

Fundamentals of Electric Vehicles: Technology and Economics
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Lecture - 57
Three Phase AC

So, so far we have talked about the controller, and the controller, controller's job is to control. To control means to maintain something. What can be maintained is only a DC entity, V_d and V_q are DC, that is a greatest advantage of V_d and V_q . So based on all my computation, I say, I want V_d to be 20, I want V_q to be 10, something. And the PA loop can execute the error computation cycles and ensure that those levels are maintained.

But what is actually getting applied to the motor is not the DC values, but AC. So let us look at how the AC world plays out and that is what is going to be happening in the real world. And in the interactions between the motor and the controller, we will constantly be switching from the AC to the DC and again back from DC to AC.

We will keep doing this exchange because we can do control only in DC but the motor will run only in AC. So let us quickly recap about AC, alternating currents.

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The slide is titled "Sinusoidal waveforms" and is divided into several sections. On the left, there is a list of characteristics of a wave: Amplitude (peak value) A_{pk} , Frequency f , Angular frequency, $\omega = 2\pi f$, and Phase ϕ . Below this list, it states "Current and voltage are alternating sinusoidally in an AC circuit" and provides the equation $A = A_{pk} \cos(\omega t + \phi)$. On the right, there are three diagrams: a circular diagram showing a sine wave on a coordinate system, a vertical diagram of a pendulum, and a graph of a sine wave with a vertical axis labeled 'y' and a horizontal axis labeled 'x'. At the bottom of the slide, there is a footer with the text "Fundamentals of Electric Vehicles: Technology & economics" and a URL "animations.physics.unsw.edu.au".

When I say alternating current, it can do alternating voltage, anything that is sinusoidal. So any sinusoidal waveform is characterized by what is called the amplitude and that is a peak value, a frequency, and phase. Frequency can also be represented as angular frequency by

multiplying it by 2 pi. So an easy way to visualize it is that if I have a oscillator, a simple harmonic oscillator, which means a mass tied to a spring, which is bobbing up and down.

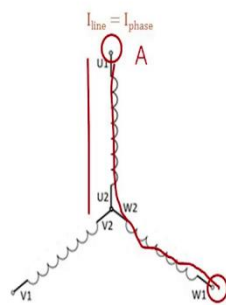
Then if I plot its height versus time, its position versus time that will be a sine wave. Another way of visualizing a sinusoidal waveform is that if I have something that is going with a uniform angular speed in a circle then the shadow that it casts, the length of that shadow with respect to time is a sine wave. Fine?

And I can also represent the sine wave using a cosine equation that is why I have written it there as cos omega t, but cos means when t is 0, the amplitude will be maximum. Whereas what you are seeing in the picture is that, when time is 0 the amplitude is small. So I have to give a phase offset. I can put 5 is equal to negative 90 degrees and it will describe this equation.

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Three Phase circuit



Peak vs. RMS

For a sinusoidal entity,

Peak value = $\sqrt{2}$ * RMS value

For a star winding connection

Line vs. Phase

$I_{line} = I_{phase}$

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Now, let us look at what is called a three phase circuit, we will be looking at what is called as star connected because that is what we use in the motors before which I will tell you that there is a, when there is a sinusoidal waveform, it is going up and down, it does not have a constant magnitude, magnitude is rising-falling, then rising in the negative side, coming back to 0. So it is not having a magnitude, which is constant.

So as a measure of how big the wave is, if I ask you what is the average magnitude, you will say it is 0 because it is equally plus and minus. So in order to get rid of this and if you say the average value is 0 it does not give me any information, it does not give me a sense of how big

the wave is. So in order to get rid of this absurdity, what I will do is I will simply square the wave.

So if the height is 1, when I square it, it becomes 1. And if the height is negative 1, when I square it, it also becomes 1. And then if I add, I get 2, that is a measure, of how big it is. If the wave was, to begin with, 2 in the plus direction and minus 2 in the negative direction, when I square it, I will get 4 and I will get for the negative part also 4, and if I add I get 8. So in the first case, when the amplitude was 1 when I squared and added I got the answers as 2.

In the second case when the amplitude was 2 when I squared and added, I got 4 plus 4, 8. But then, it gives me the wrong impression that the second wave is four times bigger than the first, which is wrong. And I get that impression because I squared it. So to compensate, I will take the square root again. So for the first wave, I will say its magnitude is not 2, but root 2.

