

**Fundamentals of Electric Vehicles: Technology and Economics**  
**Professor L. Kannan**  
**Professor of Practice**  
**Indian Institute of Technology, Madras**  
**Lecture - 59**  
**Thermal Design - Part 1**

(Refer Slide Time: 00:33)



## 6.8 Thermal design

Evacuating heat: Calculating the resistances – conduction and convection. Thermal circuit for estimating temperature profile

6.8

Fundamentals of Electric Vehicles: Technology & economics

79

So, let us look at thermal design. Thermal design, normally, people think it is very complicated from a, for a motor and to get any understanding of what is happening, they resorts to a very brute force, computationally intensive method called finite element analysis and go through a very elaborate stimulation which is good because you get fairly accurate results when you do FEA.

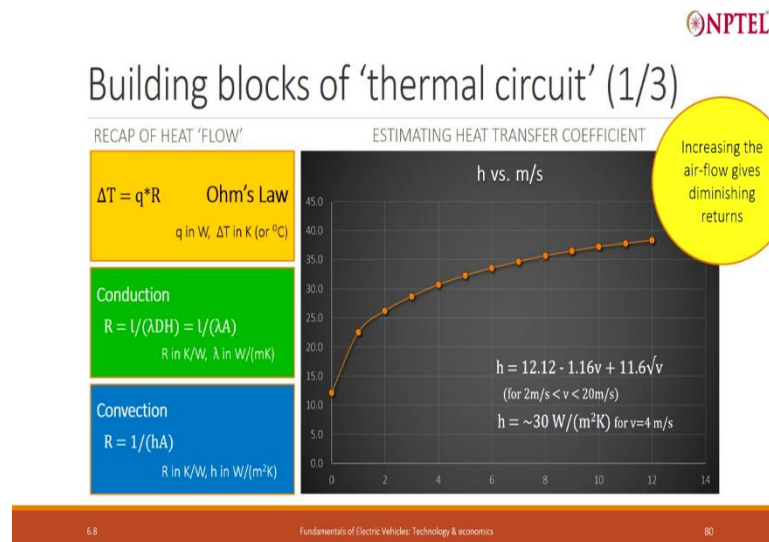
The problem with it is that in the early stages of design, we are not near anywhere near the optimum thermal design and we want to know if I make this a little bit thicker what will happen, if I increase the airspeed a little bit what will happen, if the ambient temperature goes up a little bit what will happen? And if each of these things have to be done by repeated cycles of simulation, it becomes a very tedious process.

So we have developed I and two-three of my colleagues have developed a very simple way of thermally modeling the motor using simple things that we already have studied in the very first section on heat flow, and I will take you through it step by step.

It is very simple and whenever I have a design of a motor whose geometry and other things are known, I can using some simple spreadsheet, like excel or something I can just model the

whole thing and then just be changing numbers as I want I can modify the design and I can see the impact on how the thermal profile is, the temperature profile in the motor is.

(Refer Slide Time: 01:53)



So before we get into building the whole thermal circuit of the motor, I will start with a few building blocks. Firstly, let us recap what we already know Ohm's law applies. Temperature difference is like the voltage which drives the heat current which is in watts, and the ratio between the two is the thermal resistance.

And this thermal resistance gets applied both in conduction and in convection. In the case of conduction, the material property is well known, what is called lambda, the coefficient of thermal conductivity. And the resistance depends on the material property lambda as well as on the geometry which is the length and area. This we already know.

When it comes to convection, there is no notion of length because it is just air circulating around the object so there is no l there is only a 1 by A term and the material property is given by this mysterious number called h, which is the heat transfer coefficient. What does this heat transfer coefficient really depend upon? In the case of lambda, I can just look up a material property handbook and I know what is the lambda, but in the case of h, there is no single value of h if you are talking about air.

So we need to find a way of estimating, there are some empirical formulas. I will give you one formula, h is equal to 12 minus 1.16 v plus 11.6 root v. This is broadly a formula that holds true for velocity in the range of 2 meters to 20 meters per second. We are not going to

be applying anywhere near 20 meters per second because it will be like a cyclonic storm the speeds that we encounter for the airflow will be much smaller than this.

