

**Fundamentals of Electric Vehicles: Technology and Economics**  
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**Lecture - 50**  
**Torque Production - Part 1**

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## 6.3 Torque Production

Force on a wire in a magnetic field, Torque on a loop  
Commutation, Magnetic torque, Reluctance torque, Saliency  
Phase-advance angle, MTPA

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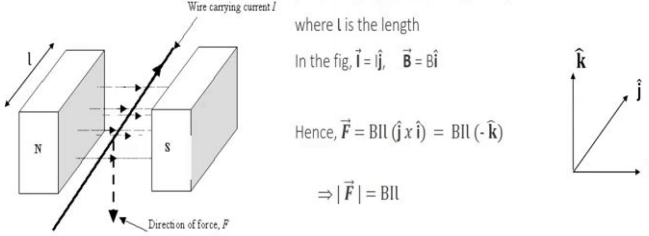
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So now, we are going to do something what we have not done so far. A little bit exciting. We are actually going to open up the motor and see what really is happening inside it. And when we talk of a motor, we are actually talking of only two things of interest, primarily. The reason we are interested in a motor is it delivers torque, it delivers speed.

So the first thing we will look at is torque and how this torque is produced, what makes the motor produce the torque, and how can we make it deliver the torque that we want. These are the questions we will explore in this chapter. I will take you through it step by step. It can get a little complicated at some points, so please be attentive to this.

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## Force on a Wire



$F = I \vec{l} \times \vec{B}$  [Vector cross-product]  
where  $l$  is the length  
In the fig,  $\vec{l} = l\hat{j}$ ,  $\vec{B} = B\hat{i}$   
Hence,  $\vec{F} = BIl(\hat{j} \times \hat{i}) = BIl(-\hat{k})$   
 $\Rightarrow |\vec{F}| = BIl$

The minus sign indicates that the force is pointing downward

Credit: [Quora](#)

The first thing we will look at is what causes a wire to experience force because it is the first simple illustration of how electrical gets converted into something mechanical. I am passing current through a wire as a result of which it experiences a force. So electrical to mechanical conversion, the simplest example is force on a wire; we start with that.

There is a wire, which is carrying a current  $I$ , and perpendicular to it is a magnetic field. The field strength is denoted by  $B$ . You already know its unit is Tesla and the total magnetic flux is  $B$  multiplied by the area. All of this we have seen in the earlier lessons. And the length of the path where it is cutting the magnetic field is given by  $L$ . And what will be the force?

You may have learned in high school about left-hand rule and right-hand rule and many things. I am sure many of you do not remember those and I want you to forget them. We are engineers, here and in future everywhere, we will use consistently what is called vector notation. And vector notation is actually very simple. If I am looking at a piece of paper, east is X-axis, north is Y-axis, and upwards, out of the paper is the Z-axis. Very simple convention and once we use that, we avoid a lot of confusion.

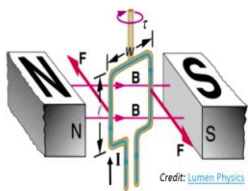
So you can see in this diagram that  $B$  is along the X-axis, and the unit vector in that direction is denoted by  $\hat{i}$ . The current is in the Y-axis, and the force is given by  $I$  cross  $B$  multiplied by  $L$ . So if I were to ignore all this left-hand, right-hand, and all these rules and just do a vector multiplication, the  $I$  vector is  $I$  times  $\hat{j}$  and, the  $B$  vector is  $B$  times  $\hat{i}$ . So,  $I$  cross  $B$  will be having a vector representation of  $\hat{j}$  cross  $\hat{i}$ , which is negative  $\hat{k}$ . Its magnitude of course will be  $B$  into  $I$  into  $l$ .

So the magnitude of force is  $B I l$ , and it is in the negative  $k$  direction, which is downwards. So this is very simple, non-confusing way and we will use vector notation everywhere. We are not going to use all this, different fingers pointing in different directions.

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### Torque on a loop

Torque  $|\vec{\tau}| = F * w * \sin(\delta)$ ;  
Where  $\delta$ , the torque angle =  $90^\circ - \theta$

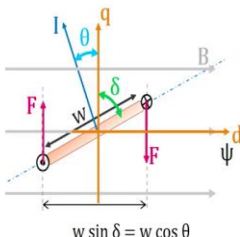


Credit: Lumen Physics

This is a PMDC motor involving split rings and sliding carbon brushes for commutation

$|\vec{\tau}| = B I l * w * \sin(\delta)$ ;  
 $|\vec{\tau}| = B I A * \sin(\delta)$ ; where  $A = l * w$   
Where  $N$  is the number of turns  
 $|\vec{\tau}| = B A N * I \sin(\delta)$   
Where  $\psi = B A N$   
[Phase advance angle,  $\theta = 90^\circ - \delta$  ]  
 $\vec{\tau} = \vec{I} \times \vec{\psi}$   
 $|\vec{\tau}| = I * \psi * \sin(\delta)$   
 $= I * \psi * \cos(\theta)$

$\delta = 90^\circ - \theta$



$w \sin \delta = w \cos \theta$

**How to increase torque?**  
Use Stronger Magnet (B)  
Increase the current (I)  
Make loops of larger area (A)  
Increase the turns (N)

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Now, we literally add a twist to the tail by twisting the wire and making it into a loop. Now, what is happening is that this wire is having current flowing in the upward direction and this wire is having current flowing in the downward direction.

Nothing has changed between the two wires except the direction. So one will experience a force in this direction, other will experience the force in the opposite direction, but the forces are equal in magnitude. So there is no net force, this loop is not going to travel away but it will rotate because the two forces are not acting on the same line. And that is the birth of torque. This is how torque gets produced in a motor.

