Fundamentals of Electric Vehicles: Technology and Economics
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Lecture - 53
Speed and Back EMF

So, so far we have talked about torque. And we have also seen that in the context of torque, we have talked about current. Torque is related to current, I want you to all remember. In a motor, current and torque are related, more the torque demand, more the current should be supplied, greater the current supplied, more the talk will be delivered.

And how to optimize that relationship is the angle theta, which is what defines for us the algorithm called MTPA, maximum torque per angle. But all the while, we did not talk about speed. And another thing from the electrical side that we did not talk about, we talked about current, but we never talked about voltage. So we are going to look at those aspects now.

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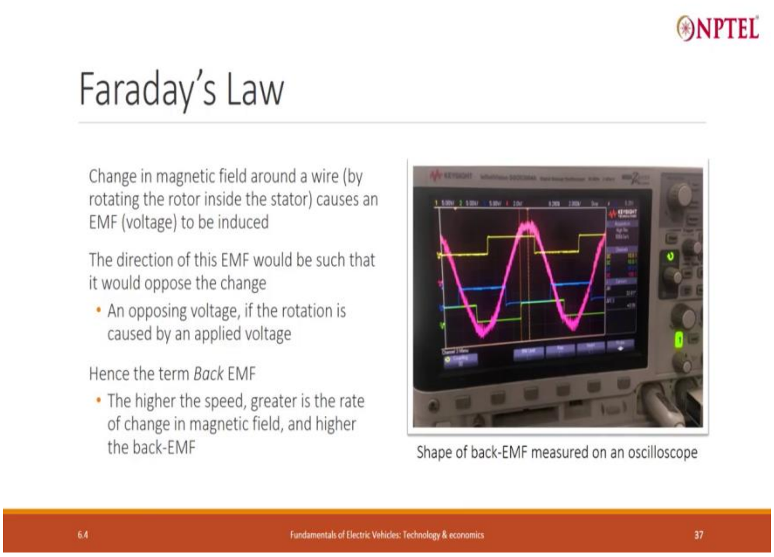

To begin with, I will refresh your memories by talking about what is called Faraday's law. Faraday's law says that when flux changes, it results in the production of an EMF, EMF is just a fancy word, electromotive force for voltage. And how much voltage will the change in flux produce?

The greater the rate of change of flux, the greater will be the magnitude of that EMF. And the direction of that EMF will be opposed to the nature of change of the flux, means the EMF that is getting produced, will try to suppress the change in the flux. That will be the direction.

So these are two parts of the Faraday's law. First one deals with the magnitude of the voltage and the second deals with the direction of the voltage. The second part of Faraday's law is sometimes called Lenz's law and it just is a corollary of conservation of energy because if the voltage were to amplify the change in flux and, in turn, the change in flux will amplify the voltage, then you will have a runaway explosion of energy, you will have creation of energy out of nothing.

So it has to oppose it and suppress the change, otherwise, you will be violating conservation of energy. So Lenz's law or the second part of Faraday's law is just a direct consequence of conservation of energy.

So if I just take a motor, I do not connect it to any power and take the wires coming out of it and link it to a oscilloscope, and if I rotate the motor shaft by hand, then on the oscilloscope, I will see a sinusoidal voltage which you see in this picture, which is called the back EMF profile of the motor. This just comes from Faraday's law.

When I am rotating, the flux is changing because the direction is changing. The direction of the magnet, magnetic field is changing and this change in direction is causing a change in the flux and that is generating a back EMF.

If I rotate it faster, the ups and downs will become bigger, the peak will also go up. At the same time, the rapidity with which it goes up and down will also increase. Because the X-axis is the time axis, I am completing more cycles in one second than I was doing before, so the frequency will also increase and the amplitude will also increase.

So this is just a explanation of the Faraday's law that we just now discussed. Because it is opposing the change, we use the word back, along with EMF; back EMF. And as we just now discussed, at higher speeds the back EMF is greater in magnitude, it is also higher in frequency.

