Fundamentals of Electric Vehicles: Technology & Economics
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Lecture - 54
The d-q Equivalent Circuit - Part 1

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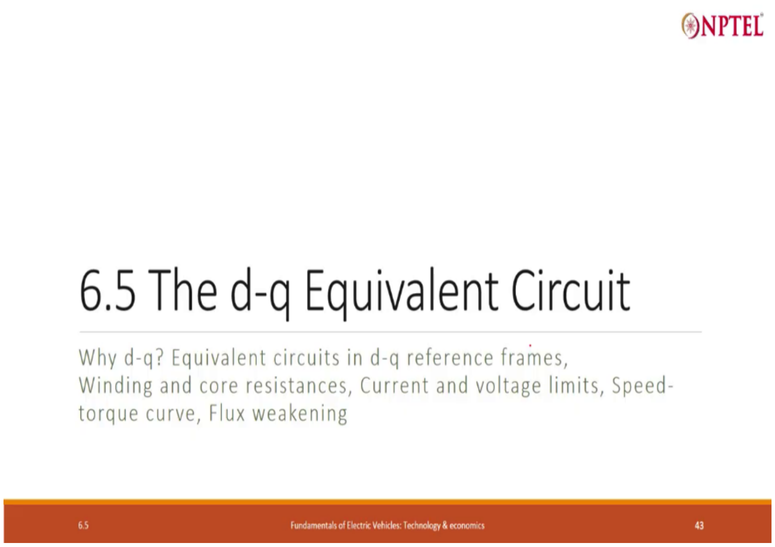

So now we have continuously talked about d-axis, q-axis; d-axis, q-axis. We have used that to calculate torque. Why are we obsessed with this d-axis, q-axis? Where is d-axis, q-axis? There is a motor in my hand, it is a PMSM motor, IPMSM motor, where is it?

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If I look at an actual motor, I will not find any d-axis or q-axis physically, nothing is marked there. And yet, we always talk about d-q frame, we are somehow in love with that. But when I actually go to the motor and see, there is a rotor which is rotating, it has so many poles. There are windings around it, there is not one winding. Typically, there will be three windings, there are three different phases.

The windings are not rotating but the windings are carrying current which is oscillating up and down, sinusoidal. So the currents are all dancing up and down around a rotor and the rotor itself is rotating as a result of the dancing currents. And the rotor has a number of pole pairs with alternating north and south poles.

Now, I cannot even make sense of what is happening there, it is such a confusing picture. What do I do? How do I change anything to get more torque, higher speed? I cannot even track it, leave alone control it.

But it is actually the interaction between all these dancing currents, the flux induced by them through the teeth, and the pole which is rotating; all of it together is what is causing the usual function. But it is so horrendously complicated, I just cannot follow what is happening. So I do not know whether I can make it do anything better or should I be just happy with what it is doing, leave it alone.

So in order to make sense of what is happening, what we do is that we do something extremely clever. Instead of trying to stand on the ground and look at the motor which is running, which means I am standing on a reference frame fixed to the earth on which the stator also is fixed. That means I am on the stator reference frame and I find everything very confusing. Because with respect to the stator, nothing is still, everything is going up and down, everything is changing magnitude, direction.

So instead, now what I will do I will fix the reference frame on the rotor. It is as if I have taken a chair and planted myself in the center of the rotor, at the axis of the rotor. The rotor is rotating, the stator is around it I have taken a chair and put it in the center of the rotor and I am sitting there.

Now, no matter how the rotor rotates, as far as I am concerned, if I see that there is a north pole it is always there because I am rotating together with the rotor. I am on a stationary reference frame with respect to the rotor. And now, I will look at what is happening to the currents.

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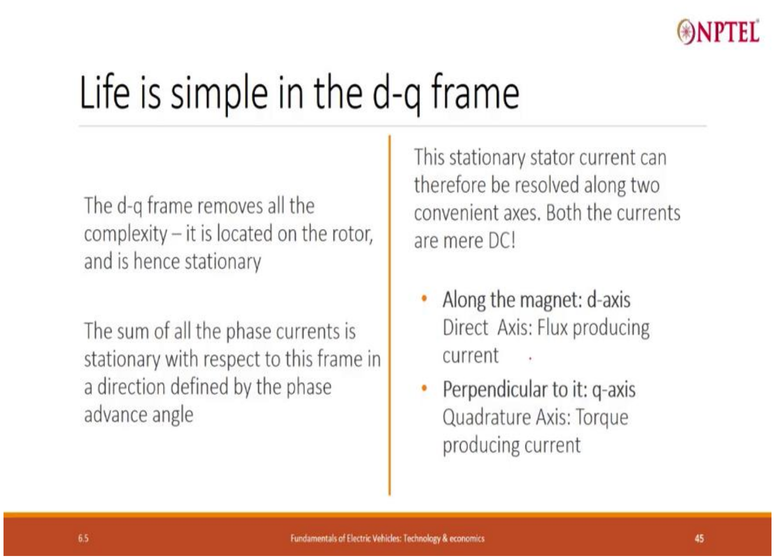

Everything becomes suddenly much simpler by doing this. So I locate my reference frame on the rotor, make it stationary and with respect to that when I look at all the three phases current, they vectorially add up to a single value in a single direction.