And for the second wave, I will say, the magnitude is not 8, but root of 8, which is $2\sqrt{2}$. Then if I compare root 2 with $2\sqrt{2}$, I know that the second wave is two times bigger. So this way of squaring, adding, and then finding the square root is called RMS, root mean square.

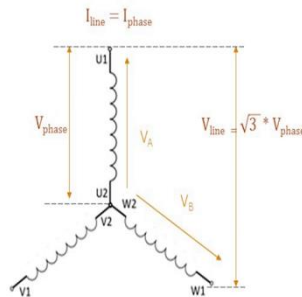
Now, if I want to measure current, what I will do is I will place an ammeter over here. When the voltage is sinusoidal, the ammeter is not going to be rapidly moving up and down, because the frequency with which it is going up and down is very fast. The ammeter will be calibrated in such a way that it can show me some average value and as we discussed, there is nothing called average with sinusoidal, what we mean by average is the RMS value.

So the ammeter will show me the RMS value of the current that is flowing. If I place another ammeter here, that will show me the average, meaning the RMS value of the current that is flowing in the other phase, there are three phases. The three phases are the same except that, they all have a phase difference of 120 degrees, 120 degrees, or $2\pi/3$. But when I place an ammeter, which is only showing me the RMS value, I will actually measure the same current in all the three, the line current and the phase current are the same.

Line means what is flowing like this and phase is just what is flowing from here to here. From here to here is the phase and what is from one end to the other, is the line.

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Three Phase circuit $V_A - V_B = V \cos \omega t - V \cos (\omega t - \frac{2\pi}{3})$



Peak vs. RMS

For a sinusoidal entity,

Peak value = $\sqrt{2}$ * RMS value

For a star winding connection

Line vs. Phase

$I_{line} = I_{phase}$

$V_{line} = \sqrt{3} * V_{phase}$

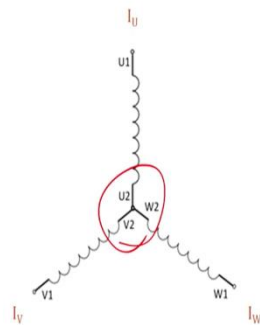
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The currents are the same but when I look at voltage, this is V_A , let me say, and this is V_B . If V_A is $V \cos \omega t$, V_B is $V \cos \omega t - 120$ degrees and the total voltage is V_A minus V_B , line voltage. So V_A minus V_B , V_A is some voltage that we know, the RMS value is V , but the V_A minus V_B , which is the line voltage is $V \cos \omega t - V \cos \omega t - 120$ degrees, which is 2π by 3 radians.

If you expand this trigonometrically, you will get that the line voltage is root 3 times the phase voltage. If it were not an AC current, you will think that no it will be double. Because there are two of them in series. But the two of them are in series but they are not of equal magnitude, when one is high, the other is small, when the other is small, this one is high. So on the average, if you see, it is only root 3 times more, it is not 2 times more.

