Fundamentals of Electronic Vehicles: Technology and Economics
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Lecture - 51
Torque Production - Part 2

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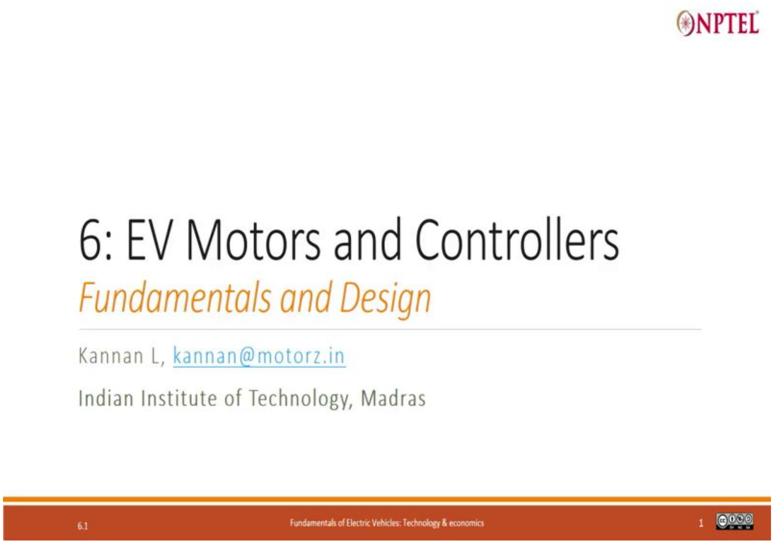

NPTEL

6: EV Motors and Controllers

Fundamentals and Design

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Good morning, everybody. Before moving ahead with what we have to cover today, I thought I will take you on a very quick recap of what we did last time.

The summary was that we looked at a number of different flows that happen across different physical phenomena and the flows are all characterized by three common features. One is Ohm's law, where the force which is causing the flow and the thing that is resulting in the flow are related by a resistance or sometimes the reciprocal of it; the conductance. And the second law that relates them is that the thing that is flowing is conserved, it is somewhat like conservation of mass.

And the third law that relates them is that the difference in potential is a conservative field, which is independent of the path. So whichever path we take, the difference in potential is the same and that is another way of saying conservation of energy.

So Ohm's law, conservation of mass, conservation of energy. Conservation of mass is called Kirchhoff's first law or the continuity equation, and conservation of energy is Kirchhoff's second law.

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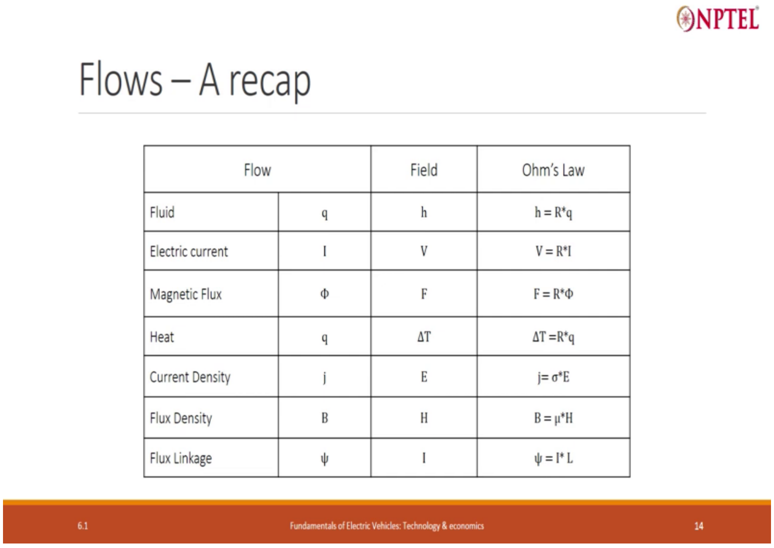

Flows – A recap

Flow		Field	Ohm's Law
Fluid	q	h	h = R*q
Electric current	I	V	V = R*I
Magnetic Flux	Φ	F	F = R*Φ
Heat	q	ΔT	ΔT =R*q
Current Density	j	E	j= σ*E
Flux Density	B	H	B = μ*H
Flux Linkage	ψ	I	ψ = I* L

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So this is common to all the flows and in this manner, we looked at a number of different flows which are shown here. So we looked at flow of fluid, flow of electric current, magnetic flux which is caused by what is called the magneto-motive force; heat, where the flow of heat is caused by difference in temperature.

