Fundamentals of Electric Vehicles
Technology and Economics
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Lecture 8
Vehicle Dynamics

Welcome. We have so far introduced electrical vehicles in context of India. Today we begin with a very important chapter called Vehicle Dynamics. This chapter is strictly speaking common to a electric vehicle as well as a internal combustion engine vehicle, there is not much of a difference. What does it try to do? It tries to for example look at when a vehicle drives what is the power that it requires.

What is the peak power that it requires? What is the speed at which you are moving? What is the torque that is required? And finally how much energy you will be required? These are very important aspects because only when we understand the vehicle requirement we will be able to sort of say well we want a motor which such and such specifications, which can give so much peak power, so much average power, so much peak torque.

So much speed, we have to figure out each of these parameters in the vehicle being driven will reflect onto the motor. And finally it will also reflect on the batteries, is the battery capable of giving me the power that we really need? Is the battery capable of giving me the total amount of energy that I will need for certain drive?

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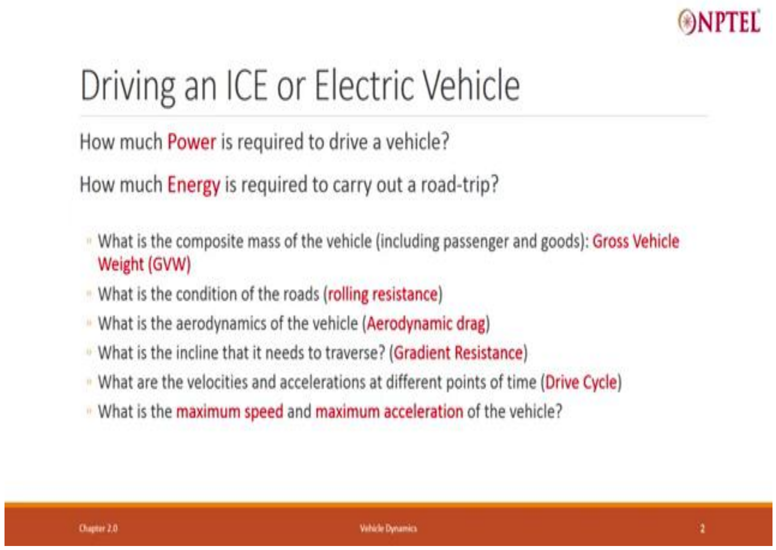

So in this chapter we will attempt to actually compute a simply vehicle dynamics; how much power is required to drive a vehicle? How much energy is required to carry out a certain trip from one place to another trip? So for us to be able to answer this we need to answer a few things about the vehicle. The first thing that I have to answer is what is the gross vehicle weight, weight of an empty vehicle plus weight of all the people and goods loaded on it.

Because that will impact the power, the energy everything will be impacted by the composite mass of the vehicle also called GVW; the Gross Vehicle Weight. It will depend on what is the road condition. Is the road very rough, is road very smooth, what is it made off? And there will be important parameter called rolling resistance.

A rolling resistance that of the road, that road will offer to the vehicle will determine many of the other things that we are talking about. Similarly, for a vehicle what is the aerodynamic drag? As the vehicle moves it cuts through certain air, air apply certain amount of pressure. So how do you compute the aerodynamic drag?

We also need to know does it have to travel incline? And if it has to travel in incline at what speed it travels? An incline will make a lot of difference in all the parameters that we are talking about, it is called gradient resistance. Just like rolling resistance on a flat road or on a any road aerodynamic drag due to the air and the gradient resistance what it requires to climb a vehicle.

We are then going to look at a very important component called drive cycle. How does the vehicle travel? Does it have what kind of velocities at which it travels. Does it have resistances? Does it have accelerations? So all these questions I have to answer and define what is called drive cycle of a vehicle. And of course I have to always worry about the maximum speed of the vehicle, maximum acceleration of the vehicle. As we start understanding these parameters we will be able to understand using the vehicle dynamics how much power, energy etc., that are required. And this is what we will do in this course.