So the magnet north pole is there or magnet south pole is there and the vector sum of all the three currents is there and the angle between them is what? What is angle between them? That is the phase advance angle, theta; that we talked about.

So the current that I see, I can resolve it into two components. One which is aligned to the magnet axis and another, which is perpendicular to it. So suddenly, instead of having three alternating currents, I have two conveniently oriented stationary currents. They are not alternating currents, they are DC currents.

So the whole world view has become very simple just because I took my chair from the stator and put it on the rotor. Whatever is the component of current that is perpendicular to the magnetic field, we already know that is what causes the magnetic torque. So that is called the torque producing current.

The other component does contribute to reluctance torque but that is a relatively recent invention. In the traditional motors, that was a wasted component of current. It only weakens or strengthens the magnetic flux depending on the direction in which it is acting. So that is called a flux producing torque, a flux producing current.

So the d-axis current is commonly called the flux producing current, the q-axis current is called the torque producing current. But we know that when we have reluctance torque, both the components of current will contribute to reluctant torque but the magnetic torque comes only from the torque producing current.

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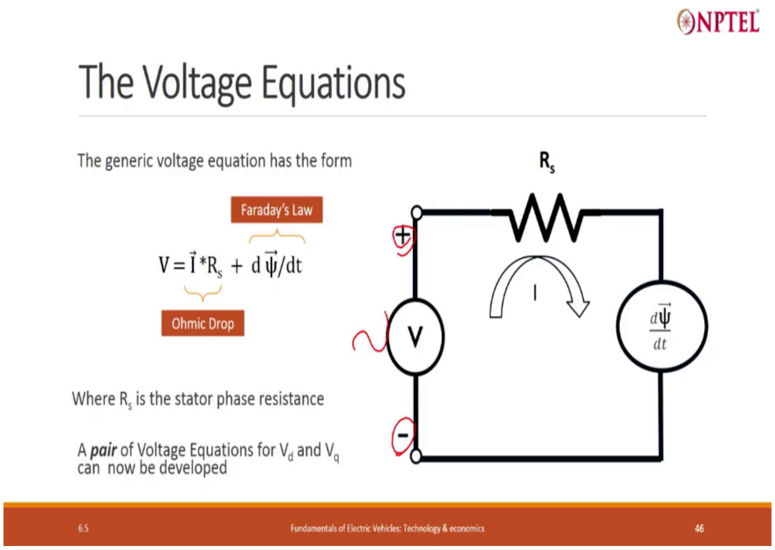

Now, let us look at the voltage. We have already seen how current is related to the torque or the torque is related to the current. Now, we will develop some equations that connect, we already done that for the PMDC, motor voltage minus back EMF is equal to I into R, very simple. But what will happen in this IPMSM motor, for that we have to develop what are called the voltage equations.

The starting circuit for it is very similar to the PMSM the only difference being, to the PMDC, the only difference being that here everything is actually alternating. Actually, there is a small mistake here, I should not be using, Okay. I should not be using plus and minus here, plus or minus at one instant, next instant, it will become minus plus. So really, I should be saying like this. It is an alternating voltage. The current is alternating, the flux is alternating, everything is alternating. That is the only difference between the IPMSM and the PMDC.

And again, the applied voltage is equal to the ohmic drop plus the voltage that Faraday's law uses d psi by dt. So far it is clear? Because suddenly, I am going to explode it into something that looks very nasty. If you have any doubts till now, please ask. You are fine?

So the voltages has these two parts. Rs is the resistance of the stator, which comes from the copper winding. What I am going to do is since I do not like this alternating, it is always messy to handle. We already discussed that we can get rid of alternating current and change it with a pair of direct currents along the d-axis and the q-axis when we go to the d-q frame; that is what I am going to do now.