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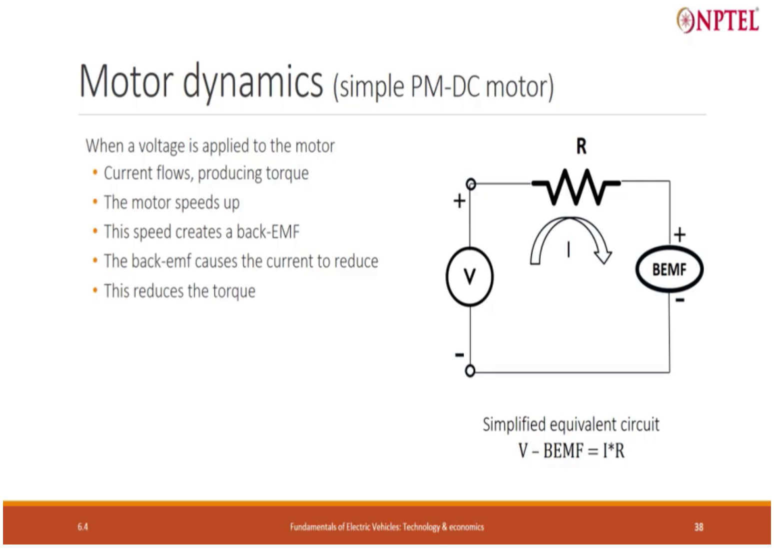

Now, for the sake of understanding, let us go back to the PM-DC motor that we started with. Because PM-DC motor is easier to understand, it is somewhat simple architecture, not because we are going to use it. We will again come back to PMSM and within PMSM, we will come into, IPMSM. But we will start with the PM-DC as we did for torque.

So the PM-DC motor has two poles of a magnet on the stator and there is a loop of wire which is going round and round as we saw earlier. This is a circuit, simple circuit diagram which represents what is happening. There is an applied voltage, which is causing current to flow through the copper wires.

Copper as you know is a very conductor, so the resistance will be very small and if I design the motor carefully, I will try to minimize the resistance. I will try to use copper in such a way that the resistance is as low as possible because I want to avoid losses and an important loss is the copper loss, which is proportional to I square into R. So the, if we keep the R low, I will have less losses, but there will be some loss.

And after, as a result of flowing through the winding, the current is causing the rotor to rotate. And when the rotor rotates, as we saw, the direction between the flux and the magnetic, magnetic flux and the current is changing and this change is resulting in a change of Psi. Since the direction is changing, the Psi is changing, although the magnitude of the current may not change.

And proportional to the rate at which this is changing, a back EMF is generated by the motor, which can be visualized in the circuit diagram as a battery with an opposing voltage. So that block that you see there called BEMF, is like a battery with a plus sign, which is opposed to the applied voltage, and a minus sign, which is again opposed to this minus sign. So in this drawing what you can see is that the net voltage that is driving the current I, is the difference between V and the back EMF. That is one thing that you can straight away see.

If the difference is greater, more current will flow. If the difference is low, less current will flow. But the difference, in turn, depends upon how big the back EMF is. Assuming I am applying some arbitrary voltage V, will this back EMF be constant or not? For this, we have to try to visualize the dynamics.

Initially, the motor is simply at rest, I am applying some voltage, some 5 volts. The motor is not rotating, it is at rest, to begin with. Since it is at rest, there is no change in the flux. Since there is no change in the flux, there is no back EMF. To begin with, there is no, 0 back EMF.

So a large current will flow because the difference between the applied voltage and the back EMF is large, all the 5 volts will get applied. A large current will flow. This large current will result in a large torque, which will cause the coil to rotate and cause it to accelerate because torque will translate into acceleration.

So the speed starts increasing, as the speed starts increasing, the back EMF starts increasing. As the back EMF starts increasing, the difference between the applied voltage and the back EMF decreases. Because that decreases, the current is decreasing. Because the current is decreasing the torque is falling. What started off as a high torque motor, because high current is flowing will slowly have lesser and lesser torque.

Finally, it will reach what we call as a steady-state. A steady-state is when the speed is neither increasing nor decreasing. At steady state, maybe the voltage is 4 volts and it is going at some thousand rpm and I am, from beginning to end, I have been applying only 5 volts. And what happens in the difference between 5 volts and 4 volts, 1 volt will drive a current through the resistance, and that current is delivering a certain torque, which is just enough to overcome the load, whatever load is connected. That load could be air drag, it could be some frictional resistance, whatever. It could be the torque needed for going up a gradient, something.

So where the torque is exactly balanced by the torque supplied, where the load torque is exactly balanced by the torque supplied by the motor, that is when steady state is reached. And the speed at which that balance is achieved will actually define the speed of the motor for that voltage. So it depends not only on the voltage I apply, it also depends upon the load that is connected. Hope you are able to get a physical intuition of this dynamics of the motor.