And we found that it is often convenient to express the flow per unit area and the forcing function as per unit length. Then we get rid of the geometry parameters of the medium that is offering resistance. So the conventional Ohm's law can be expressed as the relationship between current density and the potential field. Likewise, flux density and the H can be related and the flux linkage can be related to the current.

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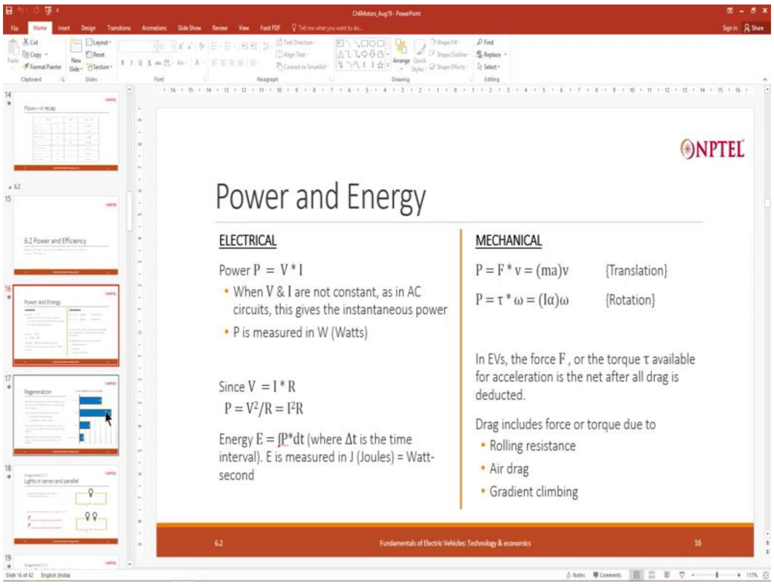

Power and Energy

ELECTRICAL

Power P = V * I

- When V & I are not constant, as in AC circuits, this gives the instantaneous power
- P is measured in W (Watts)

Since V = I * R

P = V²/R = I²R

Energy E = ∫P*dt (where Δt is the time interval). E is measured in J (Joules) = Watt-second

MECHANICAL

P = F * v = (ma)v {Translation}

P = τ * ω = (Iα)ω {Rotation}

In EVs, the force F , or the torque τ available for acceleration is the net after all drag is deducted.

Drag includes force or torque due to

- Rolling resistance
- Air drag
- Gradient climbing

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So this is broadly what we covered last time and then we went on to examine what is power, how is it related to energy, how is mechanical power related with the electrical power? And the losses along the way, which contribute to loss of power, and therefore, the efficiency is always less than 100 percent.

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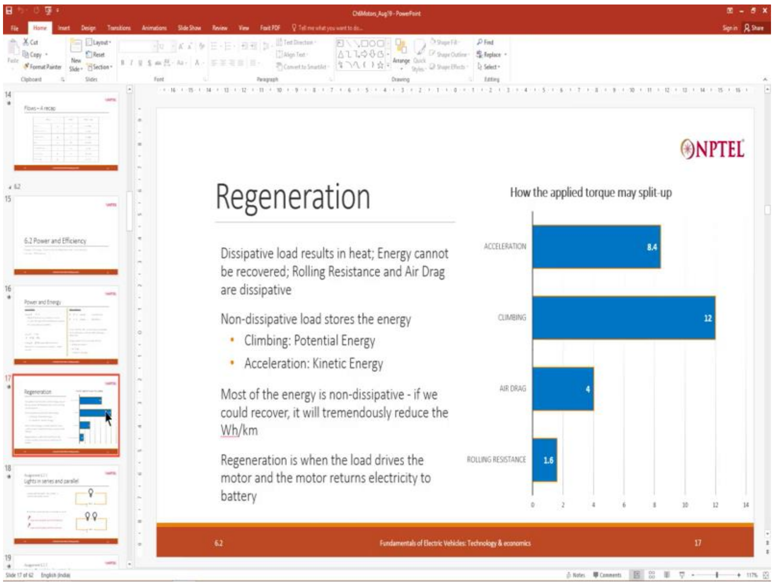

Regeneration

Dissipative load results in heat; Energy cannot be recovered; Rolling Resistance and Air Drag are dissipative

Non-dissipative load stores the energy

- Climbing: Potential Energy
- Acceleration: Kinetic Energy

Most of the energy is non-dissipative - if we could recover, it will tremendously reduce the Wh/km

Regeneration is when the load drives the motor and the motor returns electricity to battery

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And we examine what is called regeneration, which comes from the idea that some losses are dissipative; whereas, some others are not dissipative. They result in storage of energy. We could possibly recover that and put it back in the battery.