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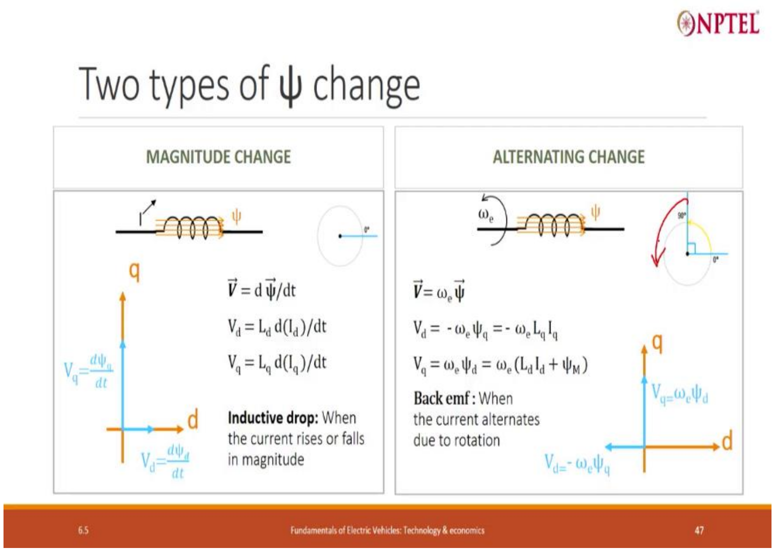

Before doing that, we will look at only the d psi by dt term. The flux is changing, there are two ways in which the flux can change; very important the slide is for you to understand the next equation.

One is the flux can change in magnitude and the other is that the flux can change in direction. We have to distinguish between the two carefully. What will cause a magnitude change? If the current changes. If current increases, the flux will increase; if current is falling, the flux will fall. This is magnitude change. And the change in the flux will be in sync with the change in the current. The flux will rise or fall together with the current. That is what is shown here that there is no phase difference between the flux and the current. Fine?

So I can say that V is equal to d psi by dt. I can break it as Vd separately, Vq separately by using the respective current components, very simple. And this is called the inductive drop; just like we had the ohmic drop, this is the inductive drop. The vector sum of Vd and Vq will be the inductive drop, very similar to ohms law for inductance.

And the vector diagram of this also very simple. The voltage in the d-axis is proportional to the rate of change of psi in the d-axis. The voltage along the q-axis is equal to the rate of change of psi in the q-axis. And the respective rate of change is equal to respective rate of change of current multiplied by the respective inductance.

So d psi q by dt is Ld into d Id by dt. d psi d by dt is Ld into d Id by dt. That is what I have written here in these equations. This part is fairly straight forward and simple. If you have any doubts, feel free to ask me. No problem? The googling will come when you have a voltage change because of not change in the magnitude of current but because the direction of current is changing due to rotation.

We already know that the back EMF is proportional to the electrical, the voltage is proportional to the electrical speed. Back EMF, when we use the term as back EMF constant, we measure it in relation to the mechanical speed. But the mechanical speed multiplied by the number of poles is the number of cycles of back EMF that we get.

So vectorially, the voltage is equal to the electrical speed multiplied by the change in psi. But this change in psi is not magnitude of psi but direction of psi. That is the important difference. And when it is due to this change in direction, there is a 90-degree phase shift which I want you all to observe.

That means if the psi is changing on the d-axis, it will cause a voltage in the positive q-axis. And if psi is changing in the positive q-axis it will result in a voltage in the negative d-axis because it will further go like this. A change in the positive q-axis of psi will result in a negative Vd and a positive change in the psi in the d-axis will result in a voltage in the positive q-axis.

So there is a direction change which is ultimately resulting in a negative sign also in the case of Vd but not in the case of Vq. This will make the equation somewhat messy, please observe it carefully. The voltage in the d direction is negative what I have shown here by the red arrow.

Omega e into psi q, remember, psi q is constant in magnitude. The change is only coming because of omega. And we know that psi q is equal to Lq Iq, but with a negative sign. That is very important. In the case of Vq which you will be seeing in a moment, this negative sign will not be there but there will be something else that messes it up. Because the psi in the d-axis comes not only from the Id but also comes from psi M, the magnetic flux. Fine?

So these are the two ways in which psi can change and the impact that they have on the voltage in the d and q directions. I can again show this vectorially. This is the back EMF term because the back EMF is what we see as directly related to the speed of rotation.

The other one is due to change in the magnitude. The change in magnitude usually happens when I am trying to accelerate, more current is being sent, or I am trying to slow down, less current is sent. But if I am going at constant speed what I get is the back EMF. So I have represented whatever we discussed now as a vector self-explanatory drawing.