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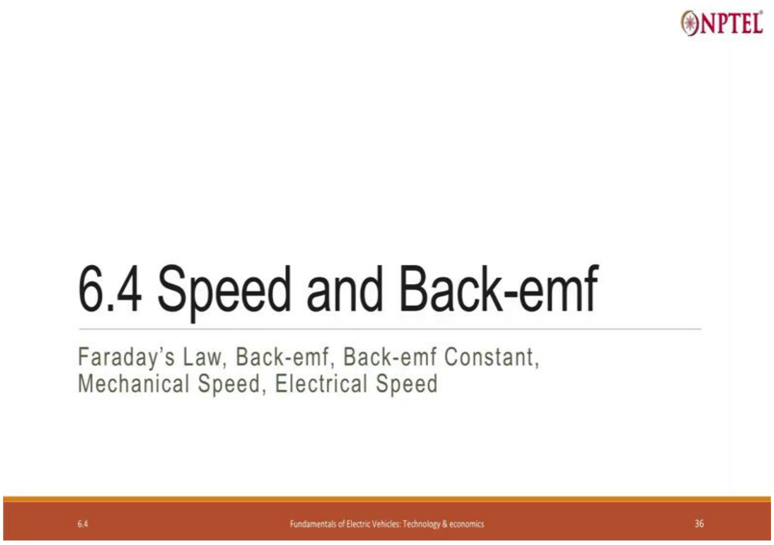

So what we just now discussed is what I have written here. And this equivalent circuit is very simple. The difference between the voltage and the back EMF is what will drive a current and the drop, ohmic drop we call it because it follows Ohm's law, I into R; the ohmic drop will be equal to the difference between the applied voltage and the back EMF.

Is this clear? We are talking so far only about the PM-DC motor. There will be some, an extra layer of complexity when we talk about the IPMSM. So, so far, is there any doubts, please feel free to ask me? Shall we move on?

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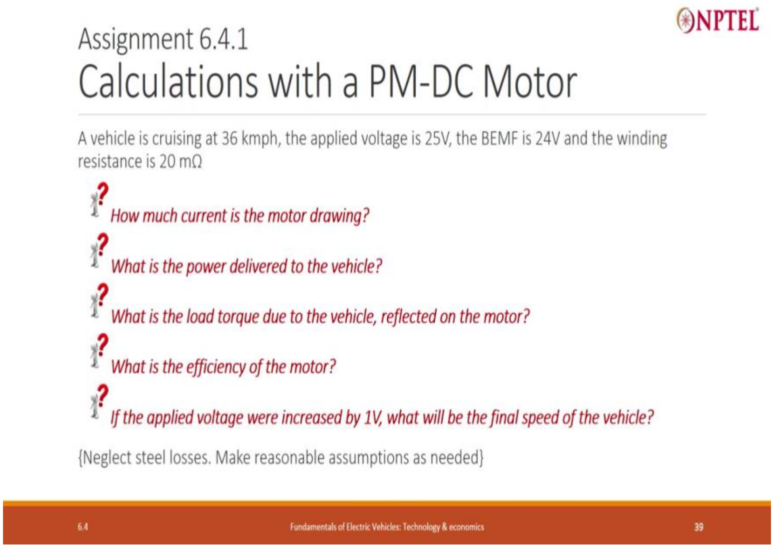

Now, you will be surprised that you have just seen a very simple equation in the previous slide. The difference between two voltages is equal to the current multiplied by the resistance. But already, you know a lot about the motor, which you will discover when you do this assignment. Seemingly very complicated questions about the motor and its behavior, you will be able to answer.

There is a vehicle, which is cruising at 36 kilometers per hour, the applied voltage is 25 volts and when I say cruising, the meaning of the word cruising means going at a constant speed, I am not slowing down, I am not going faster. I am going to constant speed of 36 kilometer per hour, the applied voltage is 25 volts. The back EMF, I can measure it is 24 volts. The winding has a resistance of 20 milliohms, this much I know. Knowing this, this is a series of questions I want you to answer.

How much current is flowing? We have already seen that the difference between the applied voltage and the back EMF is what drives the current, you can do that calculation. How much power is getting delivered to the vehicle? Think carefully, power delivered to the vehicle is the output power of the motor not the input power of the motor. You must understand the wordings carefully, we talk about output power from the motor.