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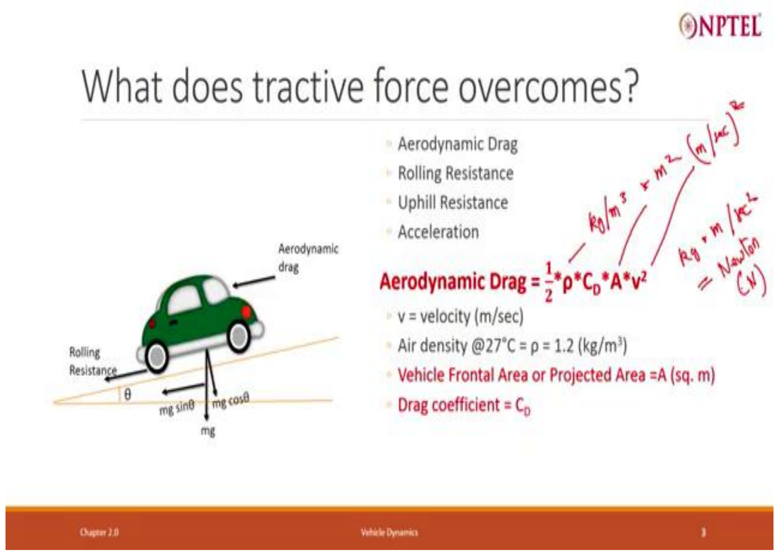

A vehicle, see this vehicle here, this vehicle actually undergoes one of the forces that it undergoes. There is aerodynamic drag, the vehicle is moving in this direction, there is aerodynamic drag which is trying to push the vehicle back, this is called aerodynamic drag.

The wings are rotating they will offer a rolling resistance, here I will have to worry about rolling resistance. And then of course the weight of the vehicle mg. This of course I have shown on an incline with an angle theta, now theta may be 0 in which case it will be a flat road. And the weight mg will have two component, one along the surface of the road that is the mg sin theta another is perpendicular to the surface of the road and is mg cos theta.

Now if theta is small we will see mg cos theta is approximately mg, cos theta is 1 so we can kind of ignore that but we have to actually worry about all these parameters. The one of the first that I have to, the force that I will define is what is called aerodynamic drag. Now this is a standard textbook equation that aerodynamic drag is defined by half into rho, rho is the density of the air.

And at air density at 27 degree centigrade rho is 1.2 kg per meter cube, so in this course pretty much we can take it rho as 1.2 kg per meter cube. What is a parameter called CD, the drag coefficient? The drag coefficient is going to play important role. What is the projected area, see air is going to hit the vehicle what is the area which it will hit?

It will be given in square meter, the velocity and what is the velocity at which it will move? That will be given in meter per second, drag coefficient Cd which is does not have a any units, air density, kg per meter cube and vehicle frontal area projection is A square meter and the aerodynamic drag is half into rho into CD into A into V square.

So what will be the unit of aerodynamic drag? We can actually calculate the aerodynamic drag, so rho is given in terms of what? Kg per meter cube, CD does not have units, area is into meter square and velocity is meter per second whole square. So what is that unit works out to be; this will come out to be kg there is a meter cube below, meter square on the top it will cancel, so but there is 1 meter square here.

So it will into meter and divided by second square. So kg into meter per second square which is equal to newton the or called Newton, the unit of the force. So aerodynamic drag is the force, is the force applied, it is half into rho into CD into the area multiplied by the velocity square. Now it is a velocity square we will see we will actually talk about it as we go on the implication of it.

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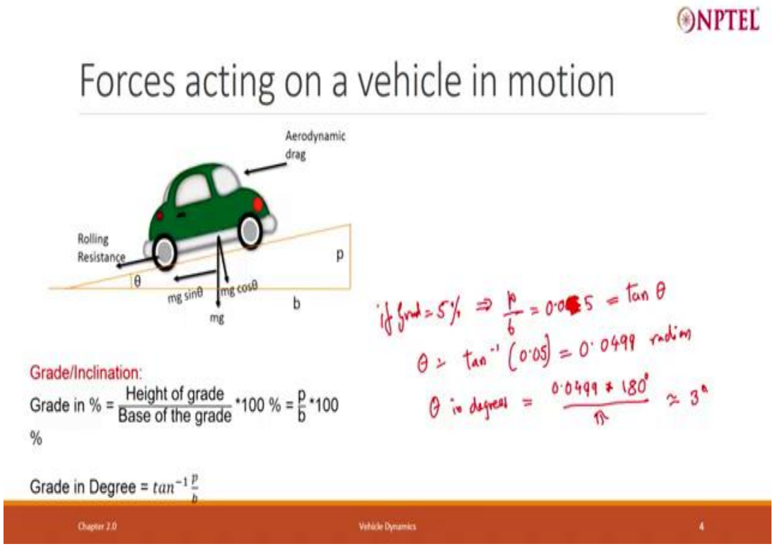

