiver.

Shot noise Variance  $\sigma_s^2 = \langle i_s^2(t) \rangle = \langle i_s(t) i_s(t+\tau) \rangle$

$$= \int_{-W}^W S_s(f) e^{j2\pi f \tau} df = 2q I_p \Delta f$$

$\sigma_s^2 = 2q I_p \Delta f$

$S(f)$  vs  $f$  graph: Shows a constant signal intensity  $q I_p$  across a frequency range  $f$ .

03:04

So, the first source of noise that I would like to actually consider is what is called shot noise. So, what is shot noise? Well, when we think about light sources, so even as we speak certain light that is incident on me and then that is being captured using the camera. Now, we look at these light sources, as constant intensity as a function of time. So, we look at, whatever is emitted by an LED lamp or fluorescent lamp as being constant intensity. But in reality, that is not the case. So, you have essentially the emission of the photons from this light source happening at random time intervals.

So, we may look at, what is coming into the receiver. If this is your, say your optical receiver, we say what is coming in is let us say particular intensity as a function of time, as being some uniform intensity. But in reality, if you look at this from photon picture, it is not. So, if you basically say, if you split this in terms of certain time slots, it is not like you have uniform number of photons coming within that time slot, especially when you go to time slots in the order of nanoseconds or picoseconds, where you get to observe only a few photons coming through. You will find that the photons are not like this.

So, this is what we would expect if it is uniform intensity like some constant number of photons per unit time per slot that we are looking at. Instead, what we will see is it could be, so, once again if we look at these time slots over here, in reality, what happens is it could be 4, it could be 5, it could be 3, it could be 2, it could be 3 again and so on. So, in reality, when

we look at the corresponding signal, the signal might actually look like and so on. So, this is basically the number of photons per unit times window may be varying with respect to time. So, this is a function of time and what we are looking at is?

Now, the photocurrent that is generated  $I_P$  of  $T$ , that photocurrent may actually be varying with respect to time because of the random arrival times of photons. So, this short noise is essentially due to the random arrival times of photons at the receiver. So, and it is actually characterized by Poisson statistics.

So, yeah, you would say, if you have, you can only talk in terms of mean number of photons per timeslot. So, the mean number of photons, in this case, maybe still three. But when you actually look at what is happening per slot, it could be something like 2, sometimes it could be 5 times that can be even 0 sometimes, and, and so on.

So, you would have, wide fluctuations in the number of photons that are coming in, and that fluctuation is more when you have a more mean number of photons. So, that is what we are talking about, what we are characterizing as short noise. So, let us actually try to quantify that. So, if you were to quantify that, you would typically look at the variance of this noise.

So, the short noise variance, let us call it  $\sigma^2$ . So where do we get that from? Well, we can actually get it from the photocurrent that is generated because of the arrival of the photons. So, you can say, it is an ensemble average of  $I^2$  square of  $T$  where  $I$  corresponds to the signal that photocurrent that we are generating.

Now, if you want to quantify that, you can basically say that this is nothing but the autocorrelation of this photocurrent. So, you have  $I$  of  $t$  correlated with  $I$  of  $t + \tau$  when  $\tau$  goes to 0. And whenever we talk about autocorrelation, we actually use what is called the Wiener Atkinson theorem. So, we know that the autocorrelation and the power spectral density are Fourier transform pairs.

So, you can express this as, over a bunch of frequencies from minus infinity to plus infinity of the power spectral density  $S_S$  of  $F$  corresponding to the short noise process,  $E$  power  $J^2$  pi  $F$  times tau. Now, of course, we want the variance, which means that tau is actually equal to 0, so you can neglect this, and then you have just this quantified in terms of the power spectral density.

Now, when we talk about the power spectral density of this poisson process, we can just draw it is over here. So, it will tend to be, so,  $S_S$  of  $F$  as a function of  $F$ , it will be sort of a white

noise. So, it will be corresponding to just some constant value across all frequencies. And that constant value is going to be dependent on the amount of power, is dependent on the mean number of photons like I said, it is seldom this, it is not the mean number it is not the same, it is varying as a function of time and the corresponding power spectral density is having a value given by Q times I P, where I P is the photocurrent that we are generating.

So, more the photocurrent we are generating, or more as the number of photons that is incident on the photodiode, more will be the corresponding variants. So, this is integrated over all frequencies, but the frequency of interest as far as the optical receiver is concerned is only, the optical receiver has a certain bandwidth, delta F.