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Three Phase current



$$I_U = I_{pk} [\cos(\omega t)]$$

$$I_V = I_{pk} [\cos(\omega t + 2\pi/3)]$$

$$I_W = I_{pk} [\cos(\omega t + 4\pi/3)]$$

$$I = I_U + I_V + I_W = 0 \quad \text{Kirchhoff's First Law!}$$

$$I = I_U + I_V \cos(2\pi/3) + I_W \cos(4\pi/3)$$

$$= 1.5 * I_{pk} \cos(\omega t)$$

The resultant current is a rotating vector that is 1.5 times bigger

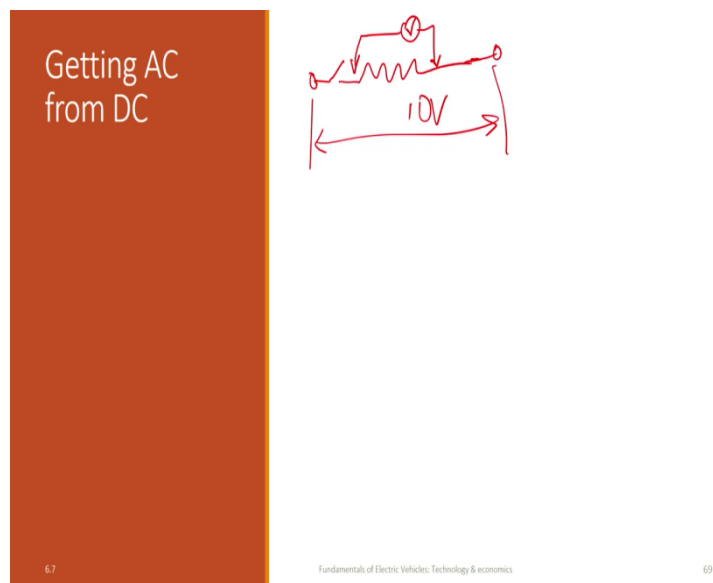
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Now, let us look at the currents a little more carefully. The three phases are commonly represented as U, V, and W. They can also be described as a, b and c; small a, small b, small c. These are all different conventions. So if the U phase current is represented by ωt , then the V and W are 120 and 240 degrees apart.

The interesting thing is if you add all of these, you can of course expand them trigonometrically and add them. But without doing all of that you can just look at Kirchhoff's law and say that the total current should be 0. And you will get the value to be 0 when you expand it.

But if you expand them as vectors, what are called phasors, then you get a different kind of expression. And when you add them you get 1.5 into $I \cos \omega t$, which is essentially one and a half times bigger than this. And it is a rotating vector, it is a rotating current. We will see this graphically going forward.

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Before getting into that, we know that the electric vehicle is powered by a battery and the battery gives only DC, but the motor needs AC. So we will take a small detour and see how we convert DC into AC. Just imagine that I have a battery and I have connected it to some load, some resistance, and I am putting a voltmeter and I am measuring the voltage across the resistance.

Let us say the battery is 10 volts, if I switch it on and measure, what will the voltmeter read? There is a resistance and from here to here, I am applying 10 volts. So if I stick a voltmeter, it will read 10 volts. If I open the switch and then connect the voltmeter, what will it read? What will the voltmeter reading be? I have opened the switch, it will show no voltage, 0. There is no voltage across the resistance. My voltmeter is from one end of the resistance to the other end of the resistance.

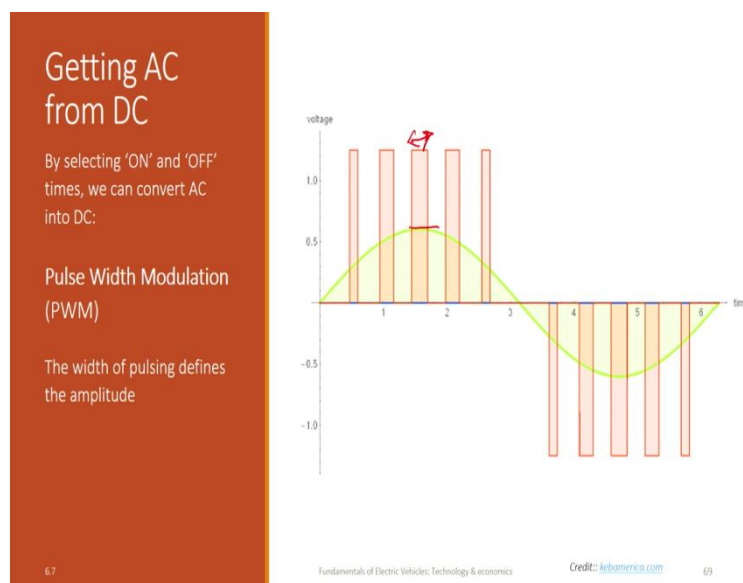
So my voltmeter will read 0 when the switch is open, if I close it, it will read 10 volts. But imagine, I have a certain superhuman ability, I can turn on and turn off the switch 10,000 times every second. And what I do is, what I have done is I have sliced 1 second into 10,000 intervals. In each interval, I have the ability to turn, to flip the switch if I want, understand?

And what I do is I flip by waiting for 3 slices of time I keep it on, each slice is 1 by 10,000th of a second. For 3 slices of time I keep it on, and then for the next 7 slices of time, I keep it off. And I keep repeating it till the 10,000 slices of time are over. And during that entire 1 second, I have connected the voltmeter, I am trying to see what its reading is, what will it read? It will read 3 volts because on the average 30 percent of time 10 volts was on.

