Fundamentals of Electric Vehicles
Technology and Economics
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Lecture 9
Aerodynamic drag, Rolling Resistance and uphill Resistance

I will now start my next part and I am going to get into more into Aerodynamic drag, Rolling Resistance and uphill resistance, give you enough examples and work out some numbers. But just want to point out as so far we have learnt that a vehicle when it moves it needs to overcome the Aerodynamic drag and Rolling Resistance.

It also has to overcome the uphill resistance and it will have to spend certain amount of force and certain amount of power for acceleration. We computed each one of them and we determined number over the things from the force we actually also computed the torque required at any time.

We also computed the total power required at any time, force into velocity and we finally computed what is the energy required which was a integration of power over time. We also studied the regeneration factor which helps us recover the energy during deceleration and during climbing down. Today we are only able to recover part of the energy may be 30 percent, so the factor R of the actual energy consumed is only about 70 percent. But tomorrow one can try to increase that.

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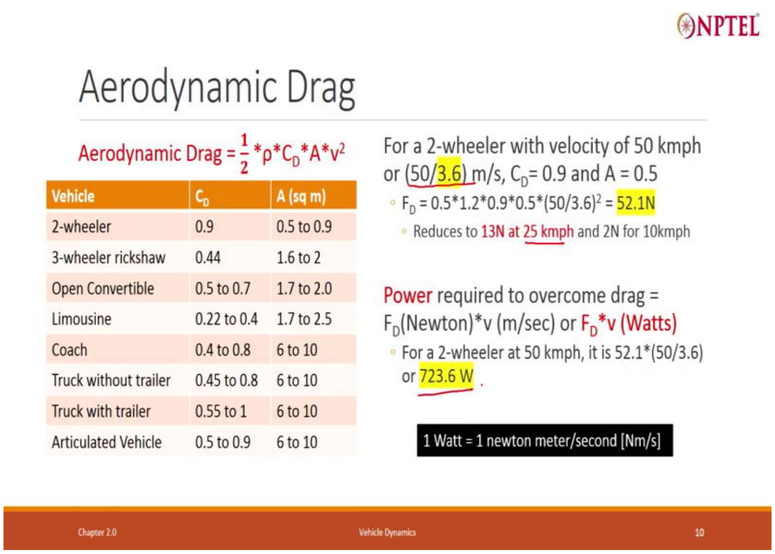

What we will now do is take actual examples by taking some numbers which are the numbers which are commonly used. Let us start first with Aerodynamic drag and as I, as we have studied Aerodynamic drag which is half into Rho, Rho is the air density, CD the drag coefficient, the projected area A into velocity square. What is a typical value of CD and area for different devices?

If you see, projected area of two wheeler has a very small projected area, three wheeler has a larger, car has still larger and if you take a coach or a truck or a truck with a trailer or something like that number will go bigger and bigger. This is what it is, that is a amount of air it will cut and therefore that much force will have to be applied. The drag coefficient on the other hand is 0.9 for two wheeler, three wheeler is only 0.44, for the cars it is more like 0.5, 0.6, for limousine, it is less 0.22 to 0.4 for a coach between 0.4 to 0.8.

So you have to figure out what these two numbers are. But I have given you some typical numbers and we will actually use that in our work. So now we actually calculate and this is something that we are done I will take the same two wheeler that we talked about except I will take a 50 kilometer per hour. At 50 kilometer per hour which is equal to 50 by 3.6 meter per second is the velocity, CD is 0.9 I will take 0.9 for two wheeler and area is 0.5. And I compute the force and it comes out to be 52.1 Newton. This is at a velocity of 50 kilometer per hour.

Now, instead of 50 kilometer per hour, suppose I go to 25 kilometer per hour what will happen? My velocity is becoming half, so my force is going to become by 1 by 4 so the force is not 52 Newtons but only 13 Newton. On the other hand, if I go to 10 kilometer per hour so from 50 kilometer if I go to 10 kilometer per hour, I am going one fifth the velocity my force will go by 1 by 25 so I will only get 2 Newtons.