So typically, we can say that this equation is valid for the normal design condition in which our motor will operate. What you see is that the, as the speed increases initially, if I put  $v$  equal to 0 then  $h$  is equal to 12, 12.12. And the moment a slight wind starts blowing, it is becoming close to 25. It actually becomes 25 when it is 2 meters, 25 or 26. But after I cross 4 meter per second, any further increase in speed results in a very marginal increase in the  $h$ .

So beyond a point, it is not useful to increase the speed. I will be pumping more and more energy into making the airflow faster and faster but I will not get commensurate returns. So any speed more than about 4 meters per second is really not a useful thing as far as improving the  $h$  is concerned.

(Refer Slide Time: 04:51)



## Building blocks of 'thermal circuit' (2/3)

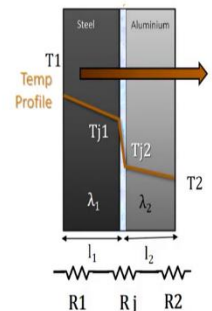
### CONDUCTION ACROSS SLAB JUNCTION

The junction resistance can be represented by a 'h' like convection.

- 'h' has no notion of thickness
- 'h' acts on the area available
- $R_j = \frac{1}{hA}$
- $T_{j1} - T_{j2} = q * \frac{1}{hA}$
- $T_1 - T_2 = q * (\frac{l_1}{\lambda_1} + \frac{1}{h} + \frac{l_2}{\lambda_2}) * (\frac{1}{A})$

Junction resistance could come from weld bead, air-gap, adhesive bond...

### TEMPERATURE GRADIENT DUE TO JUNCTION



Now, the second basic block of how to build a thermal circuit. We know how conduction takes place across a slab and I will take you to the next level of complexity there, where I have two slabs that are stuck together.

They may be stuck together by means of some interference fit in which there will be some air gaps and other things or they could be bonded by some kind of a glue, then there is a new material called glue that comes in between. Or I may just stick them, press them firmly together and weld them. Then there will be weld beads in some places and air gaps in some places.

So you see that there is, for example, steel and aluminum. This kind of an interface will happen between the stator and the housing. The stator is made of steel and the housing is made of aluminum and there is a junction between them which is shown as this thin blue strip. And it is shown here as a uniform strip but it may not even be of uniform thickness. It is, if it comes, for example, from interference there will be some places where the two metals are digging into each other so the gap is 0. In some other places, there will be some clearance and the magnitude of clearance will be varying and so on.

So you cannot describe the junction by the usual conduction equation usually because the thickness is not known. So we replace the description of the junction by an equation that is similar to convection and just define it by a number  $h$ . And say that the junction resistance is  $1/hA$ , where  $h$  is the heat transfer coefficient of the junction. But for the slab on the left and the slab on the right, we can continue to use the normal conduction equation.

So if I apply the equation like convection for the junction, I can see that on the left-hand side there is a higher temperature and on the right-hand side of the junction, there is a lower temperature. And that temperature drop is taking place across a small fraction of a millimeter, it is less than even 0.1 millimeter.

Across a very small boundary, there is a steep fall in the temperature and how much the temperature will fall is defined by the number  $h$ , apart from of course the area to which the heat current is exposed.

Knowing this I can describe the temperature drop from the left to the right as  $T_1 - T_2$ , which is equal to  $q$  into the sum of three resistances that are in series  $R_1$ ,  $R_j$ , and  $R_2$ . And  $R_1$  is a conduction, so I will have  $l_1/A_1$ ,  $l_1/\lambda_1$ . The  $A$  is common to all of them, so I have taken them out I have taken the  $1/A$  out.

So the resistance of the first term will be proportional to  $l_1/A_1$ , the junction resistance is  $1/h$ , and the slab on the right which is the aluminum has  $l_2/A_2$ . And a subtle point I want you to notice is that the slopes of the temperature gradient as heat is flowing through it is different. A steep slope means the resistance is high. A gentle slope means the resistance is low.

So as you can see, the junction offers the highest resistivity; not resistance, resistivity. And the aluminum is almost horizontal which means, is a very good conductor, its conductivity is very good and steel is poorer than aluminum but not as bad as the junction. That is what the

slopes tell us. So as I said, this junction resistance can come from some adhesive bond, it can come from welding bead, it can come from air gap, all sorts of things. It is difficult to characterize it in a strictly geometric way.