Before I proceed, I want to talk about this theta, how is this gradient defined? Suppose the gradient is defined in terms of 5 percent, suppose if gradient is 5 percent, it is 5 percent that means p by b is 5 that is what it means. So 5 percent implies p by b is equal to 0.005, sorry yes 0.05 I made a mistake, 0.05. If the p by b is 0.05 what is a gradient? This is tan theta is p by b, so this tangent of theta.

So what is theta equal to? Tan inverse 0.05 and this works out to be very-very close 0.0499 radiance. So theta is same 0.05 radiance if I want to convert into degrees, theta in degrees is equal to 0.0499 that is in radiance multiplied by converting radiance into degree is 180, degree is by Pi and if I do this this will be approximately 3 degrees.

So this is a something that you need to be able to do very-very commonly. 5 percent gradient you may get the answer in 5 percent which basically means p by b is 0.05 or the angle theta is 0.05 radiance which approximately is 3 degrees and you may have to often do this conversion sometime theta may be given in degrees, sometime it may be given in radiance, sometime it may be given as percentage. Slope may be given as percentage. The slope is important in wherever the vehicle is going to climb and we need to do this.

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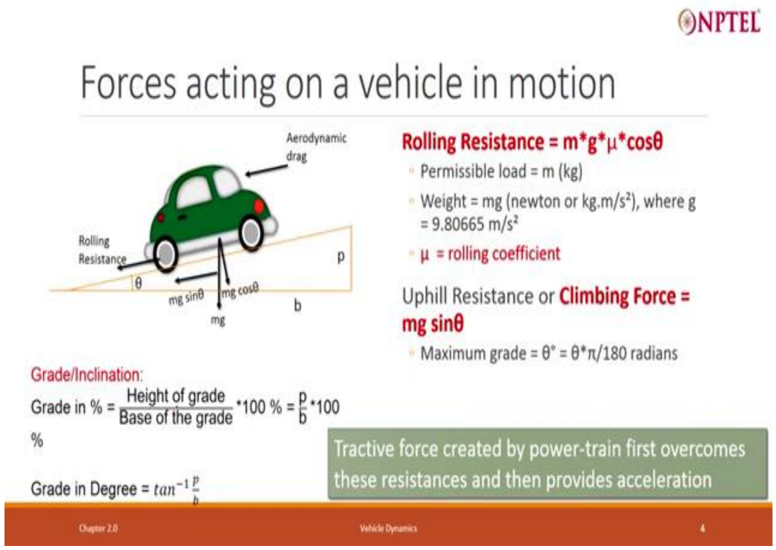

So basically I have sort of say you will often get a slope vehicle has to travel up a slope and when it travels up a slope you will have to define theta, theta can be defined in percentage, very often it is gradient in percentage; 5 degree, 8 degree, 10 degree, (sorry) 10 percent, 5 percent, 8 percent.

You have to be able to convert it into theta in radiance and theta in degrees this is what I actually just now did. So given that this is the gradient what are the slopes that the vehicle has to go through? The rolling resistance it has to face is mg into Mu into cos theta and m is the mass in kg, mg is the total weight where g is 9.8065 meter per second square and Mu is the rolling coefficient.

And as I pointed out unless theta is large cos theta will proximately be equal to 1 and we can often ignore that. So this is the second force that it has to worry about, rolling resistance it has to worry about, earlier I had pointed out that it has to worry about the aerodynamic drag, the rolling resistance and finally the climbing force, mg sin theta. So mg sin theta will be working rolling resistance will be working, aerodynamic, these are three resistance.