So, this drag is totally dependent on the velocity and the drag coefficient will keep on going up with the velocity very, very rapidly because this is v square. What are our power requirement? Power requirement is force multiplied by velocity. Remember, force is in Newton, velocity is meter per second so Watts is given as Newton meter per second, Newton meter per second that is how Watts is given.

So, for a two wheeler at 50 kilometer per hour 52.1 Newton is the force and you have to multiply it by the velocity 50 divided by 3.6 remember kilometer per hour to meter per second and that comes to 723 Watts. So, for two wheeler at 50 kilometer per hour it only consumes only 723 Watts. This is for the drag coefficient this does not include anything else.

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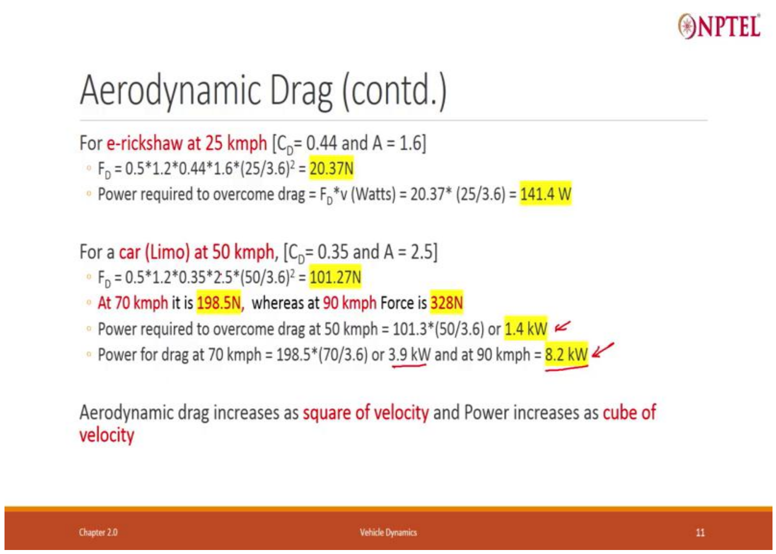

What about E rickshaw? A E rickshaw is commonly used. E rickshaw is limited to 25 kilometer per hour and the drag coefficient is 0.44 and this could be taken as (1.6), area can be taken as 1.6 larger. And we recompute the force 0.5 into Rho into drag coefficient into area multiplied by 25 by 3.6 kilometer per hour being converted to meter per second whole square and this works out to be 20.37 Newtons. This is a 25, 25 kilometer per hour.

Power required to overcome on the other hand you have to multiply one more velocity, one more time the velocity so one more time 25 by 3.6 and E rickshaw requires only 141 Watts for the drag. It does not require that much power for the drag. Because the velocity is very slow, small remember that.

For a car a Limo at 50 kilometer per hour, the drag coefficient goes down a bit 0.35 area goes up 2.5. And therefore the force is 0.5 into 1.2 into 0.35 multiplied by 2.5, 50 by 3.6 square and it comes to 101.27 Newtons, 101.27 Newtons. What about at 70 kilometer per hour you have to multiplied by 7 by 5 square so which is 49 by 25 which is approximately double so approximately 200 Newtons.

At 90 kilometer per hour, it further increases to 328 Newtons. So, it shoots up, starts at low value with the power keeps on shooting up. Remember the power is a function of v cube, why? Force is a function of v square and then power is force multiplied by velocity so it is a v cube. So, as the velocity keeps on going up it keeps on shooting up like anything.

So if I want to compute the power required to overcome drag at 50 kilometer per hour it comes to 1.4 kilometer, kilowatt and for a car and at 70 kilometer per hour it comes to 4 kilowatt and at 90 kilometer per hour it is 8.2 kilowatt. But this is just for the drag. So, just imagine in a E rickshaw I can manage with 1.41 Watt and for a car at 90 kilometer per hour it is 8.2 kilowatt. How much times more? 7 times, it becomes 1 kilowatt 56 almost 55, 60 times the power. Why?