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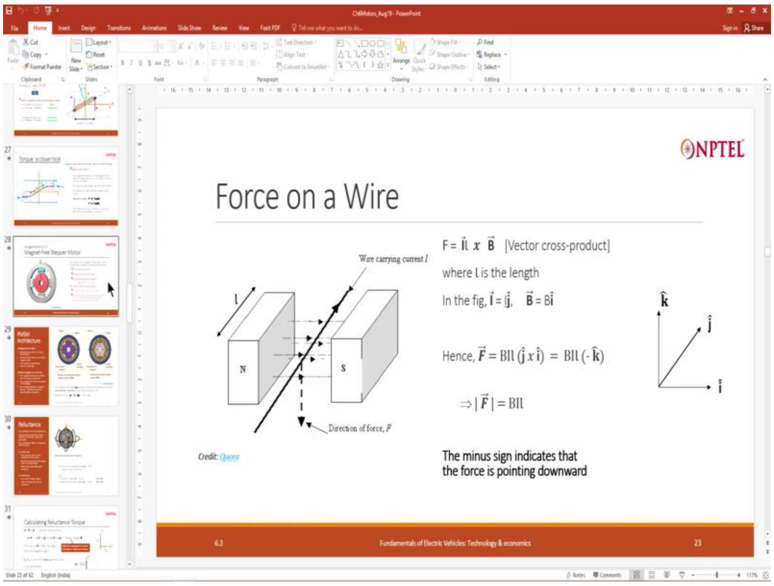

Force on a Wire

$F = \vec{I}l \; x \; \vec{B}$ [Vector cross-product]

where l is the length

In the fig, $\vec{I} = I\hat{j}$, $\vec{B} = B\hat{i}$

Hence, $\vec{F} = BIl\,(\hat{j}\,x\,\hat{i}) = BIl\,(-\hat{k})$

$\Rightarrow |\vec{F}| = BIl$

The minus sign indicates that the force is pointing downward

Credit: Quora

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And finally, we started looking at torque production. The most fundamental case being where a current-carrying conductor in a magnetic field experiences a force, which is perpendicular to the direction of both the magnetic field and the current. And this can be then exploited by turning the wire on to itself like a loop and it will result in torque production.

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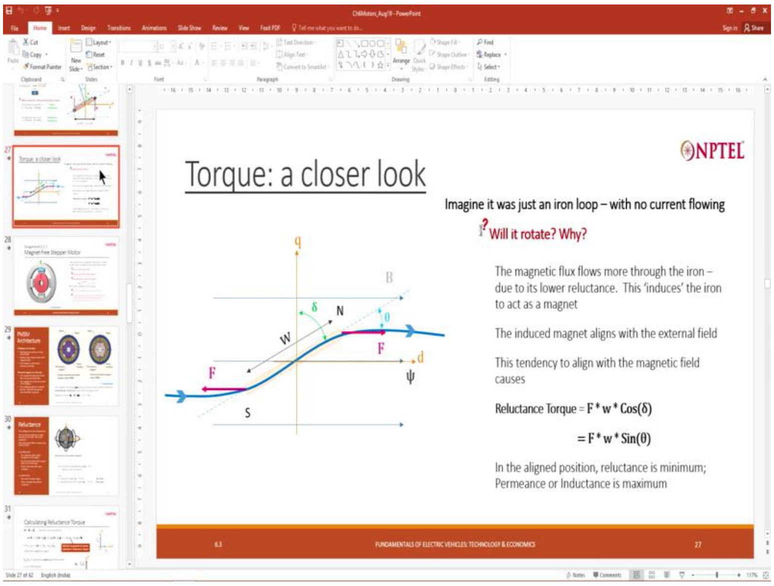

Torque: a closer look

Imagine it was just an iron loop – with no current flowing

Will it rotate? Why?

The magnetic flux flows more through the iron – due to its lower reluctance. This 'induces' the iron to act as a magnet

The induced magnet aligns with the external field

This tendency to align with the magnetic field causes

Reluctance Torque = F * w * Cos(δ)

= F * w * Sin(θ)

In the aligned position, reluctance is minimum; Permeance or Inductance is maximum

And finally, we looked at what is called magnetic torque and what is called reluctance torque. Reluctance torque is something that we get along the way, it is not intentionally driven. By far, the most prominent torque with a permanent magnet motor is the magnetic torque.

But in addition, the fact that there is steel in the form of the rotor core along the way of the magnetic field means that it will tend to be turned in some direction. And that is what contributes to reluctance torque.