So when it is climbing it has to overcome these 3 resistance and mg sin theta well if theta in degree or theta in radiance we can actually do that and that will give us the climbing force. So there are three forces that the vehicle will always encounter. Now if it is a flat road theta will be 0, and mg sin theta will be 0.

But the rolling resistance and gradient and aerodynamic force it will always encounter. So tractive force created by a power train will have to first overcome these three resistances always. The aerodynamic, rolling resistance and gradient resistance and then the vehicle will start moving and it can accelerate. So then it will have to have a acceleration power.

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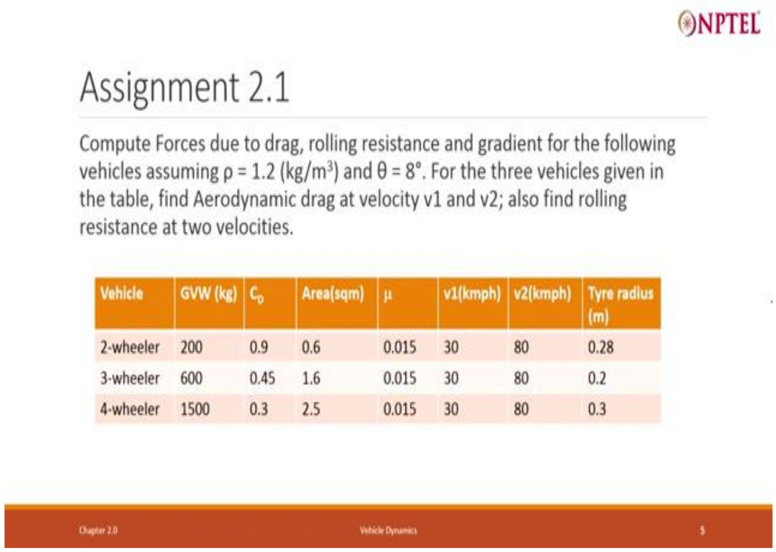

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Assignment 2.1

Compute Forces due to drag, rolling resistance and gradient for the following vehicles assuming ρ = 1.2 (kg/m^3) and θ = 8°. For the three vehicles given in the table, find Aerodynamic drag at velocity v1 and v2; also find rolling resistance at two velocities.

Vehicle	GVW (kg)	C_D	Area(sqm)	μ	v1(kmph)	v2(kmph)	Tyre radius (m)
2-wheeler	200	0.9	0.6	0.015	30	80	0.28
3-wheeler	600	0.45	1.6	0.015	30	80	0.2
4-wheeler	1500	0.3	2.5	0.015	30	80	0.3

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So this is the smallest assignment that I am actually giving it to you more to get to familiar with this. I have taken three vehicles 2-wheleer, 3-wheeler and 4-wheeler and a typical 2-wheeler can have a weight off around 200kg of course there are, this is with person. 3-wheeler will be a little bit heavier weight there will be 3-4 people, so it will be 600 kg and 4-wheeler will may be 1500 kg.

Thus see the drag coefficient we swill study that later on. I have put some value for the drag coefficient I have put some of the area of projected area. And I have also given a value of Mu, various Mu value. The velocity can be different and I have taken two different velocity. Remember that aerodynamic resistance is depend on velocity square.

So that is going to play a role and the tyre radius also I have given as 0.28 for 2-wheeler, 0.2 for 3-wheeler and 0.3 meters for 4-wheeler. Now given this compute forces, individual forces due to drag, due to rolling resistance and due to gradient assuming theta equal to 8 degrees. Assume rho equal to 1.2 kg per meter cube.

For these 3 vehicles find aerodynamic drag at both velocity 1 and velocity 2 and the rolling resistances and the gradient resistance for the 3 vehicles. So this is of assignment problem you need to solve them and give the answers. Let us continue, let us take first of this 2.1 A for the 2-wheeler and let me just solve this for you.