Because first of all the velocity from 25 I have gone to 90 kilometer per hour and remember the velocity cube comes in so if I had a factor of 4, 4 cube would have been 64. So, a factor of 50 just come because of velocity. Because if I look at the other parameters 0.5 remains same, 1.2 remains same it was 0.44 into 1.6 it is 0.35 into 2.5, they are roughly the same.

So, the whole factor actually come due to the velocity. And this is something that you must remember and we will see the implication of it as we go on. So, aerodynamic drag increases square of the velocity, power increases the cube of velocity. This is something that we have to worry about all the time.

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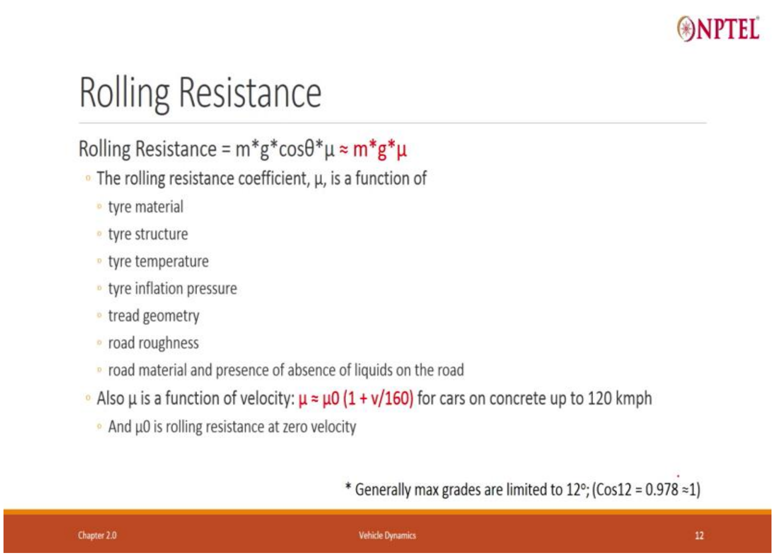

Let us now come to the next force the rolling resistance m g Mu, m g Mu cos theta, cos theta is approximately equal to, to 1 this is what I have seen, even a 12 degree, cos 12 degree 0.978 is approximately 1 so I will ignore that. So, that is m g Mu, this is not a function of velocity, it is a constant. Power will be a function of velocity because power will be force multiplied by velocity, rolling resistance is the force.

The rolling resistance coefficient depends on the Mu, depends on tyre material, tyre structure, tyre temperature, tyre inflation pressure if you have less air in the tyre you will see the rolling resistance increases, thread geometry, road roughness and road material and presence of liquids on the road. All these things will change the rolling resistance. So, very often it is talked about a smooth tarred road so that is what we often use but you know you all do not get smooth tarred road many times in which case rolling resistance will tend to increase.

What about Mu? Is Mu a constant, yes and no? Mu is normally a constant at low velocity but as the velocity increases there is a factor of 1 plus v by 160 where v is in kilometer per hour. So, v is 80 kilometer per hour there will be of Mu increases by 1.5 . But if v is 50 kilometer or 40 kilometer per hour it has a small impact on v Mu. So very often this extra term is ignored but of you need to take into account you can take that into account.