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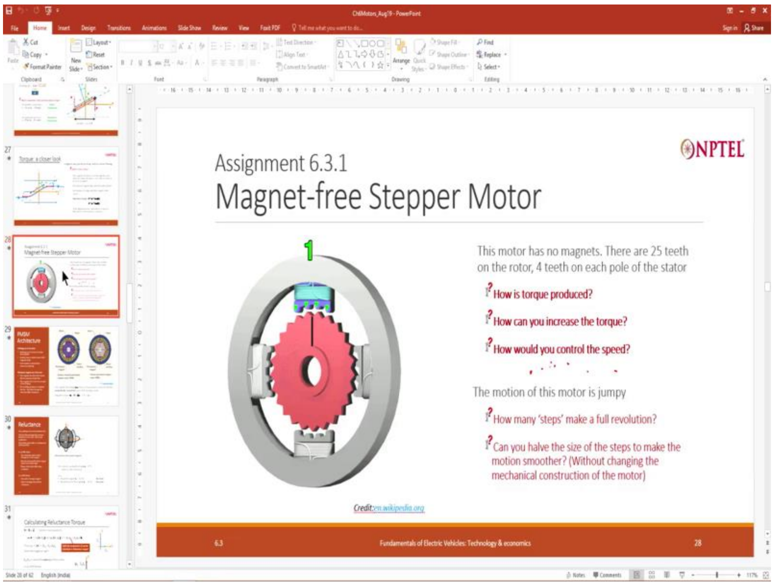

Assignment 6.3.1

Magnet-free Stepper Motor

This motor has no magnets. There are 25 teeth on the rotor, 4 teeth on each pole of the stator

How is torque produced?

How can you increase the torque?

How would you control the speed?

The motion of this motor is jumpy

How many 'steps' make a full revolution?

Can you halve the size of the steps to make the motion smoother? (Without changing the mechanical construction of the motor)

Credit: en.wikipedia.org

And we signed off by looking at the example of a motor, which has no magnets. It is called a stepper motor, where only reluctance torque is used to produce torque.

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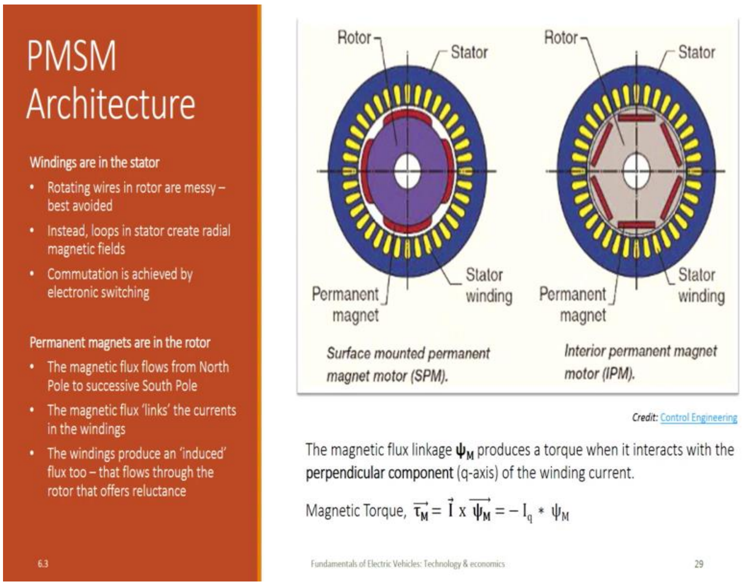

PMSM Architecture

Windings are in the stator

- Rotating wires in rotor are messy – best avoided
- Instead, loops in stator create radial magnetic fields
- Commutation is achieved by electronic switching

Permanent magnets are in the rotor

- The magnetic flux flows from North Pole to successive South Pole
- The magnetic flux 'links' the currents in the windings
- The windings produce an 'induced' flux too – that flows through the rotor that offers reluctance

Surface mounted permanent magnet motor (SPM).

Interior permanent magnet motor (IPM).

Credit: Control Engineering

The magnetic flux linkage $\boldsymbol{\psi}_M$ produces a torque when it interacts with the **perpendicular component** (q-axis) of the winding current.

Magnetic Torque, $\overrightarrow{\tau_M} = \vec{I} \times \overrightarrow{\psi_M} = -I_q * \psi_M$

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So now, we will go on with the subsequent topics that we are going to cover today. If you ever think that I am going a little too fast, please feel free to raise your hand, slow me down. Because some of the sections will be a little bit mathematical and I can cover it more slowly so that you grasp the import of it.