So, it is only over that now, although we say minus infinity to plus infinity, it is effectively over minus delta F 2 plus delta F is what matters. So, you essentially have 2 Q I P multiplied by delta F. So, let me just write it out again, sigma square is given by 2 Q I P multiplied by delta F. This is what we call as shot noise.

Now, that is actually representing the noise that we get when light is incident on the photodiode but, you could also have noise similar, the rest characteristic is similar to the shot noise through thermally generated carriers. So, even when there is no light falling on the photodiode just because of thermal excitation you could have electron-hole pairs and you could have a certain photocurrent because of that, and that is what we call as.

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Dark current noise → due to thermally generated electron-hole pairs

$$\sigma_s^2 = 2q(I_p + I_d) \Delta f$$


APD multiplication noise → multiplication process is random accentuated by initiation by both carriers


Excess noise factor ( $F_n$ )

McIntyre et al. 1966

$$\sigma_s^2 = 2q M^2 F_n (I_p + I_d) \Delta f$$

$F_n = K_M M + (1 - K_M) \left( \frac{K - 1}{M} \right)$





Dark current noise, which is due to thermally generated electron-hole pairs. So, when you have a condition where your photodiode is cooled to a very low temperature, then the

corresponding thermal energy is low. And because of that, the dark current noise is also low. So, that is a very nice way of making sure that the dark current is a relatively low value. But if you include this in the other expression, we say that the short noise variances. Not just  $I_P$ , plus you have this  $I_D$ , which corresponds to the dark current noise as well. So, that is something that you have to take care of.

And it is, actually in some ways a fundamental limitation, because even when you have no light incident on the photodiode, you still have this dark current noise. And so that will be the noise background that you are working with here. So, you are not going to be able to go below that noise background.

So that is short noise, you can say that is what you would see or expect when you have a PIN photodiode. But we have been talking lately about APD photodiode. So, what is the short noise corresponding to APD? When we talk about APD, we have what is called an APD multiplication noise. That is also coming into the picture. So why do we have an APD multiplication noise?

Well, we already recognize that the multiplication process, the avalanche process, the multiplication process is random in nature. We said you may have a mean multiplication factor of say 10. But it is not 10 all the time, at some instant, it may be 10. But some other instant it may be 7 or some of other instant it may be 12, and so on.

So, there could be changes in that multiplicative gain value because you are not able, I mean, it is a random event, the impact ionization and the avalanche gain that you get, because of that is a random event. And, what we also saw was that, if you have both carriers participating in the multiplication process, the corresponding variation in the gain is even more.

So, this scenario is accentuated by the initiation of the gain of the multiplicative gain by both carriers. So, remember, that is where  $K_A$ , the value of  $K_A$  becomes important,  $K_A$  which is the ratio of the impact ionization holds to the impact ionization coefficient for electrons, we want that to be as far away from one as possible. So, that is also playing a role in this. And of course, we saw that this was characterized through what we call excess noise factor. We called it  $F_A$ , a couple of lectures ago, we were looking at what is the  $F_A$ ? What is the excess noise factor?

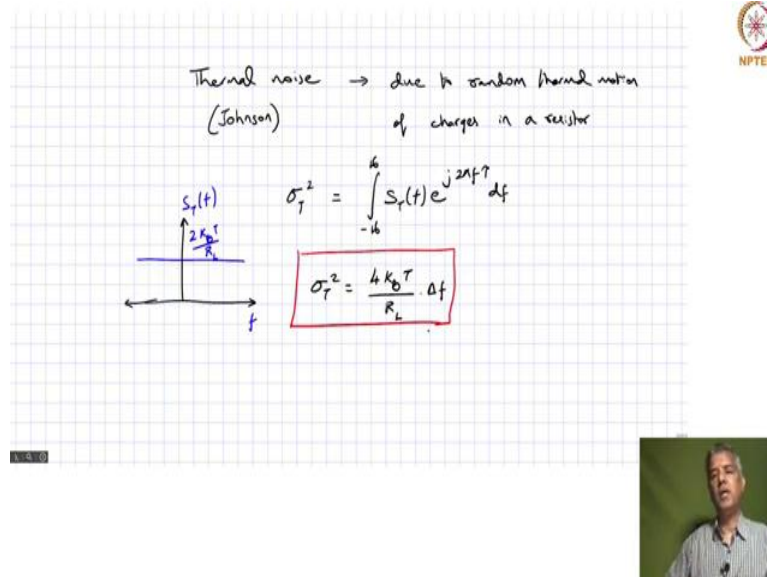
And quantifying all of this is, so when we look at all of this, this has been discussed in this paper, it is a fairly old paper in 1966. I think it is McIntyre, McIntyre et al in 1966 had looked at this, in the noise in an APD. And what they had reported was the noise would correspond to  $2Q$  times  $M$  square  $F_A$ , multiplied by  $I_P$  plus  $I_Q$ , sorry. what I meant is  $I_D$ . That is a dark current noise multiplied by  $\Delta F$ .