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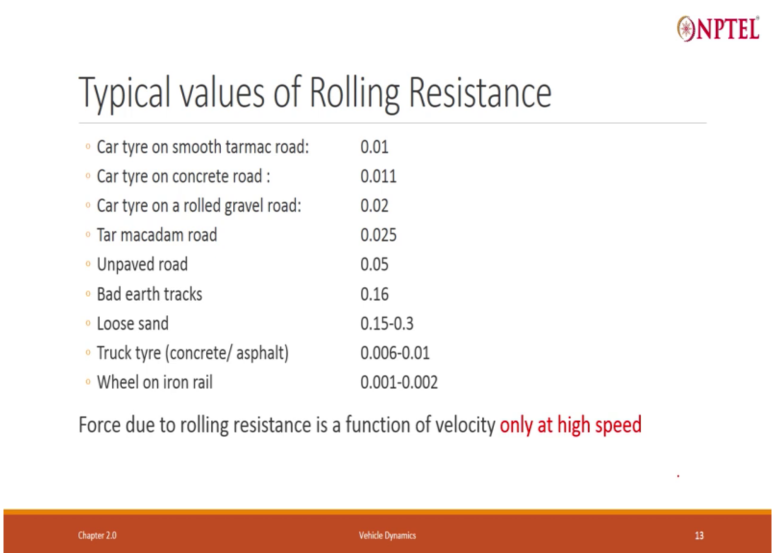

Now just to give you a value of Mu, Mu 0 the term that we have use for car tyre on smooth tarmac road is 0.01, on concrete road its 0.011 slightly higher, car tyre on the gravel road increases quite a bit 0.02 as there are various other roads where it can go up 0.25, unpaved road it can go to 0.05, the bad earth track it can go further but as wheel on iron rail this is more for the train we are computing, it is actually 0.001 very little rolling resistance to 0.002.

But for most purposes it will be 0.01 to 0.011 and that is the value that we will commonly use. Force due to rolling resistance is function of velocity only at high speed. Remember, that Mu is Mu 0 into 1 plus v by 160. So, we will generally ignore function of velocity.

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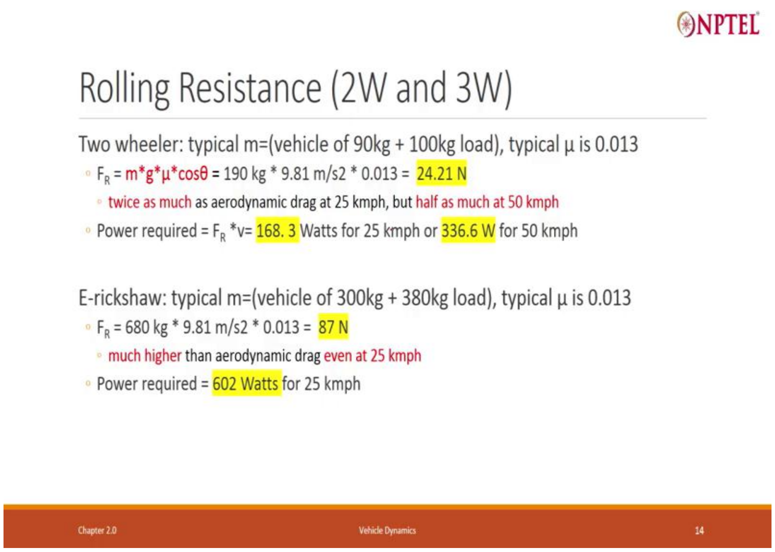

Let us however compute the value. Force is equal to m g cos theta, so let us take a vehicle two wheeler, 90 kg plus 100 kg load 190 kg. Take a value of 0.013 which is quite common and m g Mu cos theta is approximately equal to 1 mass is 190 into g is 9.81 meter per second square and 0.013 it comes to 24.21 Newton. So, actually if I now remember for a two wheeler, for a 25 kilometer I got almost half the force. So, it is twice as much as 25 but half as much as 50; at 50 kilometer because there was a v square factor the it had gone to 50 Newtons here it is only 24.21 Newtons. We will again compare that later on.

Power requirement again you can calculate multiplied by velocity and it comes to approximately 336 Watt sorry, 168 Watt for 25 kilometer or 336 Watt for 50 kilometer per hour. Let us redo this thing same thing for E rickshaw mass is much larger ignore, mass is much larger mass is 300 kg plus 380 kg of load Mu value is similar 0.013 and you calculate therefore the force comes out to be higher 87 Newton, much higher than aerodynamic drag even at 25 kilometer per hour. Power requirement you have to multiply it by the velocity it comes to 625 602 Watts for 25 kilometer per hour.