So far, we have talked about what is called the PMDC motor, where a field is created by a magnet which is stationary. And there is a wire which is in the magnetic field in the form of a loop and it is rotating.

And we already saw that a rotating wire loop is a messy thing. Because you have to then do commutation by using mechanical split rings and carbon brushes and then take the leads on to the source of power. So normally, it is convenient in electric motors to invert this topology and that is called PMSM, Permanent Magnet Synchronous Motor.

The way it will look is the exact opposite of the topology of a PMDC motor. Here, the rotor has the magnets and the windings are on the stator, in the non-rotating part. And again, you see two kinds of options here. One, where the magnets are pasted on the surface of the rotor, and another where they are embedded inside slots, inside the rotor.

The one, where it is on the surface is called an SPM motor, Surface Permanent Magnet. And one, where it is inside the steel of the rotor it is called a IPM motor, Internal Permanent Magnet. So this architecture is somewhat different from the PMDC motor that we earlier saw and it is called the PMSM. We will predominantly be discussing the PMSM architecture in all the subsequent parts of this chapter because this is the preferred architecture, you will soon be seeing why this is a better one.

To begin with, we are already aware that having mechanical brushes that are sliding over split rings is a disadvantage. Apart from the friction, there is also a risk of sparking because you are disconnecting from one split ring and jumping on to the other, and during that, there could be sparks if you are in a place where there are combustible fumes; that can be a hazard.

So here, because the windings are on the stator, you do not have this problem of the wires going round and round. So the wires are stationary and whenever you want to do communication, you can do that through electronic switching. So this is one important advantage of the PMSM architecture.

The other thing is that now that the magnets are in the rotor and of course, it will not be just one pair of magnets, there could be multiple pairs of magnets. So normally, the north pole will be alternating with the south pole. You can see here if we say that this is a north pole, then this will be a south pole and this will be a north pole and this will be a south pole.

In this picture, we say that we have two pole-pairs. So the pole-pair is an important word I want you to remember. So if there are four magnets, you can conclude that there are two pole pairs. Likewise, in the picture here, you have got six magnets. They will be typically alternating in polarity, north alternating with south. And in this picture that you see, the IPM magnet has three pole pairs.

And the magnetic field will go from every north pole to the successive south pole, that is a way the field will travel. And along the path, it will link the wires in the stator through which current is flowing and when a flux linkage happens due to a magnetic field that is cutting across them, you know that that will result in the torque production. And this torque production you are already familiar, it is called the magnetic torque.

And we can resolve the current flowing through the wire into two components. One, which is parallel to the magnetic field, another which is perpendicular to the magnetic field. And the component that is perpendicular is what results in magnetic torque production. All of this, we have already covered in the case of the PMDC motor and similar thing will be happening here, there is no difference.

But there is one important new phenomenon that is happening. If you see the windings are wound around small projections in the stator. These projections are called teeth. So there is a, there are a number of teeth. You can see that every blue projection is a tooth, there are a number of teeth. And in between the any pair of teeth, there is a empty space. And that empty space is called a slot.

So the stator is made of teeth and slots. And the copper wire is taken through the slots and wound back around a tooth. Sometimes I can bring it around two teeth. These are all different winding patterns that are possible. That is my choice. But essentially, the windings go around the teeth.

And when the winding goes around the tooth, the fact that current is circulating around the steel tooth means that the steel tooth itself becomes a magnet. So there is a flux that is emanating radially towards the center which is emerging from the tooth and some other tooth on the other side will have the opposite magnetic polarity into which it will flow.

So this is a very interesting phenomenon because we are having a new, so to speak, secondary magnetic field in addition to the magnetic field caused by the magnet. Now, the magnetic field produced by the magnet is interacting with the current going in the windings and it is producing magnetic torque, we have already discussed that.

But what happens to the magnetic field that is coming from the teeth, because of the winding, current in the windings. It has no other current to interact with but it passes through the rotor, and when it passes through the rotor it encounters reluctance. We will examine what is the consequence of this reluctance in the next slide. I just want you to notice that there is an additional magnetic flux apart from the magnetic flux of the permanent magnet.