What is the load torque due to the vehicle reflected on the motor? In mechanical engineering, we use this term reflected torque because the actual torque at the vehicle will be something but through the gearbox, which is doing a torque multiplication. Finally, the torque that needs to be delivered by the motor to apply the torque required at the vehicle is called the reflected torque. So we do not know about the gearbox and all those ratios. So I am only talking about what is the output shaft torque of the motor. That is what is the meaning of reflected torque.

Now, what is the efficiency of the motor? For which you need to know the output, power, and the input power. And then, if I were to slightly increase the voltage, coming from the battery, I have a potentiometer or something, throttle; instead of 25 volts, I slightly increase it and make it 1 volt more. You already know that therefore, more current will flow; therefore more torque will be produced, so the vehicle may move faster, and then it will balance everything out at a higher speed. What will be that higher speed because of the extra 1 volt?

All of this you can calculate based on what you already know. The questions may appear very challenging but actually, it is all very simple. Whatever we saw in the previous slide, you can answer this. So do this it will help you develop the intuition better. You can neglect steel losses. And any other assumptions that you want to make feel free to make, as long as they are reasonable.

If you do not know what is the weight of the vehicle and if you need to know it, assume; it is something that is reasonable. If you do not know how big the wheel is, assume; any number that is reasonable and you will be able to answer all of these questions.

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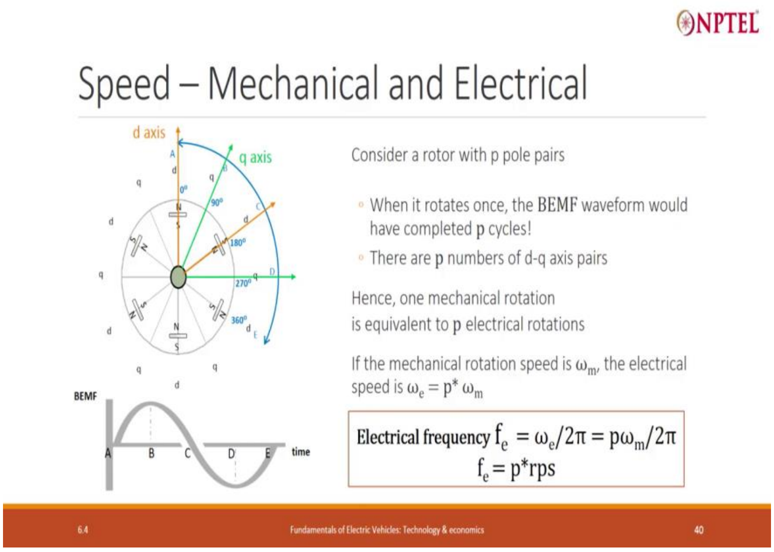

Now, we have, we have some idea of what speed depends upon. It depends upon the applied voltage and on the torque, load torque, which is on the motor. There is an important variant of this idea called speed. We distinguish between two kinds of speed, what is called mechanical speed, which is what you can see when the motor is rotating, and there is something called electrical speed.

Look at this rotor. In this picture, I have shown a rotor with three pole pairs because there are six magnets. Every time one magnetic cycle is completed, one back EMF cycle is also getting completed. If I rotate this motor by connecting to the oscilloscope as I showed in the first slide on this section, I will see the back EMF waveform.

But the back EMF waveform will complete one cycle during just this arc of revolution because one magnetic cycle is got completed. And then the next magnetic cycle, and then the next magnetic cycle. There are three magnetic cycles in the whole mechanical cycle. Can you all visualize this?

So mechanically, when I only cover one-third of a circle, which means 120 degrees, in a magnetic sensor I have already covered 360 degrees, a full cycle has been completed. And moving from A, going via B, C, D, and reaching E, up to this point, if I look at what is the back EMF waveform, it is like this. A full-wave has been completed.

So what this tells us is the electrical cycles are p times, the mechanical cycles, where p is the number of pole pairs. So simply applying the ratio p will help us convert mechanical speed into electrical speed. Is this fine? It is a very important concept very simple, just a number p you have to multiply by. But why do we do it is something that I want you all to grasp.