What I want you to see here is that the voltage in each axis is influenced by the current or the flux in the perpendicular. So till now, we had actually visualized a very simple picture that all the complexity of a PMSM motor I can break it into neatly two compartments. One is the d-axis DC motor, another is a q-axis DC motor, which is correct. But the d-axis DC motor, its voltage is influenced by the current flowing in that motor. And the voltage in the q-axis DC motor is influenced by the current flowing in this motor the d-axis motor.

So they are not really independent, there is what is called cross-coupling between them. So though the circuit diagrams are correct the way we visualized, they are not completely independent because one term here is influencing that and one term there is influencing this, which is what we will see in what is called the equivalent circuit diagram of an IPMSM motor.

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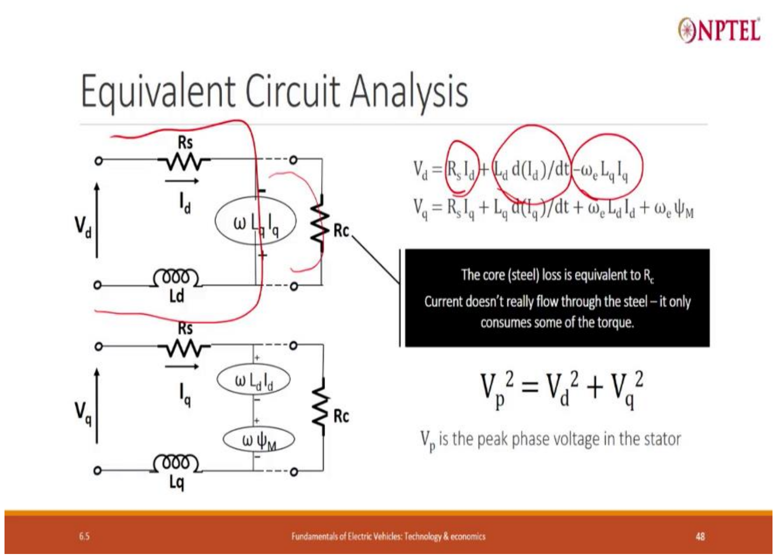

So now, we can create what is called an equivalent circuit. If you recollect, we had earlier created an equivalent circuit for a PMDC motor where there is a, there is a voltage source, there is a resistive drop due to the armature and then there is a back aim of term. It is a simple thing that we had created. What we are going to create now is very much similar to it.

There is a voltage source and there is a resistive drop and there is a back EMF term. There is also an inductor at the bottom. That inductor is for the times when there is a transient change in the current, the first of the two psi changes that we described. And the back EMF term is proportional to the electrical angular speed multiplied by the change in the psi. And you can see that in the d-axis circuit, d voltage, Vd voltage, the back EMF term is coming from the q-axis current.

So this is how the equation for the voltage in the d-axis will appear. This is the ohmic drop. This is when the current is rising or falling. If I am turning the throttle up or down that is when this term will appear, at all other times when I am going at a steady speed that term will be 0, because d I by dt is 0. And the last term is what is coming due to the flux change in the q-axis. The fact that there is a 90-degree shift is the reason for the minus sign there.

Likewise, we can draw a circuit for the q-axis and the equation will be ohmic term plus the transient term plus omega times the change of flux in the d-axis. And here, it is coming with a plus sign because the 90-degree phase shift from the d-axis will create a positive voltage in the q-axis.

So this is a simple way of arriving at the equations and the important thing I want you to remember is that there is this cross-coupling. Though they are like two independent circuits with two independent equations, what is happening in one affects the other as far as voltage is concerned.

There is also another thing here called Rc, which is just a way of representing the steel losses. It is not as if there is actually a resistance that is connected through which current is flowing but in terms of voltage, what this tells us is that it is not going to affect the voltage at all. That is why Rc does not come in the voltage equation. If you remember Kirchhoff's law, if I want to find what is Vd, I can simply take a path like this and there is no Rc in that path.

So the steel loss does not affect the voltage. What the steel loss does is that whatever is the torque that is getting produced, some of it is getting consumed over here. So what appears on the shaft of the motor is lower. So the electrical way of representing it in the circuit is by showing it as a resistance connected in parallel. It does not affect the voltage but it reduces the torque.

And if I know what is Vd and Vq then the sum of the squares of it will be the, will help me find the resultant actual voltage in the armature, which is what I can measure using a voltmeter. If I stick a voltmeter into the motor, I will not be able to measure Vd or Vq. What I will be measuring is the phase voltage.

What is the phase voltage? What is different from phase voltage and line voltage is something we will see subsequently. But for now, remember that Vd squared plus Vq squared is equal to the square of the phase voltage peak. Again, there is something called peak, there is something called RMS, we will discuss all of that in subsequent slides. This is what I want you to remember.