So yeah, why is that  $M$  square well basically because your photocurrent goes as is amplified as  $M$ , but what we are tracking is the variance. So, the photocurrent would correspond to the root mean square RMS value, but if you  $(\ )^2$  in the variance, then it is a square of the photocurrent.

So, that is why we have an  $M$  squared over there. And we saw that  $F_A$ , we define that previously, where  $F_A$  is given by  $K_A$  multiplied by  $M$  plus  $1$  minus  $K_A$  multiplied by  $2$  minus  $1$  over  $M$ . So, we looked at what is the effect of  $F_A$ . So, this is a generic expression, which holds code for both APDs as well as PIN. For a PIN photodiode,  $F_A$  equals  $1$   $M$  equals  $1$ , so then you just get, what we have over here, this  $2Q$  multiplied by  $I_P$  plus  $I_D$  multiplied by  $\Delta F$ . So, this is one of the key noise processes that happens in the receiver and this is happening right at the photodiode.

So, in some ways, you can say that whatever noise that you accumulate here is going to be propagating through the inter optical receiver, because, beyond this, you have the T I A and maybe other voltage amplifiers and so on, they are only going to multiply this noise. So, this is one of the fundamental limitations, as far as an optical receiver is concerned, you are not going to be able to get around that, and this once again is because of the fact that we have random arrival of photons at the receiver. And, that is what is initiating this noise process. So, now let us move on to another type of noise that we get to see in typical receivers.

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Thermal noise  $\rightarrow$  due to random thermal motion of charges in a resistor (Johnson)

$S_r(f)$

$\frac{2k_B T}{R_L}$

$f$

$$\sigma_r^2 = \int_{-\infty}^{\infty} S_r(f) e^{j2\pi f \tau} df$$
$$\sigma_r^2 = \frac{4k_B T}{R_L} \Delta f$$


This is what we call as Thermal noise. It is also called as Johnson noise in certain books. So, what is thermal noise? What is it due to? Well, it is due to random thermal motion of charges, say electrons in resistor. As it goes through a resistor. So once again, what we are saying is, when you have a current going through the resistor, we would like to think of it as if you take any small interval.

And you look at the number of charges that are present within that interval, we like to think of it as a constant value, but when it goes through the resistor, what comes through the resistor is going to be not a constant value, it is going to have some variance in the number of charges that are coming through.

So, this is what the thermal noise is all about. And like the name suggests, it is dependent on temperature. So, if we go through what we did previously, we said, you can quantify this thermal variance as once again minus infinity to plus infinity  $S_T$  of  $F$ ,  $E$  power  $J$  omega, sorry,  $J 2 \pi F \tau$ ,  $D F$ , if we can quantify this, well, that is, of course, what you get when  $\tau$  equal to 0, so, this exponential goes away and then you are just integrating over  $S_T$  of  $F$  and  $F$ ,  $S_T$  of  $F$  is once again it is characterized by white noise. So, you will find that  $S_T$  of  $F$  as a function of the  $F S$  is like this.

And the value of this is going to be given by  $2 K B T$  over the value of the resistor which we generally call us  $R_L$  where  $K B T$  corresponds to the thermal energy, that the resistor is experiencing. So, in certain ways you say that, if you can lower the temperature at which these components are kept, these resistors are kept, then you can reduce the randomness in

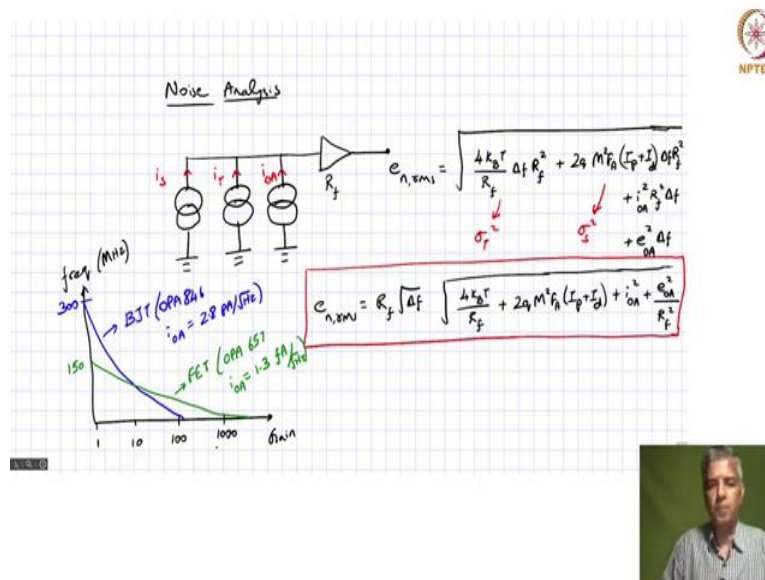
the in a flow of charges and the number of collisions essentially that is happening in this resistor, you can reduce that, and through that, you can reduce the overall variance.

