

Fundamentals of Electric Vehicles: Technology and Economics
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Lecture 6
Torque Production - Part 3

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The d-q plane and Total Torque

$$\vec{\tau} = \vec{I} \times \vec{\Psi}$$

$$= (I_d \hat{i} + I_q \hat{j}) \times (\Psi_d \hat{i} + \Psi_q \hat{j})$$

Here, $\Psi_d = \Psi_M + L_d I_d$

Hence, $\tau_{total} = (L_q I_q) I_d - (\Psi_M + L_d I_d) I_q$

$$\equiv -\Psi_M I_q + (L_q - L_d) I_d I_q$$

Thus, $\tau_{total} = \tau_M + \tau_R$

Since $L_q > L_d$

τ_R adds to τ_M only if I_d is negative!

So, let us now examine the IPMSM of motor in some more detail. We look at the reluctant torque in isolation. Let us look at what is total torque I will get. Again we can draw to a drawing like this where we are concern with the total flux, we are not worried whether it is coming from the winding and the reluctance or whether it is coming from the permanent magnet. I simply trade everything as flux.

And we know that the permanent magnet flux is along the d axis and it is not the permanent magnet flux is not depended on the current and it is acting along the d axis. In addition, there an $L_d I_d$ which is the d component of the reluctance flux which is also acting along the d axis. And perpendicular to the d axis that means on the q axis I have the flux due to the q component of the current which is $L_q I_q$ so the total flux there is acting is in this form, it is vector sum of the 2 things.

Now, again I can say torque is equal to $I \times \Psi$ I is nothing but I_d plus I_q with the unit vector appropriately. And I do a cross product with Ψ_d and Ψ_q . The Ψ_d is nothing but is this, this

I_q is of course equal to $L_q I_q$. So now I can just expand this term by doing the cross product. And now for clarity sake.

If I work to isolate the magnet, magnetic flux related term separately and then add all the other things in another bunch. Then I will find that I have got the magnetic torque as 1 term, ψ_m and I_q we already saw and we have got the L_q minus L_d into I_q which is the reluctance torque that we saw. So, the total torque is the sum of the magnetic torque and the reluctance torque. Is this clear? But now in this there is a problem.

The magnetic torque has a negative sign in relation to the reluctance torque, it means when I go and pay for 12 cakes, I get only 11. When I thought I will get 13 this is the problem because the 2 are in opposite directions. Now, without changing the direction of the magnetic torque which is the dominant part. I want to change the sign of the reluctance torque. How can you change it? There are only 3 terms, 1 is L_q minus L_d , another is I_d , another is I_q .

We already know that L_q is more than L_d , therefore L_q minus L_d will be positive, we cannot do anything about that. I can change I_q by changing the direction of the current but then that will change the direction of magnetic torque also because I_q is in both terms. So, there is no point in changing I_q I want to change the sign of the 2nd term which is the reluctance torque without changing the sign of the first term.

So, my only option is this I_d , if I make I_d alone negative then the 2 torques will add up I will get 13 cakes instead of 12 but if I_d is positive I will get only 11 cakes. Do you get this? So, this is the very important idea we will go into it a little more detail in the next 1 or 2 slides. But I want you to remember that in IPMSM motor we deliberately make I_d negative, so that reluctance torque adds up to the magnetic torque instead of canceling out against it.

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Phase-advance Angle

Now we know that I_d must be negative for the reluctance torque to add to the magnetic torque, rather than act against it

The phase of the stator current is therefore *advanced* as shown

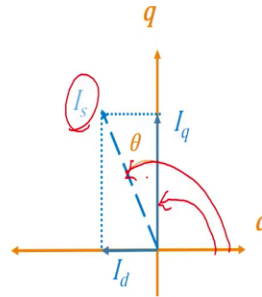
$$\tan(\theta) = I_d / I_q$$

$$I_s^2 = I_q^2 + I_d^2$$

$$I_q = I_s \cos(\theta)$$

$$I_d = I_s \sin(\theta)$$

θ is called the Phase-Advance Angle



So, we come to a very important concept called the phase advance angle which is based on the idea we just now discussed. I have I_q as positive as you can see here but I have deliberately made I_d negative. Which means it is in a direction that is opposed to the magnetic field. And the total current which is flowing in the stator I have shown that by I_s this is the vector sum of I_d and I_q .

So, that vector sum will make an angle theta from what we already know about trigonometry, we can say that I_s will be the Pythagorean sum of I_d and I_q which means I_s squared is equal to I_q squared plus I_d squared. And $\tan \theta$ will be equal to I_d by I_q , since I have advanced the direction of I_s . Normally we went on saying in the earlier discussions that the current has to be perpendicular to the magnetic field to get maximum torque.