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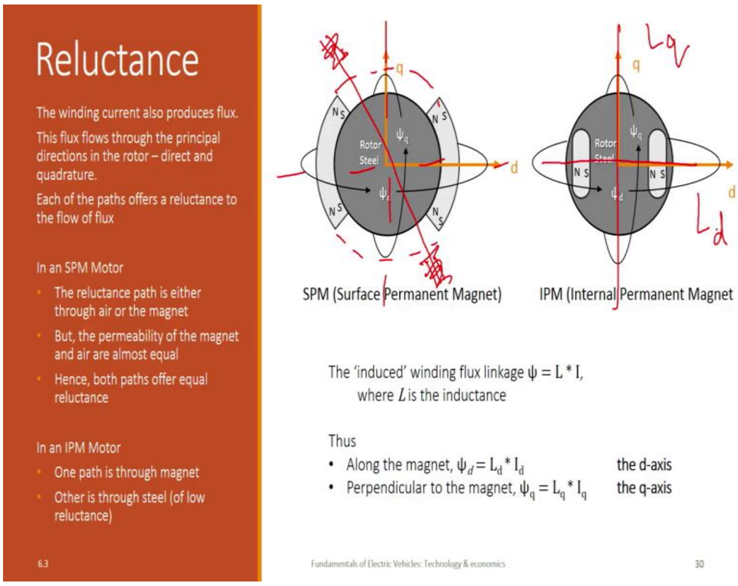

Reluctance

The winding current also produces flux.

This flux flows through the principal directions in the rotor – direct and quadrature.

Each of the paths offers a reluctance to the flow of flux

In an SPM Motor

- The reluctance path is either through air or the magnet
- But, the permeability of the magnet and air are almost equal
- Hence, both paths offer equal reluctance

In an IPM Motor

- One path is through magnet
- Other is through steel (of low reluctance)

SPM (Surface Permanent Magnet) IPM (Internal Permanent Magnet

The 'induced' winding flux linkage $\psi = L * I$,
where L is the inductance

Thus

- Along the magnet, $\psi_d = L_d * I_d$ the d-axis
- Perpendicular to the magnet, $\psi_q = L_q * I_q$ the q-axis

So we will look at this reluctance a little more closely. The winding current produces a flux, which is different from the flux due to the permanent magnets. Now, this flux will be in some direction depending on the teeth between which the flux is flowing. That direction is neither parallel to the magnet nor it is perpendicular to the magnet. It will be at some arbitrary direction.

We can resolve the flux into two components. One, which is parallel to the magnet and another which is perpendicular to the magnet. So when we do that, let us look at what happens in a SPM motor, Surface Permanent Magnet motor. For simplicity, I have taken a rotor with a single-pole pair. Later on, we will see what happens when more pole pairs come.

So in this, I could, for example, say that there is a tooth here. So there is a pole here. There is a tooth and there is a winding. And there is a similar tooth over here with some winding. And the field may be flowing like this. The flux produced by the windings passing through the teeth may be flowing in some direction like this.

So I can resolve this flux in a direction, which is aligned with the q-axis and another component of the flux which is aligned with the d-axis. So if I view it that way, one part of the flux is going like this and another is going like this. And when the flux is flowing through any material, the material offers reluctance which tries to limit the amount of flux that is flowing through it.

How much is the reluctance that the flux in the two directions will encounter? It depends on all the materials that it is passing through. So here if you see, if I were to just complete this imaginary circle, the flux is passing through the air up to a certain thickness and then through the steel across the diameter and then again through air and coming out. So the total reluctance is diameter of steel plus thickness of air path.

If I look at the q direction, it is passing through one magnet and then through the diameter of the steel and then through another magnet. So the total reluctance is the diameter of steel, reluctance offered by the diameter of the steel plus 2 times the thickness of the magnet.

The difference between the two reluctances is the diameter of the steel is common in both cases. What is different is that in one case, a certain length of path travels through the magnet. In the other case, same length is passing through air. But it so happens that the reluctance offered by a magnet, the permeability of a magnet is almost the same as the permeability of air.

So there is actually no difference in the reluctance between the two parts. Is this clear what I am saying? You are able to visualize the geometry? Because this is going to be very different when we will look at the IPM, Internal Permanent Magnet.

Here, you will see that this is one path and this is another path. In the q direction, the reluctance comes from the steel diameter entirely. Whereas, in the d direction, there is steel to a certain part of the diameter. In addition, some portion of the path is equivalent to air because it is magnet. And air offers far greater reluctance and a magnet also offers far greater reluctance than steel.

So what we will find is that the reluctance along the d-axis is much higher than the reluctance alone the q-axis. And we already know that the reciprocal of reluctance is called permeance. And permeance, multiplied by N squared, is commonly called, all of this we studied in the first chapter on flow; 1 by reluctance is the permeance and when we multiply permeance by square of the number of turns, N squared, it is called the inductance.