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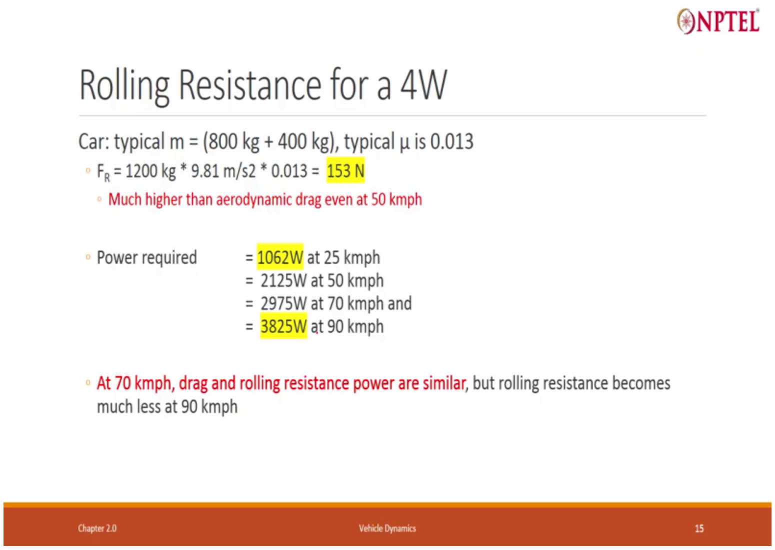

What about for the car? Car weight is higher 800kg plus 400kg of people. So, typically 1200 even it can more. Mu is similar 0.013 because Mu is similar the car weight is higher, Mu is similar, g is 9.81 so the force comes out to be 153 Newton, much higher than aerodynamic drag at 50 kilometer per hour because the weight is much higher.

Power requirement on the other hand shoots up. 1062 1 kilowatt at 25 kilometer per hour but at 50 kilometer per hour it goes up to 2125, 2975 and 90 kilometer it comes to 3825 at 90 kilometer per hour. At 70 kilometer per hour drag and rolling resistance for a car is roughly similar. But at 90 kilometer per hour the drag crosses the rolling resistance. We will actually put all these things together and compare that but this gives you a good idea of what we are talking about.

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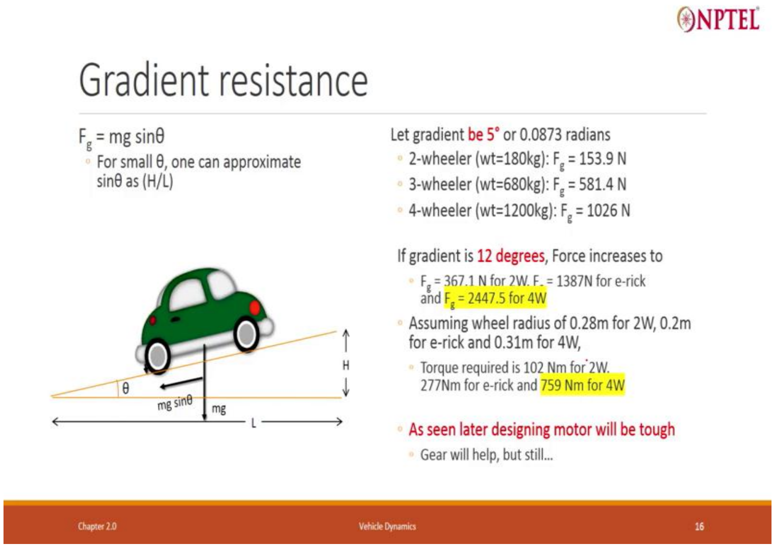

And finally the Gradient resistance mg sin theta. And for small theta one can approximate sin theta as simply H by L (()) (17:03) it is equal to theta. For a gradient of 5 degrees or 0.0873 radians for two wheeler, for 180kg mg sin theta comes to 153 Newton but for three wheeler it comes to 581, for four wheeler it comes to 1026. Means mass keeps on increasing 180kg, 680kg, 1200kg the gradient keeps on going on.