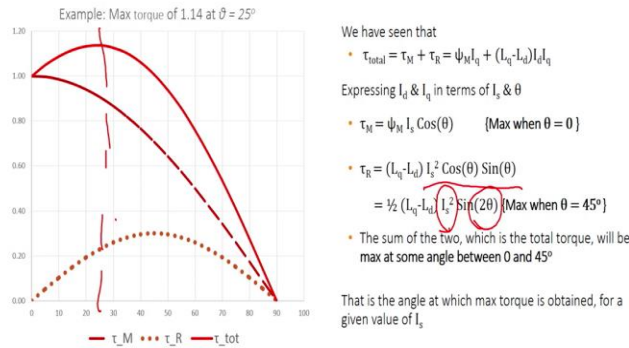
But here I am deliberately advancing its phase a little more than 90 degrees our earlier discussion was that you get maximum magnetic torque when it is 90 degrees. But since we are greedy for the reluctance torque also we are further advancing it by amount of theta. So, this theta is called the phase advance angle. So, $\tan \theta$ as we discuss is I_d by I_q I_s squared is equal to I_q squared plus I_d squared.

If I know what is I_s and theta I can expressed I_d and I_q in terms of I_s and theta, I_q will be the equal to I_s into $\cos \theta$, I_d will be equal to I_s into $\sin \theta$. Is this clear? So, this theta is the term which will keep on haunting us forever after when we are dealing with an IPMSM motor

because it is very important matrix in optimizing the performance of a motor. So please remember what this theta is and it is called phase advance angle.

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Max Torque Per Ampere (MTPA)



Based on this phase advance angle what we understand is that if I change theta, if I increase theta the magnetic torque will reduce, the reluctance torque will increase, what will happen to the total torque? We do not know; it may increase or it may decrease because the total torque is made of magnetic torque plus reluctance torque. One of them is decreasing the other is increasing but what we are caring about is not individually what is happening to the magnetic torque and what is happening to the reluctance torque.

We want to maximize the total torque how do I select theta in such a way that I maximize the total torque. So, let us look at that the total torque is the sum of the magnetic torque and the reluctance torque. If I express the I_d and I_q in terms of I_s and theta as we just now saw. I will find that the magnetic torque is proportional to $I_s \cos \theta$. And when I say $I_s \cos \theta$ I can readily find that for a given current, if I want maximum torque $\cos \theta$ can at most be 1. And that will happen when theta is 0.

So, when I do not do any phase advance I will get maximum magnetic torque. If I try to plot what the magnetic torque is with respect to theta. I will get something like this as theta becomes greater and greater the magnetic torque will keep falling and finally at 90 degrees, the current is in the same direction as the magnetic field, so I will get no torque at all.

Next let us see what happens to the reluctance torque, the reluctance torque is L_d minus L_q into I_d into I_q . I_d into I_q is I_s into $\cos \theta$ multiplied by I_s into $\sin \theta$. So, I can say that is equal to $I_s^2 \cos \theta \sin \theta$ but $\cos \theta \sin \theta$ is nothing but half of $\sin 2\theta$. So, here is the trick term the magnetic torque was depended directly on θ . But the reluctance torque because it is the product of I_d and I_q is depended on 2θ .

On the one hand I wanted observed 2 things about reluctance torque. It depends upon 2θ and it is also proportional to square of the current not just the current but the square of the current will come to that in a minute. But what we can see directly from the 2θ is $\sin 2\theta$ can have a maximum value of 1 that will happen when 2θ is 90 degrees, that means θ is 45 degrees. So, if I plot the reluctance torque with respect to the θ I will get a graph like this.

If I tried to maximize the magnetic torque, then I will have to be here I will get no reluctance torque. And the sum of the 2 will be 0 plus this, it will be equal to 1. If I tried to maximize the reluctance torque I will be here, this value is something between 0.2 and 0.4 and this value for that angle 45 degrees is something between 0.6 and 0.8 so this is 0.7 this is 0.3, if I add it I will get 1 but actually I will not get 1. I am doing only approximate visual addition if I actually add it I get actually something less than 1.

So, 45 degrees is not optimum for the total torque but it is optimum for the reluctance torque. 0 degrees is optimum for the magnetic torque but also not optimum for the total torque. To know which is the optimum angle I just have to add the torque at each value of θ and see how that graph goes, and see where the peak of that is.

As you can guess that angle will be between this peak and this peak somewhere in between them it will be somewhere between 0 and 45 degrees. In this particular case, this is what the shapes look like and if I see where this peak is coming I have drawn it very badly. But it is coming at about 25 degrees this way it will go. And that maximum torque has a value of 1.14 I am getting 14 percent more torque.

Then I would have got if I did not use phase advance to exploit the reluctance torque. If I only focus on the magnetic torque I would have got a torque of one but now I am getting 14 percent more torque. I want you to before we go on to the next slide I want you to reflect a little more on

this interesting dynamics of the reluctance torque. We only look at the theta part that the reluctance torque is proportional to $2 \theta \sin \theta$.

The part that we did not look at is that the reluctance torque is proportional to square of the current. Whereas the magnetic torque is proportional to the current supposing I want to halved the current. What will happen is what I can do control if a most P. So, supposing halved the current let us look at the things separately. I only look at the magnetic torque I halved the current that means this peak will half it will come to 0.5 because the peak value is proportional to I_s .