And where the reluctance is low, inductance will be higher because they are reciprocally related. So the inductance in this path, I can call it Lq, will be very high. And the inductance on this path is Ld, which is much lower, significantly lower, maybe it is half. So the important difference between a SPM and an IPM is that Ld and Lq are equal in the SPM but Ld and Lq are different in the IPM. So far am I clear?

So what is the consequence of this? What we have just now discussed that overall the flux path is encountering some inductance, which in some sense is a reciprocal of the reluctance. And therefore, the magnitude of the flux that is flowing will be proportional to inductance multiplied by the current.

And this can be resolved in two directions; the flux along the q-axis and the flux along the d-axis, which may or may not be equal. In fact, they would not be in general equal because the flux along the d-axis is Ld into Id, the flux along the q-axis is Lq into Iq. Since the angle that the flux is making with the d and q-axes is arbitrary, it is not symmetric. Therefore, Iq will be different from my Id. So straight away we can say that the fluxes will be different.

In addition, Ld and Lq may or may not be equal. In the case of an SPM, they will be equal. In the case of an IPM, they will not be equal. But clearly, the flux in the two directions will not be equal. In general, they will not be equal. Unless the angle is exactly 45 degrees, Id will not be equal to Iq.

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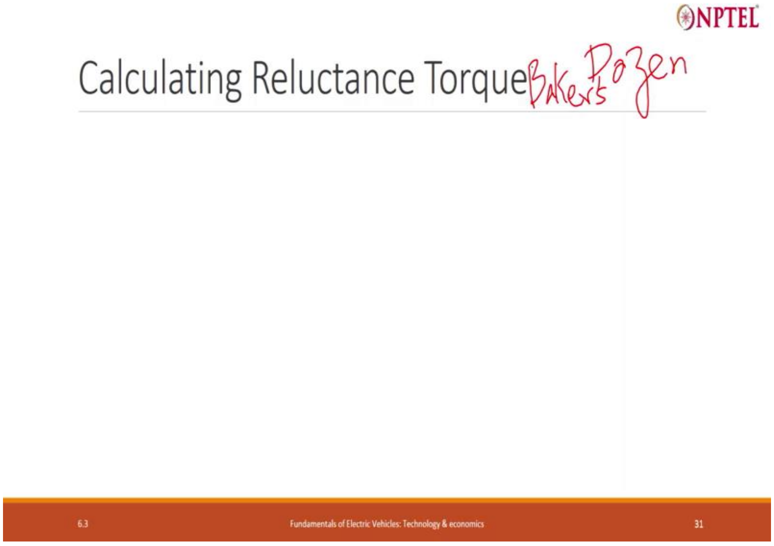

Calculating Reluctance Torque Bakers Pozen

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Now, what is the consequence of this reluctance will be very interesting to see. The reluctance in one path is different from the reluctance in the other path and the steel will try to align itself along the direction of least reluctance. In other words, it will try to align itself along the direction of maximum inductance. And this tendency will also cause a torque.

So far, we looked at the magnetic torque but this behavior where the Ld and Lq are unequal will result in an alignment along the direction where the inductance is maximum and that tendency will result in the creation of a reluctance torque.

So this reluctance torque, when there is a permanent magnet, will typically be small. It will be a smaller contributor to the overall torque compared to the dominant contribution that comes from magnetic torque. But still, it is valuable, and in electric vehicles where we are trying to maximize the performance, every little bit of torque counts.

So this is somewhat like, I do not know if you have heard this word, dozen. What is a dozen? How much is a dozen? 12. There is also a word in English that they say called baker's dozen. You know what is a baker's dozen? 13. Why is 13 a baker's dozen? If I go to a baker and ask him to give me, say, 4 pastries. I will pay for 4 pastries, I will get 4 pastries. But if I ask him to give me a dozen pastries, it is a large number, I will pay for 12 pastries but he will give me 1 extra pastry free.

So this little extra that you get free that is the reluctance torque. You have not really done any extra effort but as a bonus, you are getting some extra torque in the form of reluctance torque. So that way, the reluctant torque is a very valuable thing because that little extra will suddenly bump up the efficiency of the motor and make it a much better motor than you had earlier thought.