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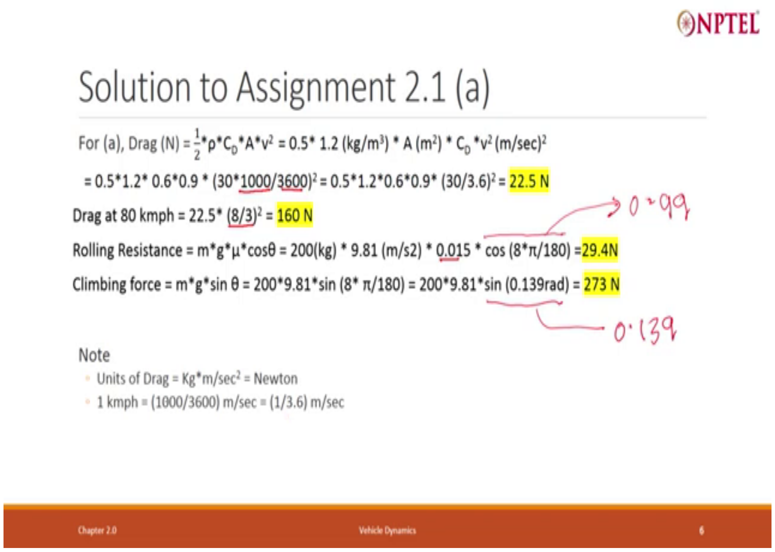

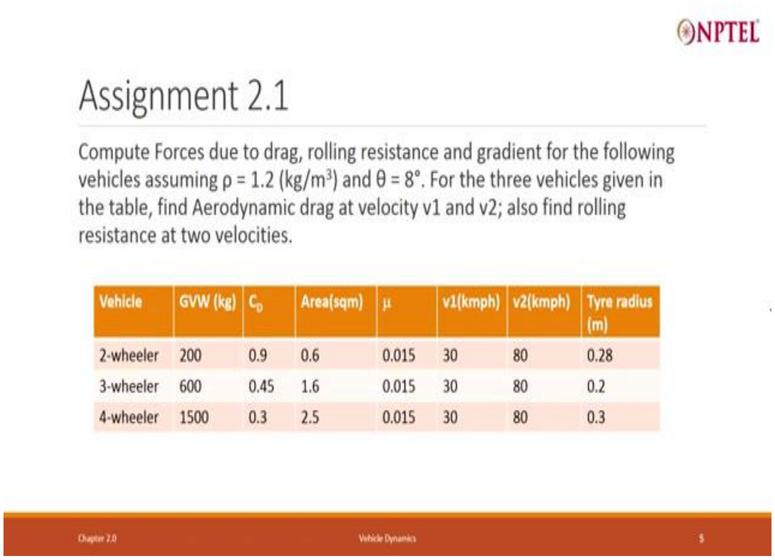

For 2.1 A, the drag what is a drag coefficient? I have mentioned it was half rho CD A into v square, half is 0.5 rho is always 1.2 kg per meter cube area into CD into V square and therefore I have put here 0.5 into 1.2 and area for the first vehicle is 0.6 square meter. So I am actually

putting 0.6 and then comes sorry CD is 0.6. The area is 0.9 I think that is what it is that is a double check.

Well CD is 0.9, area is 0.6 so I have put area first multiplied by the CD and then the velocity square. What is the velocity? Velocity I have to work at 30 km per hour. Now 30 km per hour is a kilometer per hour I need to convert it into meter per second. So 30 to convert it into instead of kilometer to meter I have to multiply it by 1000 and per hour I have to convert it to per second so I have to take 60 into 60 or 3600.

So this is the expression for the drag which comes out to be 0.5 into 1.2 into 0.6 into 0.9 and this 1000 by 3600 it will always come from kilometer per hour to meter per second, it is actually 1 by 3.6. So it is 30 divided by 3.6 whole square and this comes out to be 22.5 newtons. So our 2-wheeler will give you a 22.5 newtons at 30 kilometer per square.

Now what will it give to you at 80 kilometer per hour? 80 kilometer per hour the rest of the expression will remain same rho so CD A does not change, velocity changes from 30 to 80. So what I have done, I have taken 8 by 3 whole square. So this is what I have done 8 by 3 whole square and 8 by 3 whole square is 64 by 9 which is approximately 7 times I multiply 22.5 by proximately 7 times and I get 160 newton.