So, if you plug this in here, once again, what you get sigma T square is equal to four K B T, where K B corresponds to the Boltzmann constant divided by R L and multiplied by delta F. So, the other factor of two that we got here is just because of the fact that it is a two-sided spectrum that we are looking at. So, we are integrating over that.

So, this is another fundamental limitation that we have, as far as an optical receiver is concerned. So, this is, when you are looking at this, you are saying, it looks like higher the resistor value lower will be my variance and, so the, so what I want to pick is a higher resistance value.

So, for example, in a trans-impedance amplifier, you say that the feedback amplifier is my load resistor. And then if I increase that feedback resistor, it is good. But mind you, that is not a true picture to look at. Because when we are looking at this noise, this is referred to as the input. So, what is it? These are all what is called input-referred noise. So, if you want to do a proper noise analysis.

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Let us look into that in a little more detail and say that when we consider the noise analysis of the optical receiver, you want to take whatever noise that you have in the receiver and refer everything to the input. Why do we want to do that? Well, if you have done all your noise reference at the input, then you can just multiply that by whatever gain of your receiver and then that is what you get at the output.



So typically, like we have been talking about, it is a trans-impedance configuration, which means that all your noise is referred the input as a photocurrent, something that is adding to the photocurrent, and then you have certain trans-impedance gain and so what you eventually get the output is going to be a voltage corresponding to that whatever noise current that you have.

So, in our circuit, we have let us say, these current sources. So, what are these current sources? Well, one of these you can say corresponds to the shot noise. So, now we have defined what shot noise, is shot noise variance is. So, variance once again is nothing but ensemble average of  $I_S^2$  square of  $T$ .

So similarly, we have defined  $I_T$ , that corresponds to the thermal noise. Similarly, you can also imagine that you have certain noise, because of your Op-Amp, because at the end of the day, when you, when you look at what is inside an Op-Amp, it is a bunch of transistors, resistors, that those sorts of circuits what you have, so each of those resistors in the Op-Amp is generating noise.

So that is once again, like, what we talked about is because of the thermal motion of a  $(\text{()})_{(30:55)}$ . So, this  $I_T$  is corresponding to all the external components that we have. But this current here, I would say, is because of the Op-Amp, so I am calling this  $I_{O,A}$ . So, all these are going into some trans-impedance amplifier, with a gain of  $R_F$ , and then what you are getting at the output of this is certain voltage, noise voltage  $E_N$ . So, you can just say, this is the RMS noise voltage that you get at the output. So, this is what we call us input, referred noise, So, you have taken whatever is happening in the circuit, you have just referred everything to the input.

So now, this is just an additional term to, your photocurrent. And then it is creating this noise. So, if we were to quantify,  $E_N$  now, you would say this, this is corresponding to the multiplication of whatever noise that we looked at before, we looked at the thermal noise, and we looked at the shot noise. So, I will represent all of that, along with, it is all multiplied by  $R_F$ . So, like we talked about before, it is, if you were to consider thermal noise, it is  $4 K B T$  over  $R_F$ , because  $R_F$  is the resistor that we are considering in the feedback circuit, multiplied by  $\Delta F$  multiplied by  $R_F$ ,  $R_F^2$  square because this corresponds to the variance.

And, what we are interested in is the RMS value. So, then, this is going to be this variance, multiply by  $R_F^2$  square, and then you are taking a hold root. So, this is important to understand. These processes that we are talking about, these are all independent processes,

one is not dependent on the other. So, when you have independent random processes that are adding together, let us say you have two Gaussian random variables that are adding together, what happens?

Well, the variance of these individual processes is going to add, so now we are considering three different processes, which are independent of each other. So, the variances corresponding to each of these processes are going to add, and then you are going to take a whole root of that, and that is when you get the RMS of the combined noise. So that is what we are doing.