In the next second, if I chose to keep it on for 4 slices of time and then off for 6 slices of time and repeat it for 10,000 slices, then the voltmeter will read 4 volts because 40 percent of the time I kept it on, 60 percent of the time I kept it off. So this percentage of on-off time, it is called the duty cycle. 40 percent duty cycle means 40 percent of the time it is on. By doing thus I can get, whatever voltage I want from the battery provided, I do not want any voltage more than 10 volts. The maximum voltage I can get through the batteries is only 10 volts.

But anything less than 10 volts, if I want 6 volts, 6 by 10 is 60 percent; I will just keep my switch on 60 percent of the time. If I want 8 volts, I will keep my switch on 80 percent of the time. I can get any voltage I want from 0 to 10 volts, by changing the on-off time.

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So if I want a sine wave, like the green wave that you are seeing in the picture, at different instants of time, I want different voltages and that all of those respective times I will change the percentage of on-off time. This is called pulse-width modulation. The sine wave maybe something like 50 hertz or 100 hertz but my on-off is happening at 10,000 hertz, much higher frequency.

So whatever you are seeing here as let us say, one-tenth of a wave is actually a few thousand cycles in my switching on-off. And you see here that the widest pulse is this, this width tells me what this height is.

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Getting AC from DC

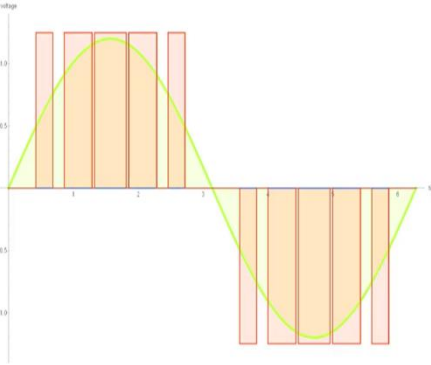
By selecting 'ON' and 'OFF' times, we can convert AC into DC:

Pulse Width Modulation (PWM)

The width of pulsing defines the amplitude

The period of pulsing defines the AC frequency.

6.7



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If I were to make all the pulses proportionately wider, I can get a wave of larger amplitude. And how quickly I run through this cycle and then come back again for the next cycle, is also another thing in my control.

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Getting AC from DC

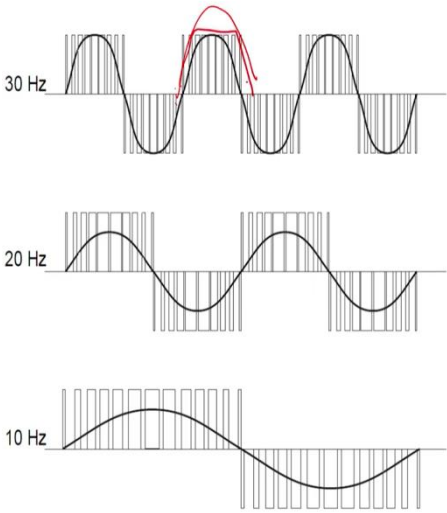
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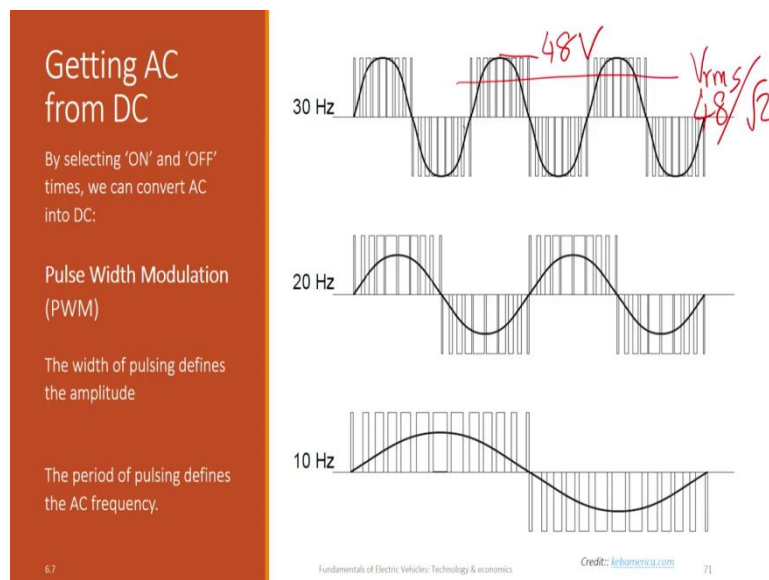


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By this, I can change the frequency of the sine wave. So the percentage of on-off, the sequence in which I vary, what is the maximum to which I go before I start coming down defines the amplitude of the voltage. And how rapidly I go through that cycle defines the frequency. And all of this is under my control just by having a switch which I can rapidly turn on and off.