So because there are p pole pairs, every successive pole pair, I told you the magnetic flux froze, flows from north to every successive south, and again to the north and again to the south. If I were to draw the d-q axes, I will find that there are p sets of d-q axes here. Here is the d-axis, from one magnet to the other, next successive one.

Likewise, there is a q-axis, which is in between them. And it looks funny because the d and q axes are not at 90 degrees, they are at 90 divided by p degrees. In this case, because p is 3, three pole pairs, the angle between the d and q-axis is 30 degrees. So when I complete one mechanical revolution, I am cutting across three pairs of d-q axes. And every d-q axes is one electrically complete revolution.

So we can say that if the rotor is rotating at an angular speed omega m, where m is a subscript that denotes mechanical, the electrical speed of revolution is p times that. So omega e is p times omega m. And omega e divided by 2 pi is called the frequency as you are all familiar. That is the electrical frequency of this waveform, what you normally measure in hertz.

And of course, instead of going into omega and 2 pi and everything, if you know what is the rpm divided by 60, you will get the rotations per second. And rotations per second multiplied by p is the frequency. That is what the last line is indicating to you. The frequency is p times rps; is a simpler form of remembering how to calculate the frequency. Is this fine?

So when we talk of speed, there are two important parameters that emerge from it. One that we have just now discussed the electrical frequency.

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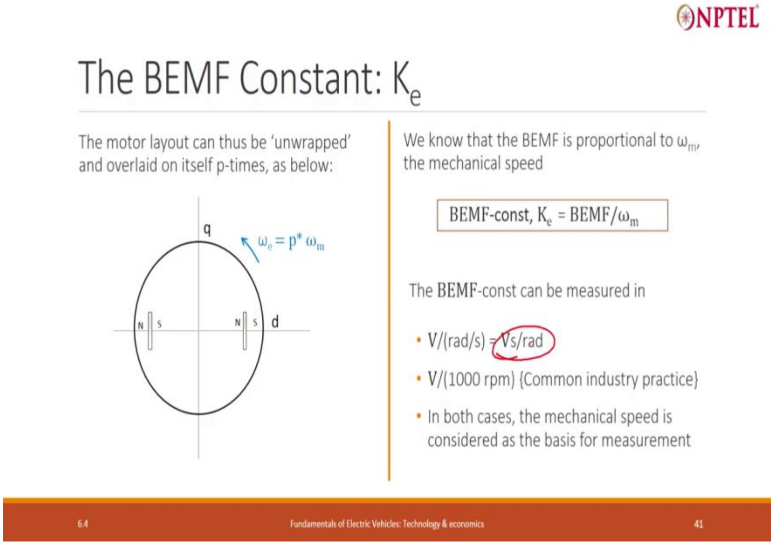

The other important parameter that comes as a result of speed is what is called the back EMF constant. Before going into what is the back EMF constant, I will show you a drawing which is less messy than what you saw in the previous slide. This is how we commonly represent a rotor. We do not like all these X and Y axes which are at 30 degrees and drawing so many of them, it becomes very messy.

So from an electrical point of view, we like to keep the diagram very clean and simple. I will show only one pole pair. I will have the normal d and q axes which are perpendicular to each other but I will show the rotor as rotating p times faster and say that the angular speed is omega e, which is p times omega m.

So it is as if I have taken the diagram that you saw in the previous slide, grabbed the ends of the d-axis which were at, they were forming an angle, which is 180 by 3, 60 degrees. I grabbed it and stretch it out threefold so that everything stretches out and the successive pole pairs all go under each other. So it is like unwrapping the rotor and making it go around itself p times. That is what this picture is. Fine?

Now, let us look at the idea of back EMF once again. We have already discussed that the faster the rotor rotates, greater the back EMF will be. And we were talking then about the mechanical speed, we will continue to talk about mechanical speed. Always when we talk of back EMF, back EMF is measured in relation to the mechanical speed, not the electrical speed.

So there is a constant of proportionality, the voltage that I will get when I measure it on my oscilloscope is directly proportional to the mechanicals speed. That means, there is a constant of proportionality. The back EMF is equal to some constant multiplied by the mechanical speed. Is this clear?

That constant of proportionality is called the back EMF constant. It is simply equal to the back EMF voltage divided by the mechanical speed. Very simple idea but a very powerful idea, we will use it in different ways as we go forward. So please grasp it clearly and remember it.