(Refer Slide Time: 09:12)

NPTEL

## Building blocks of 'thermal circuit' (3/3)

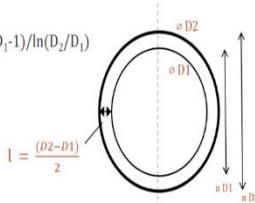
### CONDUCTION ACROSS PIPE

A pipe can be "unwrapped" as a trapezium.

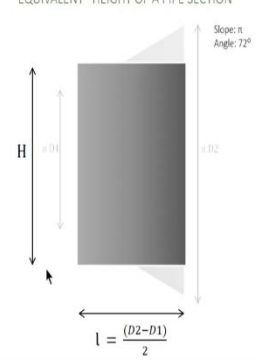
By slicing it into infinitesimal rectangles that are joined in series (with zero junction resistance), we can treat it as a slab of uniform equivalent height  $H$

$$H = \pi D_1 * \left\{ \frac{(D_2/D_1 - 1)}{\ln(D_2/D_1)} \right\}$$

$$R = l / (\lambda DH)$$

$$l = \frac{(D_2 - D_1)}{2}$$


### "EQUIVALENT" HEIGHT OF A PIPE SECTION



Fundamentals of Electric Vehicles: Technology & economics 82

The last of the basic building blocks that we will discuss is, how do we deal with conduction when it is not happening across a slab, which we all know how to deal with but instead it is happening across a pipe. Why are we so interested in a pipe? Because if I look at the cross-section of the motor, it is like a number of concentric pipes.

The innermost pipe has slots and teeth. The slots have copper and the teeth is made of steel. And after that, you have another layer of iron called the back iron in the stator, and around it, you have the aluminum housing. So it is a number of pipes that are coaxial which together make up the motor and the source of heat goes from a point close to the axis and travels outwards. And finally, when it reaches the outer surface of the housing it is taken away by the air.

So in order to be able to model how the heat flow happens, I should understand how heat gets conducted across a pipe. If the inner diameter of the pipe is at a higher temperature and the outer is at a lower temperature and heat is therefore flowing out radially, how can we describe it?

So the first thing that I will do is I will cut the pipe over here. The outer diameter is  $D_2$  and the inner diameter is  $D_1$ . If I simply cut it here and then stretch it open and flatten it out, then I will see that actually what I have is a trapezium. It is a trapezium where the longer side is

equal to the circumference of the outer circle, which is  $\pi D_2$ . And the shorter side of the trapezium is the circumference of the inner circle of the pipe, which is  $\pi D_1$ . And you can figure out what will be the slope of this.

The slope of it will be equal to  $\pi$ . I leave it to you as an exercise to figure out why. And therefore,  $\tan^{-1}$  of  $\pi$ , which is 72 degrees will be the angle of the trapezium. And we also know that the length of this trapezium slab, trapezoidal slab is equal to the difference between the two diameters divided by 2. So this length  $l$  is  $D_2$  minus  $D_1$  by 2. So so far so good.

Heat is entering from the smaller face of the trapezium and exiting from the bigger face of the trapezium. But this is still not like the slab that we know how to deal with. It is a slab all right, but a slab with a varying area of cross-section. The area inside is less and as it keeps going out, the area is increasing. So how can we model this?

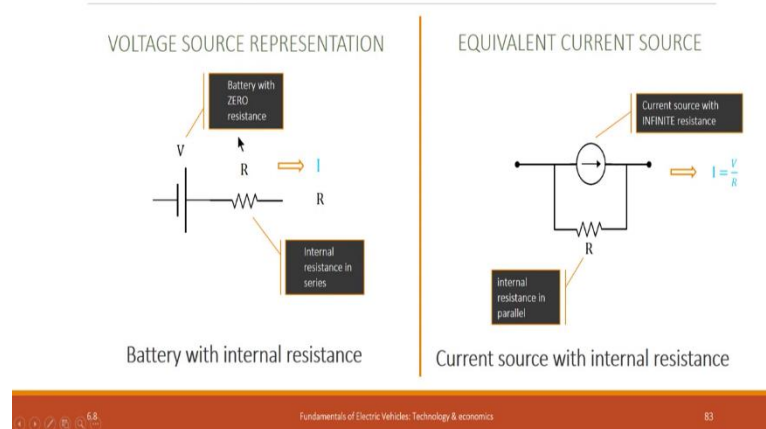
We know that because this is a smaller height by  $D_1$  and this is larger, I can model it as a rectangle whose height is something in between this and this. I will call that the equivalent height of the slab. But how much is that equivalent height? Is it the average, will it be  $\pi D_1$  plus  $\pi D_2$  by 2 or is it the geometric mean not the arithmetic mean?