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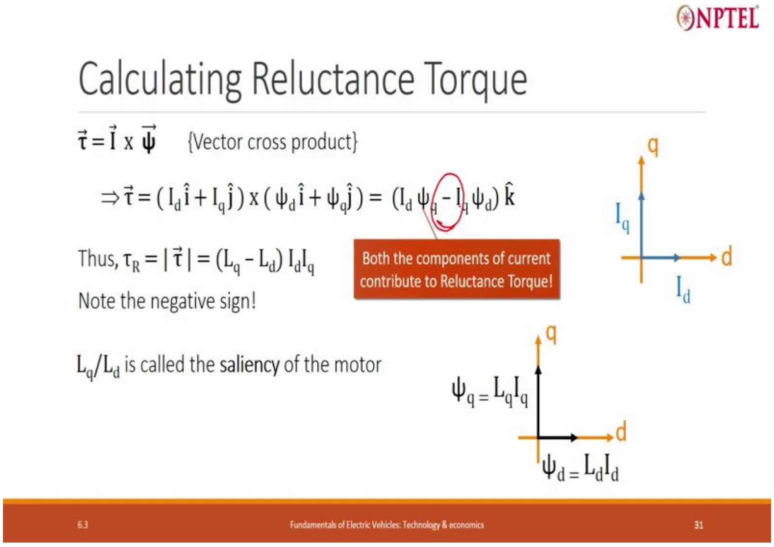

Calculating Reluctance Torque

$\vec{\tau} = \vec{I} \text{ x } \vec{\psi}$ {Vector cross product}

$\Rightarrow \vec{\tau} = (I_d\hat{i} + I_q\hat{j}) \text{ x } (\psi_d\hat{i} + \psi_q\hat{j}) = (I_d\psi_q - I_q\psi_d)\hat{k}$

Thus, $\tau_R = |\vec{\tau}| = (L_q - L_d) I_d I_q$

Note the negative sign!

Both the components of current contribute to Reluctance Torque!

L_q/L_d is called the **saliency** of the motor

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So let us look at the reluctance torque in some more detail. So we will look at how this reluctance torque plays out? We saw that the current has two components, Id an Iq. And because Id and Iq are passing through different directions where the inductance is different, because the reluctance is different, because the materials encountered along the way are different, therefore the inductance in the q direction is different from the induction in the d direction. And the flux is nothing but the product of the inductance and the current.

So I can represent psi d and psi q as a vector as I have shown in the other drawing. So far I think this no problem. Important thing is that torque is the cross product of psi and the current. So if I take this psi and do a cross product with this current, I will get a certain torque. If I take this psi, on the d-axis, and do a cross product with the q-axis current, I will get another torque.

From your knowledge of vectors, you know that the two cross products will actually have the angle in opposite directions. When I do psi cross I, psi q cross Id will be a direction opposed to psi d cross Iq. So the difference between the two torques is what will be the reluctance torque that I get.

And now that is easy to calculate; torque is I cross psi and when I apply the vector convention, using the unit vectors i and j to represent the respective quantities, and if I do a cross product, automatically I get the negative sign here. So you can see that there is a negative sign over here which comes automatically without our having to struggle with left-hand rule, right-hand rule, or anything.

And the other thing I want you to notice is that the two terms which are there in the torque, they, each of the terms has both Id and Iq in it. The first term is Id into psi q, but psi q itself has Lq and Iq in it. And the second term is Iq into psi d. The Iq term is of course the q component of the current, the psi d has hidden inside it Ld Id. Therefore, Id is also there.

So magnetic torque was contributed only by Iq but the reluctance torque is contributed by both Id and Iq. In fact, it is contributed by the product of Id and Iq. So this is how the equation will expand.

The torque is Lq minus Ld into Id Iq and I want you to notice that there is a negative sign. That is why, when Ld is equal to Lq, what will happen to the reluctance torque? If Ld is equal to Lq, what will be the value of this torque? It will be 0. Because the two opposing torques will just cancel each other out and that is what happens in the SPM motor. Whereas in the IPM, because Ld and Lq are different, you have a net gain of torque.

So this ratio between Ld and Lq is called the saliency. It is just a term which you can remember, you can also afford to forget it. Ld by Lq is called the saliency and in an SPM motor, because Ld and Lq are equal, what is the saliency? Saliency is 1. In an IPM motor, will it be greater than 1 or less than 1, Lq by Ld? Be greater than 1. So that is why in an SPM motor, you get no reluctance torque.

So we started off with the PMDC motor in the beginning of our lecture and then we said that we find the PMSM motor is a better architecture, so we have rejected the PMDC motor. Now, by looking at this 13th cake that I am getting free, the extra torque that I am getting in the form of reluctance, we can say that let us not go with a SPM motor, let us go only with IPM motor.

So the generally preferred architecture of a motor for electric vehicles would be a PMSM IPM motor. Sometimes it is contracted and we call it an IPMSM motor, Internal PMSM Motor.