So see this is a very interesting result for your first thing that you need to learn because aerodynamic resistance is v square, for a low velocity it is only 22.5 newton meter of 30 kilometer. Once you go to 80 kilometer it actually go to 160 newton very-very high, we will see the implications of these things as we go on.

How do we compute the rolling resistance? Rolling resistance is mg mu cos theta, so m is 200 kg for the vehicle we had put vehicle as 200 kg the first vehicle 200 kg, so 200 kg into 9.81 meter per second square multiplied by mu, mu as given as 0.015 and cos theta, cos theta you can calculate it is approximately equal to 1 and this comes to 29.4 newton.

Remember this is independent of velocity, so rolling resistance is not a functional velocity, aerodynamic resistance is a square of the velocity. Similarly, we can compute the climbing force, if I take theta equal to 8 degrees, it is m 200 kg into 9.8 into sine into 8 and then Pi by 180 because degrees has to be converted to radiance to give sin of radius.

And this come to 200 into 9.8 into sin of 0.139 radiance which is 273 newton you also find by the way sin of a small angle like 0.139 radiance, sin theta is approximately theta so this value sin over 0.139 is approximately 0.139 you multiply all of this and it comes to 273 newton. So if you see it is a climbing which requires the maximum force 273 newton.

At low velocity 30.30 kilometer per square your drag is there, very small compare to the climbing force and similarly rolling resistance is small compare to climbing force. But if the velocity increases to like 80 kilometer per hour it is already 160 newtons and if it is 100 kilometer per hour it will further go up.

So to conclude what can I say, I can say that we have to always when we calculate the all the forces we must always remember the rolling resistance and aerodynamic resistance at low velocity will give you a low value. Generally, the climb will take a large amount of force and at high velocity the aerodynamic resistance will increase.

And we will see the implication of it when you go further. Remember these are two units that I have used; one is unit of drag which is kg meter per second square which is same as newton which I did that in the previous slide also and the second is a kilometer per hour is always 1 by 3.6 meter per second this is something that I said 1000 divided by 3600, this is something that you will have to use again and again and again. So I am actually spending a few minutes out here to just get you to understand this.

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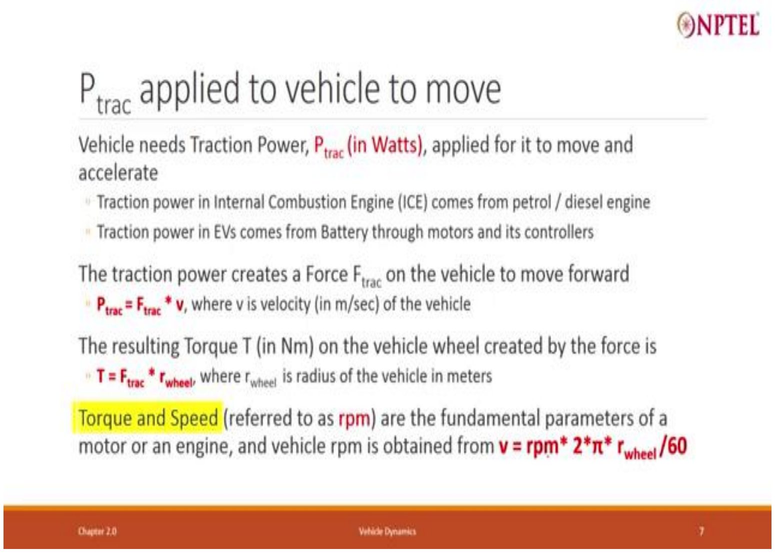

So now I am ready to say what is the total traction force. Vehicle needs a traction power applied for it to move and accelerate. So traction power has to be minimum as sum of the 3 powers; the aerodynamic resistance, rolling resistance and climbing force if there is any climb that will just be enough to keep it moving and if I want to accelerate I will need more power.

So traction power in internal combustion engine actually comes from a petrol and diesel engine. For electric vehicles it will come through motor and controller. So in some sense I am actually determining what is a kind of power that are require for my motors and controllers. So the traction force creates a force called F-track on the vehicle to move forward.