If I halved the current and look at what happens to the reluctance torque the reluctance torque is proportional to the square of I_s . Since I halved the current this peak will become one fourth. The reluctance torque will fall far more than the magnetic torque if the current is lower. So, if I plot the total torque curve again the optimum angle will move closer to 0, because the wattage of the magnetic torque is higher when the current is lower.

Are you able to visualize that? And you can visualize the same thing in the converse direction if I were to double the current then this peak which is a magnetic torque will become doubly high. But the reluctance torque peak will become 4 times as high it will become nearly equal. At higher currents reluctance torque increases in prominence which means at higher torques I will get a greater bonus in the form of reluctance torque.

At lower torques the bonus that I get in the form reluctance torque will be lower. As a result, what is the optimum angle which we saw in this example is 25 degrees, really depends upon the current at a higher current the angle will become larger. And at a lower current the angle will become close to 0. So, this is what I want you to explore as an assignment.

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Assignment 6.3.2



Plot MTPA Angle (θ) vs. Stator Current (I_s)

The total torque is of the form

$$\tau = A I_s \cos(\theta) - B I_s^2 \sin(2\theta)$$

Assume $A = 0.01$ Wb; $B = 0.001$ H

- Vary I_s from 0 to 100A (in increments of 10A). For each value of I_s , find the θ that gives max τ
- Plot I_s vs. θ (in polar coordinates)

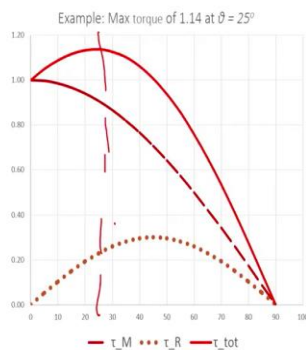
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Max Torque Per Ampere (MTPA)



We have seen that

- $\tau_{total} = \tau_M + \tau_R = \psi_M I_q + (L_q - L_d) I_d I_q$

Expressing I_d & I_q in terms of I_s & θ

- $\tau_M = \psi_M I_s \cos(\theta)$ (Max when $\theta = 0$)

- $\tau_R = (L_q - L_d) I_s^2 \cos(\theta) \sin(\theta)$
 $= \frac{1}{2} (L_q - L_d) I_s^2 \sin(2\theta)$ (Max when $\theta = 45^\circ$)

- The sum of the two, which is the total torque, will be max at some angle between 0 and 45°

That is the angle at which max torque is obtained, for a given value of I_s

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We have seen that the total torque is magnetic torque plus reluctance torque, magnetic torque is proportional to current into cos theta and reluctance torque is proportional to square of the current into sin 2 theta. This is the generic way in which I can express the total torque. For a particular motor that we have in hand let A, B equal to 0.1 weber.

You remember that A is actually the x_i of the magnet so it is expressed in weber and B is difference in the inductance L_d minus L_q . So, its units will be henry and I it is the value of B is 0.001 henry. This is the sample of a motor that we have got now I want you to calculate for different currents from 0 to 100 amperes. What will be the optimum theta as we already know when the current is low theta value the optimum theta value will be small.

And as the current increases it means more and more torque is being demanded by the motor. The value, the optimum value of theta will go on increasing but it will never be greater than 45 degrees. Because 45 is where reluctance torque becomes a maximum so from 0 to 45 degrees as current increases theta will increase. If I work to plot the current versus theta in polar coordinates means the length of the point from the origin is proportional to I_s .

And the direction is proportional to theta. And remember theta has to negative take that into account in the previous graph I show it as positive but theta will be negative plot it and you can use any tool like Excel or Mat lab whatever you are comfortable with it is easy to do in Excel also. And if you do that you will get a plot like this. So, we have a methodology where we measure the current. And based on the current we change the angle in the software.

So, that we get maximum torque this is called MTPA what we saw in the previous slide. maximum torque per ampere fine. So, If I select the angle correctly based on those equations then I can get more torque than I can get any other angle for the same amount of current. But if the current changes then the angle will also change accordingly. And this algorithm which will be important when we later on look at the controller design is called MTPA maximum torque per ampere. If there any questions? I will take them or we can move on to the next session.

Student: (())(19:31)

Professor: The magnet material, the question is why is L_q greater than L_d because in the path of L_d there are two magnets. In the path of L_q there is only steel. See in general the reluctance offered by steel is much lower than the reluctance offered by air. This is all you get it? The somewhat surprising fact but it is a fact of nature is that the reluctance offered by a magnet is almost the same as the reluctance offered by air.

So, magnet is actually high reluctance material, the material of the magnet actually so when you see magnet in the path of steel and you trying to visualize the reluctance you can think of the

magnet as an air bubble. Fine? So, in the d direction you have got 2 magnets which are like air bubbles offering high reluctance whereas in the q axis you do not have those air bubbles because there are no magnets. So, the reluctance is lower where the reluctance is lower the inductance is higher therefore L_q is greater than L_d . Is that clear? Any other question?