To figure this out, I will slice it into a number of slabs. Each slab can be considered a rectangle of a different height. And then if I consider that they are all so many slabs are pasted together with of course no junction resistance, 0 junction resistance between them then I can do a simple integration and find what that height will be. And unfortunately, it is not a very simple relationship like average it is a little messier than that. There is a log term and everything coming. But it is not very complicated integration to do; you can figure out.

So once I know what that height is, if the length of the pipe is capital  $D$  then I know that the area of cross-section of the slab is  $D$  into  $h$ . So I can say that the junction resistance is  $l$  which is  $D_2$  minus  $D_1$  by 2 divided by  $\lambda A$ .

(Refer Slide Time: 14:02)

## Norton's theorem



So far is it clear? If you have any doubts you can ask me. No problem? I am taking you step by step through this so that no matter what design you come up with tomorrow, you will be able to easily model it. The other thing I want to now introduce you to is the idea called Norton's theorem which comes from electricity. I do not know how many of you remember this, have any of you even heard of this? It is one of those obscure topics that we do not pay attention to in college.

Normally, when there is a battery, the battery is a voltage source. It also has an internal resistance and the way we represent it in a circuit diagram is separately as a battery with no internal resistance, and then in series with it, we show what the internal resistance is. So the battery is represented with 0 internal resistance and then to account for the internal resistance, we connect the internal resistance in series with the battery.

So what Norton's theorem tells us is that the same circuit can be alternatively represented as a current source where the current is equal to  $V$  by  $R$  and put the internal resistance in parallel with the current source. And here, the current source is represented as  $I$  with infinite resistance, and the internal resistance itself is drawn in parallel. The two are electrically equivalent.

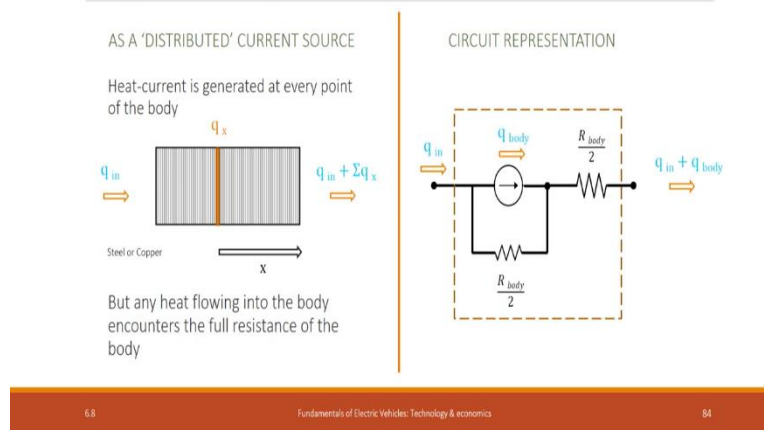
The reason I am interested in this transformation is that when we look at the thermal flow, for example, there is copper, I know how much heat is getting generated, the  $Q$ . The  $Q$  is known,  $I^2 R$  but I do not know what is the temperature difference. That is something that I am yet to find out. So I would rather model the copper winding based on what I already know rather than model it based on what I do not know.

The resistance of course comes from the geometry,  $l$  by  $\lambda A$ . So resistance can be known from the geometry of the copper. How much heat current is getting generated can be known from  $I^2 R$ . What I do not know is the temperature difference, which is the  $V$  that it is resulting in. Therefore, this way of modeling is more convenient for me than this, for all the heat sources.

(Refer Slide Time: 16:58)



## Representing a Heat-source



So I will use the Norton equivalent circuit to model all heat sources that are there in the motor. But there is a twist in the tail you can call it Condensed theorem because to my knowledge, nobody has done this before.

This as well is whatever I am going to do subsequently. It is all just done by me and my team. So when I look at a heat source like let us say, copper. Heat is getting generated at every point in the body and the heat getting generated at any one slice of the body is traveling through a certain length before it exits. So it is encountering a body resistance which is proportional to the distance where it is getting originated.