But this is the force, what about the velocity? What about the power? The power is traction force multiplied by velocity. Power is force into velocity, so we have to actually calculate the traction power not traction force is not enough, we have to multiply it by the velocity and that gives a important result and we will come to that.

Similarly, there is one more parameter it is extremely important rather than the power, I t is what is the torque required? Torque required is also dependent on the force but it is force into the wheel radius, the torque is force into wheel radius. So wheel have to actually compute the power requirement and the torque requirement, this will be two fundamental things and they are independent.

Remember while both of them are dependent on traction force one of them is traction force multiplied by velocity another is traction force multiplied by the wheel radius. So actually torque is always proportional to the force alone whereas traction power is proportional to both force as well as the velocity, wheel radius being a constant, okay. So this is what we have to actually worry about.

So we will learn throughout the course that traction and speed and by the way speed for a motor is referred to in RPM, revolutions per meter, no, minute, revolutions per minute. So that is how the speed is referred to while the speed in the units that we will refer to is meter per second but for the motor it is revolutions per minute.

And we will need to convert revolutions per minute into the meter per second are the two fundamental parameter; torque and speed. In fact, motor is always designed for certain torque-speed characteristics. So are the fundamental parameters of either a motor or engine and a vehicle rpm is related to the velocity, how is it related? Revolutions per minute, so rpm multiplied by 2 Pi into radius of the wheel this will give me the total meters that I travel. But this will give me meters per minute, now to get meters per second I have to divide it by 60. So velocity is rpm into 2 Pi r wheel divided by 60.

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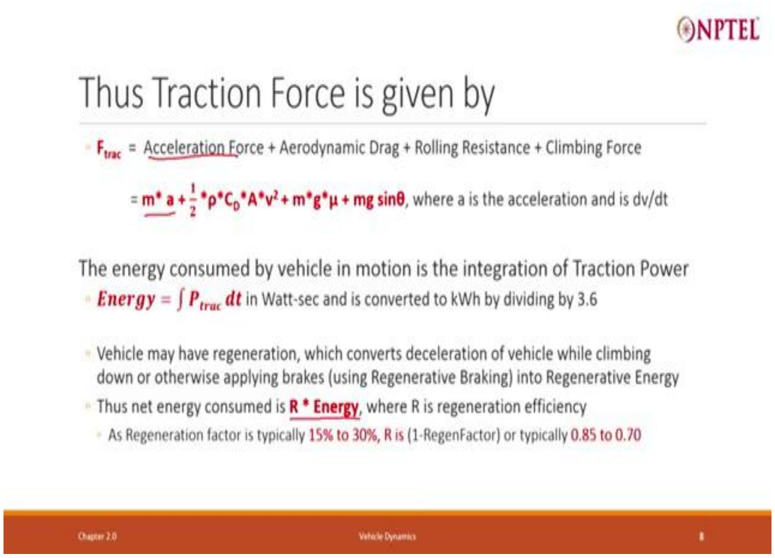

What about traction force? Traction force I have already said it is acceleration force I had not defined the acceleration force but all of us know that the acceleration force is ma plus aerodynamic resistance plus rolling resistance plus climbing force. And this is the aerodynamic resistance plus aerodynamic drag plus rolling resistance plus climbing force, where a is acceleration in dv by dt this will give me the traction force. Once I know my traction force I can know the power because power is traction force multiplied by the velocity. This of course by the way also include the acceleration. What the acceleration is we do not know yet.

How do we compute another very fundamental parameter called energy? It is a total power integrated over time. Different amount of power may be required for the vehicle at different time, why different power may be required? Because some time they will, the vehicle may moving faster, sometime it may be moving slower, sometime it may moving up the slope so we will have to compute the power every instant and integrate the power overtime.

And this will give me in watt second the total energy, power I will get in watts and energy I will get in watt second and this watt second will then can be converted into watt hour and kilo watt hour by dividing it is actually watt second can be converted to kilo watt hour by dividing by 3.6, why? Because 1000 watt, kilo watt is a 1000 watt, hour is 3600 seconds, so if I divide it by 3.6 I will get kilo watt hour.