What about gradient of 12 degrees? 12 degrees is much higher from 5 degrees if it goes to 12 degrees, the force considerably increases it becomes equal to 2447 Newtons for four wheeler. For E rickshaw it is (13.) 1387 and for two wheeler it is 367. So, approximately becomes 2.5 degrees whereas 12 degrees you get that value. Assuming wheel radius of 0.28 meters for two wheeler, 0.2 meters for E rickshaw and 0.31 meter for four wheeler. Torque requirement, torque requirement first time we are computing torque requirement.

Torque requirement is 102 Newton meter for two wheeler, 277 Newton meter for three wheeler and 759 Newton meter for four wheeler. Now this 759 (800), 750 Newton meter are very large numbers again you do not get a picture of it you will get a picture of it when we will put all these things together and we will see that really are in some amount of trouble. So, as seen later designing of motor will be tough with such kind of torque. So, torque is going to play a important role.

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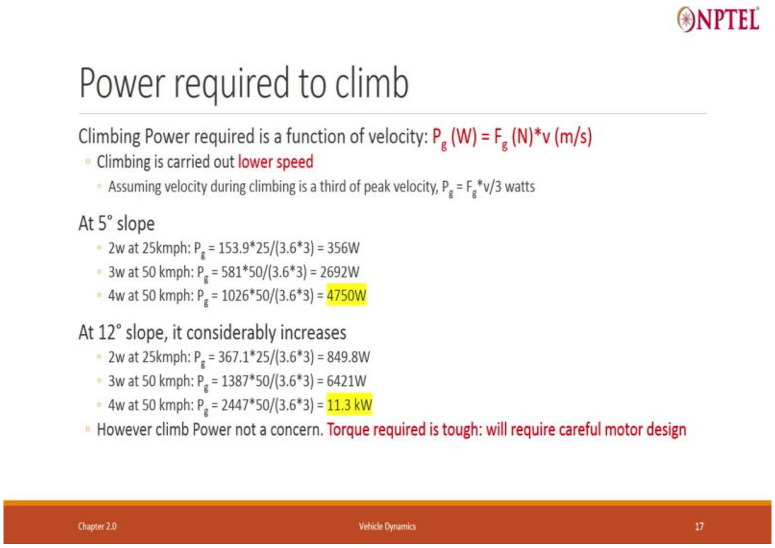

So, what is a power required to climb? Power is again force multiplied by velocity, now what is a velocity that I choose? Do I take the peak velocity? It will become huge power because force is large and if I try to climb at high velocity it is a huge power I will never be able to do that.

So, typically what one does is that on climbs set a lower velocity. You must have seen that whenever you climb up the hills you start driving slowly, so typically a third of the peak velocity. So if the peak velocity is v we will take it as v by 3. If I take power is equal to Fg into v by 3, where v is the peak velocity, I can calculate at 5 degree slope for a peak to, two wheeler it is 356 Watts not that bad, three wheeler is 2692 and for four wheeler it is approximately 5000 Watts.

And at 12 degrees slope it shoots up like anything if you see it is 850 Watts for two wheeler, three wheeler is approximately 6.5 kilowatt and for four wheeler it is 11.3 kilowatt. That is the power required to climb. Again you will find that climb power is not as much of concern because I am taking velocity as one third of the peak velocity. However, see this 50 kilometer per hour divided by 3.6 divided by 3 I am climbing not at the peak velocity but 17 kilometer per hour and that is a reason still 11.3 Newtons will come. Torque requirement will become tough as we will see.