So this is how I convert AC into DC. And you see that if I have a AC source if I have a DC source like a battery, I can get a line where I get a AC current of my choice as long as the peak is not more than the capacity of the battery. That is a really important point I want you to note here. I cannot get a wave which is like this, because the battery will simply not give. Even if you keep it on 100 percent of the time, it will only get up to here. So what will happen if I try to do that is it will go up to there and then it will flatten and then come here.

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So on this note, I will also come back to the point that we earlier discussed about 34 volts. I said 34 volts is the limit of the battery. Actually, the battery can give 48 volts. If the battery can give up to 48 volts, I can draw from it any voltage less than that through pulse-width modulation. But the maximum voltage I can get from the batteries is how much? 48 and that 48 is this peak.

If the peak is 48, what is the RMS? What is the the Vrms? It will be root 2 times 48 and that is 34, 33.9 something. I am sorry, I keep making that mistake. 48 by root 2, 48 into 0.707. So this is where we get that funny number called 34 because the battery is 48.

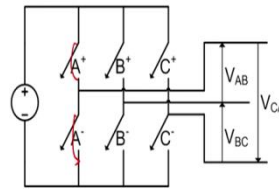
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Getting 3-phase AC from DC

PWM is applied across three phases simultaneously using a simple circuit of 6 switches:

Hex Bridge

6.7



Now, what we want is not just AC, we are greedier now. We want to get three phase AC. How can we get three phase AC from DC? Are we past time? So this topic is actually very fascinating but I will not go too much in detail into it because I only want to tell you as much as you need, to control the motor. I do not want to overload you with information.

Essentially a set of 6 switches is all that I need to convert a DC power source into a three phase AC. We have actually 3 switches on the top that is called the high side and 3 switches at the bottom, which is called the low side.

The very important thing to remember is do not close two switches at the same time in one line because then what will happen? You are short-circuiting the battery, you will have nothing short of an explosion. The entire battery will discharge in a fraction of a moment and you will have catastrophe. So normally in the hardware that you buy, it comes as a built-in standard feature that even if you try, you cannot close two of them at the same time. It is a built-in protection feature.

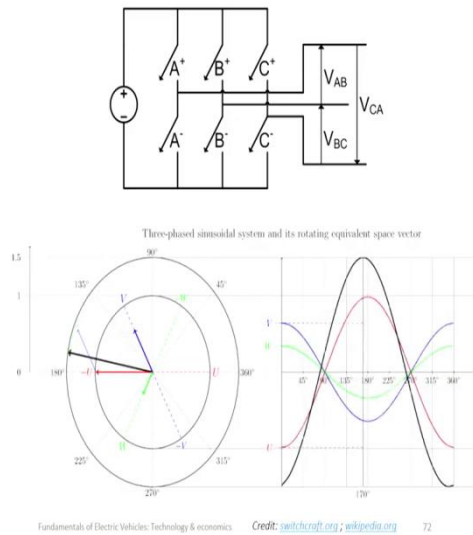
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Getting 3-phase AC from DC

PWM is applied across three phases simultaneously using a simple circuit of 6 switches:

Hex Bridge

6.7

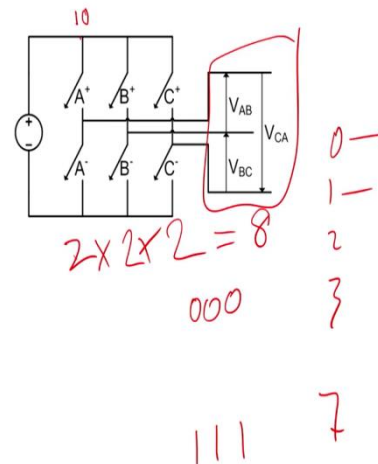


Getting 3-phase AC from DC

PWM is applied across three phases simultaneously using a simple circuit of 6 switches:

Hex Bridge

6.7



But short of that, you can do anything. And what is normally done is if the high side switch is closed, we call it a 1. By default the lower switch will be open, I have no choice about it. And if the upper switch is open, the lowest switch gets closed, I have no choice about it. So this minus is the complement of the plus. Whatever state the plus is in, the opposite state the minus will be.