So if I cut the thing again into a number of slices and see, the heat getting generated in this corner is facing the full resistance of the body before getting out. The heat that is getting generated over here is not encountering any resistance of the body at all and just getting out. And all other points along the way are encountering varying degrees of the resistance of the body, the body which is the source of heat itself.

In addition, some heat is coming into the body from some external sources which is passing through. The heat that is passing through will encounter the entire resistance of the body. But



the heat that is getting generated, how much resistance is it encountering? We can say that on the average it is encountering half the resistance of the body.

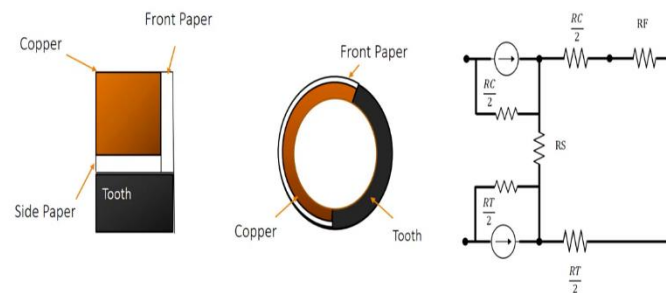
So I make a slight adaptation of the Norton circuit by putting half the resistance in parallel and half of it in series. The heat getting generated by the body will encounter the half the resistance but the heat coming in from outside into the body will flow like this and then like this. It will not flow like this, why? Why will the heat coming in not flow like this? Because the resistance in the Norton representation is infinite. So it cannot flow through this. It will flow like this and then flow here.

So the heat coming in encounters  $R$  by 2 plus  $R$  by 2, which is the total body resistance but the heat originating in the body encounters only half the body resistance. And then the exit heat current is input heat plus the heat generated by the body. So this is another important information that we will use in building the thermal model. Is this clear? Are you with me so far? No problem?

(Refer Slide Time: 20:03)



### Windings and Teeth: Heat sources in Parallel



So now, let us get into the motor itself. If I look at the motor, the innermost core, I can think of it as a pipe. It has slots alternating with teeth. The teeth are directly connected to the back iron, which is the next radial pipe. But the teeth are also connected laterally to the winding through the winding paper which we introduce to insulate the winding from the body.

And this paper is normally like a U-shaped thing inside which the windings are passing. So between the copper and the teeth, there is a layer of paper and between the copper and the back iron which is radially outwards, there is paper two. So I am calling this as the front paper

because it is facing the outward direction radially and then there is a side paper which is between the copper winding and the teeth.

If I were to look at it, I will have a number of slots and a number of teeth in between. And based on my geometric model, I can take the area of the slot to the area of the teeth as a ratio and simplify my model by saying that this much percentage of the total area is occupied by the copper and this much percentage of the total area is occupied by the teeth.

And in addition, I will introduce a paper over here. There is also paper in between but its dimension will not be equal to this, this is a very small dimension. Its dimension will be equal to the ratio between this length and this length. So I have not shown that in this diagram but we will use that in our modeling.

So using this I can construct a circuit diagram for just this portion. There is copper and there is the tooth. I know how much heat is getting generated in the copper,  $I^2 R$ . I know how much heat is getting generated in the steel as a whole and the steel is nothing but the teeth and the back iron.

So based on the area ratio between the teeth and the back iron, I can estimate what fraction of the heat is coming from the teeth and what fraction of the steel loss is coming from the back iron and I can apply that magnitude to the total steel loss and know what is the heat getting generated here.

But that is about the heat, the resistance of the copper I can calculate based on the geometry. Now that I know that this is the geometry, I will calculate the resistance of a whole pipe made of copper and then divide it by this fraction of the arc. Likewise, I can treat the teeth as a steel pipe, calculate the total resistance of the pipe and then divide it by this angle.

So I can calculate  $R_C$  which is the copper of the resistance of the copper, and  $R_T$  which is the resistance of the teeth and then show it using our Norton representation as  $R_C$  by 2,  $R_C$  by 2,  $R_T$  by 2, and  $R_T$  by 2. And between them, there is a resistance due to the paper, side paper and the copper is further facing resistance in this direction radially outward direction from the front paper. So this part of the circuit is clear? No problem?