So, this, we realize is corresponding to  $\sigma_T^2$ , and similarly, you are going to have another factor which is  $2Q$  multiplied by  $M^2$  multiplied by  $F_A$  multiplied by  $I_P$  plus  $I_D$  multiplied by  $\Delta F$ . And that again is going to go through again of multiplication of  $R_F$ . So, you have  $R_F^2$  coming into the picture here.

So, this once again, is what we are calling is  $\sigma_S^2$ . So, we have both these variances, adding with respect to each other. And then you also have like, we talked about this noise due to the Op-Amp itself. So, that is quantified as  $I_{O A}^2$ , which is the Op-Amp current.

So, here the Op-Amp characteristics are typically represented in terms of the current noise density. So, that is what  $I_{O A}$  corresponds to the current noise density of the Op-Amp. So,  $R_F^2$  multiplied by  $\Delta F$ . So, that is one term. And then the other term that you could also have is, there could be certain voltage noise also.

So, this is what we are talking about current noise, but there could be a voltage noise also, but, when we talk about voltage, the trans-impedance amplifier from a voltage gain perspective is a unity gain amplifier. So, if we are talking about voltage, this will be  $E_{O A}$  is the voltage noise density.

So,  $E_{O A}^2$  multiplied by  $\Delta F$ , is what we have. So, we have a common term coming out so  $E_N$  RMS can be written as so you take out  $R_F$  is one common term, that is the multiplicative gain, that we have for the trans-impedance gain, multiplied by this root of  $\Delta F$  is once again common across all of this.

Then you have root of, you have  $4k_B T$  over  $R_F$ , plus  $2Q$ ,  $M^2$   $F_A$   $I_P$  plus  $I_D$  that is your short noise current density, plus you have  $I_{O A}^2$ , plus  $E_{O A}$  that is the voltage

noise current density divided by  $R F$  square because you are taken out  $R F$ , so you will have to compensate by keeping that  $R F$  square.

So, this actually is representative of all the noise sources as far as your optical receiver is concerned. So, this is essentially it is contributed by the thermal noise. Once again, if you look at the overall effect of thermal noise, this is what we were talking about before it is  $R F$  square by  $R F$ . So, it is actually a multiplicative factor of  $R F$ .

So, it is wrong to think that larger the resistor you go to lower will be your noise. So, it is the other way when you go to higher value of resistances. As far as the trans-impedance gain is concerned. You are going to have more noise at the output in terms of the voltage and then you have the short noise component and then the Op-Amp contributions. So, these are what will essentially make your receiver behave in this way. So, the output voltage corresponding to your receiver is going to be generally noisy in nature. And of course, we will see how to reduce that noise.

When we consider Op-Amp noise, I can tell you, I mean we can make one generic sort of comparison. So, if you look at let us say the gain of an Op-Amp and with respect to the frequency that we have. The frequency of operation let us say in megahertz. Typically, we look at it the other way, gain as a function of frequency, but just flipping the axis, hope you do not mind that, but effectively the gain is going as 1 10 100 1000 and so on.

And if you look at what typically happens for say, a BJT based transistor, I will use a different color for that. So, I will use a different color for that. So, if you have bipolar junction transistor-based Op-Amp, they will typically have characteristics like this.

So, if you look at a particular Op-Amp, let us say BJT One example is OPA 846. So, if you consider OPA 846, you have noise current density that is in the order of about 2.8 Pico ampere root hertz. And of course, this capable of giving a reasonably good performance, this is around 200 to 300 somewhere over there, megahertz. Whereas, when you consider a FET type amplifier, the FET type amplifier is typically lower this frequency response.

So, this is typically about a value of 150. But, if you look at a FET type of amplifier, an Op-Amp field-effect transistor-based Op-Amp example of that is OPA 657. We looked at 656, 657, same family, but when you look at the noise current density. The noise current density is in the order of 1.3 pA per root hertz.

So, clearly, an FET type Op-Amp is going to have a much lower noise current density compared to a BJT. So, BJT is able to possibly give you a better frequency response, but when it comes to actually the noise performance a FET amplifier is actually much better. So, a lot of these optical sensor applications, we actually try to get a FET type of amplifier.

Especially at the front end because that is very, very important. The Op-Amp that you use for the T I A that has got to be very very low noise as low noise as possible because of the fact that any noise that is generated there is only going to be further degraded in the subsequent stages. So, we will have to be very careful about that.

So, we will look at, all these how a optical receiver works and what are the typical noise sources in the optical receiver, but we will go on to looking at how to use these optical receivers, how to mitigate this noise, and so on. And before we go to that, we will also look at what are the typical specifications for any optical sensor we will look at it next.