So from a logical point of view, I do not have 6 switches, I have only 3 switches because defining any one of those 3 switches defines the other. But each of the 3 switches can be in one of two states, either 0 or 1. So what are the total number of states possible? This can be in two states, this switch can be in two states, this can be in two states, this can be in two states.

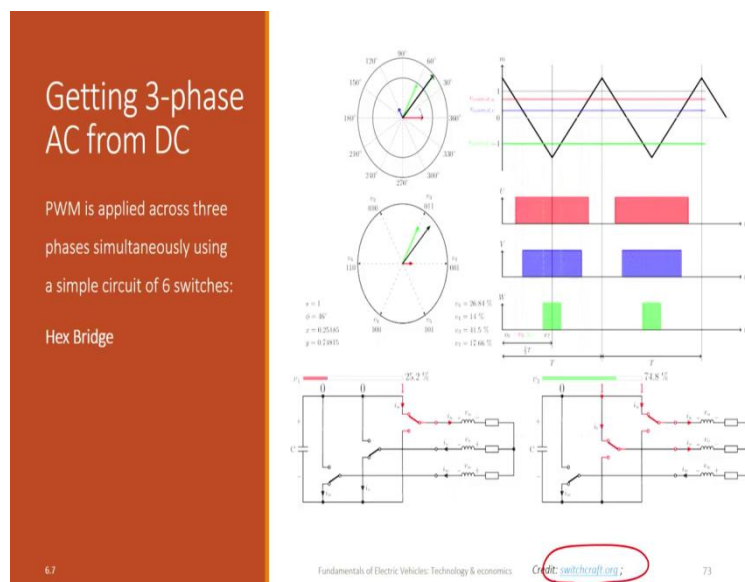
So what are the total number of states? It is 2 into 2 into 2; 2 to the power 3, 8. Out of which, so I can number them serially as 0, 1, 2, 3 up to 7, in the binary form. 0 to 7, there are eight states that the system can take, and out of that, if I keep all the three open, that is a 000 state. And the last state is when I keep all the three closed 111. In both these cases, you will find the V_{AB} , V_{BC} , and V_{CA} are 0. It is as if I am not applying any voltage.

The remaining six states V_{AB} , V_{BC} , and V_{CA} will be either 0 or positive or negative. And changing from one state to the next involves changing only one bit, binary, which means any one switch flipping is the sequence. So there is a lot of charm in the way the whole thing has been designed, it is very beautiful.

Let me erase all of this. This is called a hex bridge by the way, what is written there on the left because there are six switches. And what happens is that by doing, by switching them in sequence, the resultant vector, which I told you I_U plus I_V plus I_W is shown in black. It is one and a half times in length whereas the three phases, the U phase is red, it is oscillating, and the other two phases are green and blue, they are oscillating on different phase.

And how the sine wave look is shown on the right-hand side and the sum of them is one and a half times bigger. And it is also traveling, which means it is a rotating vector. So it is very charming, you can spend an entire day just trying to understand. It is almost lyrical and poetic, how this thing works.

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I will recommend that you visit this website called switch craft dot org. It is a beautiful website created by two young engineers from Norway. And with a lot of detailed and well-illustrated animations and diagrams, they explain how the whole thing works, but I will not get into all that detail now.

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Assignment 6.7.1

A motor is powered by a 48V battery

? What is the maximum RMS phase voltage that can be applied?

? What is the maximum RMS line voltage that can be applied?

The motor is of 6-pole construction with a back-emf constant $K_e = 8 \text{ V/K-rpm}$

? What is the maximum no-load speed it can be run at, without flux weakening?

? What would be the frequency of the voltage at this speed?

6.7

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Before signing off, here is an assignment for you. A motor is powered by a 48-volt battery. What is the maximum phase voltage and what is the maximum line voltage, which can be applied, RMS?

The motor is of 6-pole construction, and I know the back EMF constant, it is 8 volts per thousand RPM. What is the maximum no-load speed at which it can run if I do not do flux weakening? We already know that if I do flux weakening, I can make it go at much faster speeds. But if I do not do that what is the maximum no-load speed at which I can run it? And if I run it at that speed, what will be the frequency of the voltage?

They are not complicated in terms of calculation but they will require you to apply a number of concepts that we have learned at different stages in the previous lectures.