Only two things that you have to remember in the context of talking about speed. One is the frequency, which talks about the frequency of the voltage of the back EMF and the other is the back EMF of constant, which is talking about the amplitude of the back EMF voltage. How high are the waves, comes from the back EMF constant and how rapidly do they go up and down the frequency, that comes from the electrical frequency.

So this is a very important relationship. It is a fundamental label of a motor. When I want to compare one motor and another, first thing I will look at is what is the back EMF constant. By comparing them I will know many things about the motor. We will learn about that going forward.

So now what is the units of measuring the back EMF? As you can see, back EMF constant is back EMF divided by mechanical speed. Back EMF is measured in volts, mechanical speed is measured in radians per second. So volts per radian per second, which can be expressed as volt second per radian, which is what is given here.

But this is a somewhat physics way of expressing it. What is commonly used in the industry is, I will take the motor, rotate it a 1000 rpm, and measure the back EMF. Whatever back EMF I get, I call that as the back EMF constant. So then the units will be volts divided by 1000 rpm. In both cases, when we say speed, we are talking only about the mechanical speed, not the electrical speed. So these are two different units of measuring. Any doubts? If you have no doubts, I will give an assignment.

It has been a conventional industrial practice because when I want to rotate with my hand, I do not literally do it with my hand; I will couple it with some, some other motor which will be running at a known speed and I will calculate the, I will measure the back EMF. Now, if I one day couple it with a motor which is running at 1440 rpm and I find a certain back EMF, and tomorrow you do the same thing but you run it with another, by coupling to the motor which is running at 3000 rpm, then how do we establish a common basis? You will measure a higher back EMF voltage, I will measure a smaller back EMF voltage.

So we all agree that whatever voltage I get, I will express it as per thousand rpm. So this has been the historic industry practice which is followed, so it is also equally valid. So there are two different units in which we can express the back EMF constant.

So if the voltage you measure, I divide by 3000 rpm since you measured it at 3000 rpm, I will divide it by 3. Then I will say that the voltage per thousand rpm that you measured is this. And if I measure, if I divide the voltage I measured by 1.44 because I ran at 1440 rpm, I will get a number and I will say that is a voltage I measured per thousand rpm. And both the numbers will be the same if we measure the same motor. Fine? Any other question?

Student: The back EMF (())(27:13) to the rpm?

Professor: Yes, the back EMF is proportional to Faraday's law, rate of change of flux, and rate of change is proportional to the mechanical speed. So the ratio between the mechanical speed and the back EMF is constant, that is called the back EMF constant.

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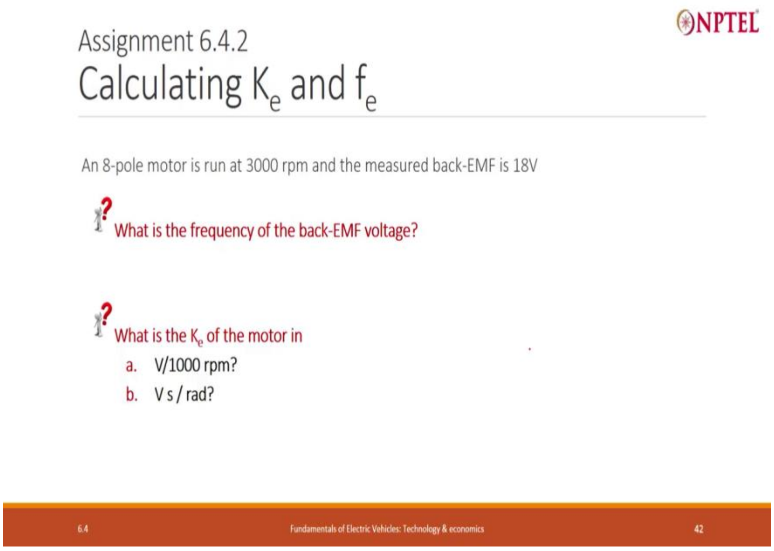

So here is the assignment for you, where you have to calculate Ke and fe. Ke is the back EMF constant, fe is the electrical frequency. So an eight-pole motor; remember, it is an eight-pole motor, the drawing I showed you earlier was a six-pole motor, this is an eight-pole motor. It is run at 3000 rpm and I measure by connecting it with a oscilloscope that the back EMF is 18 volts.

The question is what is the frequency of the back EMF? Next, what is the back EMF constant of the motor in both the units, volts per thousand rpm, and volt second per radian?