So this will give me energy, energy will always be in watt second or watt hour or kilo watt hour. Force will be in newtons, power will be in watts and speed with either will be in rpm or meter per second. These are the parameters of extreme importance and finally the torque, I forgot the torque, torque is in newtons multiplied by meter because the it is the force multiplied by the wheel radius. So these are the 5 fundamental parameters that I have to always worry.

This is so far whatever I have done is common to a internal combustion engine vehicle and a electric vehicle. But here the first difference will come when I talk about electric vehicle the electric vehicle will have something called regeneration. What is regeneration? Suppose vehicle is moving at some speed and you are applying brake to decelerate.

In a normal vehicle, in a petrol vehicle when you decelerate you apply brake what happens? That it was moving at a certain velocity, so it has some kinetic energy, now this kinetic energy will be lost as heat, the break will produce heat and it will be lost as heat that much energy you lost, you cannot do much about it.

We will later on show you, not so in electric vehicle, it is electric motor which is being driven to take the vehicle forward. When I want to decelerate I do not necessarily apply a break which is friction force against the rotor to stop it but actually I try to start applying a force on the motor reverse force, to reverse it, to slow it down and I can do it electrically.

Now when I do that, when I try to slow down electric motor just like I give a electrical power, electrical power to get the motor to move, when I want to slow it down it will convert it back into (electrical power) electrical energy that electrical energy actually can be fed back to the battery. So breaking is not all loss. Theoretically you can design something which can recover all the breaking energy back into electrical energy and put it into the vehicle.

In the regeneration so there is a concept called regeneration efficiency, regeneration efficiency is when you convert it back what is the loss percentage, loss ratio. If the regeneration efficiency is 1 it means there is no loss at all. In the regeneration efficiency is 0.8 that means 20 percent is lost, 80 percent you are able to, if regeneration factor is 80 percent you are able to get it.

But in reality of course regeneration efficiency is much smaller more like 15 to 30 percent in which case the you only get back 15 to 30 percent. But just imagine a situation where I can get back 100 percent of the energy back which means I accelerate in a vehicle, when I accelerate a vehicle I will consume certain energy then I will go at a flat speed I will not consume any I will consume only a limited amount of energy no acceleration energy is required.

Then I will decelerate, finally suppose I am accelerating from 0 to 60 kilometer per hour finally some point of time I have to decelerate from back to 0, so my acceleration and deceleration may be completely cancel out. Now if I recover all the energy whatever energy I put for acceleration same amount of energy I can get back during deceleration that will depend on regeneration factor.

If regeneration factor is 100 percent, I can do that. In reality it is not so it is actually less more like 15 to 30 percent, so actually I will lose rest of, suppose it is 30 percent that means 70 percent of energy I will actually lose only 30 percent I will recover. So a regeneration factor R is applied. The net energy used during acceleration and deceleration is R into the energy that was used for acceleration and deceleration.

This is important as we will see, this is only for acceleration and deceleration, it is also for climbing and climb down. During climbing up you will gain potential energy your electrical energy is going to convert it into potential energy. Climbing down that potential energy can be converted back to electrical energy.

And if the R factor was, if the regeneration factor was 100 percent actually I for my climbing and climbing down I consume no energy but if it is 30 percent I actually it will consume 70 percent of the energy. So a very important factor that you must remember that the net energy consumed is R into the energy that we had just computed earlier and R is the regeneration, the net regeneration.

And R is the equal to 1 minus regeneration factor because if 15 to 30 percent only I can recover, 85 to 70 percent is what I will lose and that is what we are trying to do. Is that clear? This is for electric vehicle, this is not so for ice vehicle. Though today many of the petrol vehicles have a certain amount of regeneration, sometime they called it even a mile hybrid.

And there is normally always a battery and they will try to recharge the battery so to that extent they will recover the energy it is possible to do that but with electric vehicle this is going to become more and more prominent, overtime we will learn to increase the regeneration factor from todays number of 20 percent to 30 percent to 50 percent, 60 percent, 70 percent and tomorrow it may be that the climbing up or acceleration does not consume much electricity.

But going flat what is the energy that you will consume? The aerodynamics and rolling resistance, whatever that energy I will always consume, the climbing and acceleration will disappear. So this is something that you should keep in mind when we talk about the electric vehicles.