So if I were to represent this view from the top, there is this wire on the left-hand side that you see here, where the current is flowing upwards. Upward flowing is denoted by a dot inside a circle. And here, the current is flowing downwards, which is denoted by a cross. If I apply the vector rule  $I \times B$ , then I will find that the force here is in this direction, here the force is in this direction. And so, the torque is in the clockwise direction.

Now, the magnitude of the torque is force multiplied by the distance separating them. And as you can see, that distance is  $w \sin \delta$ ,  $\delta$  is normally called the torque angle. And the complement of that angle, which is very important, we will, later on, see why it is important,

is called theta. And I can also say that the torque is equal to  $F \times w \times \cos \theta$ . Both are mathematically equivalent.

So the magnitude of the torque is this and  $B \times l \times w$ , which is the width. If I multiply  $l$  and  $w$ , I get the area of the loop. And if I do not have just one turn, but I make that loop go  $N$  numbers of times before it returns down, then every loop will face this torque. The total torque on the winding will be  $N$  times what we earlier calculated. This is when there are  $N$  number of turns.

And  $B \times A$ , as you already know, is the flux. And the flux multiplied by  $N$  is the flux linkage denoted by  $\Psi$ . So another way of expressing the torque in a motor is  $\Psi \times I$ .  $\Psi$  is  $B \times A \times N$ , and the vector way of expressing this is  $I \times \Psi$ . The force was expressed as  $I \times B$  and the torque is  $I \times \Psi$ . And this delta is called the torque angle; theta is called the phase advance angle. We will see shortly why it is important.

When I do the normal vector multiplication, it will be magnitude of  $I$  multiplied by magnitude of  $\Psi$  multiplied by the torque angle,  $\sin \delta$ . Another way of expressing it is  $I \times \Psi \times \cos \theta$ .

Now, with this very simple formula, we actually can answer a lot of questions that a designer is asked to answer. I want to increase the torque, what are my options? Very simple; that formula has  $B \times I \times A$  and  $N$ , you increase any of that, it will deliver better torque. Let us look at each of options.

If I want to increase  $B$ , I have to use a stronger magnet. The strongest magnets are what are called rare earth magnets. So generally in electric vehicles, we prefer that those magnets are more expensive but the total weight of the magnets in a motor is very small. So overall, the extra price is worth paying. Because it (has) brings with it so many other advantages.

The other option is I can make the motor deliver more torque by increasing the current. Now, there is a problem. One is, current is what I am drawing from the battery; I am draining the battery if I am going to take more current.

The other problem with this approach is the losses depend upon  $I^2 R$ . If I draw, let us say, 3 times more current because I want 3 times more torque, I am going to have 9 times more losses. My efficiency will become really poor. Plus I will have the problem of so much heat, which has to be evacuated. So that is a less preferred way.

The other option is just increase the area; that means, I have to build a bigger motor. A bigger motor will be more expensive to manufacture, it will also weigh more, it will also occupy more space, and all of these are at a premium when we are talking about electric vehicles. I cannot have a very large motor, there is not enough space there. And if it is heavy, that itself will eat into the payload of the vehicle. If it weighed less, maybe I can have one more passenger, otherwise, the capacity of vehicle is getting reduced because of the weight of the motor.

So increasing area is also not a very attractive option. Increasing the number of turns looks like a very attractive option because I just have to turn more. A little more copper turns and I will get more torque. It is actually a very good thing to do but there is a constraint. The constraint is that there is only so much space inside. I put 2 turns, 3 turns, 4 turns, at some point I am going to run out of space. I cannot put any more turns.

So what can I do in this situation if I want to increase the turns? I can use a thinner wire. So within the same space, I can pack in more number of turns. The drawback of that is that when I use a thinner wire, its resistance going to be more. So again, I have the problem that I will have more resistance, therefore, more losses. So there is a limitation. But all of these are options that I can play with to optimize the overall design.

But finally, when we, when it comes to torque, there are no other options. The magnet can be as strong as possible, the strongest magnets are the rare earth magnets. Once I have selected that, that is no longer an option, I cannot do anything more than that. And then, the rest of the design is all around how much current, how much area, and how much number of turns. So these option that we have to optimize. So really, this is all there is to designing a motor for torque. Very simple.

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## Commutation

The maximum torque occurs when  $\delta = 90^\circ$  ( $\sin \delta = 1$ )

The minimum torque occurs when  $\delta = 0$  ( $\sin \delta = 0$ )

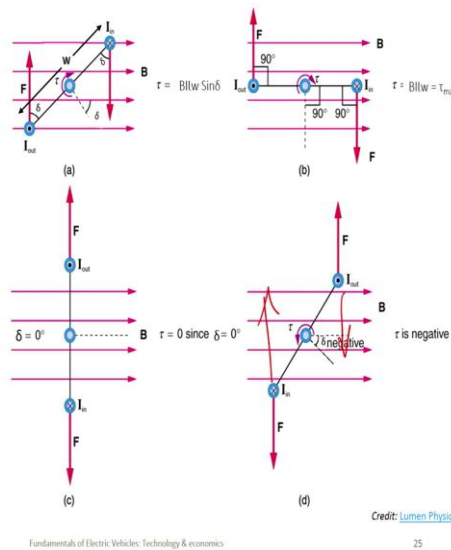
A negative torque is produced when  $\delta < 0$

Negative torque can be avoided by reversing the direction of current when  $\delta = 0$ . This is called commutation.

In a PMDC motor, this is achieved via split rings and sliding carbon brushes.

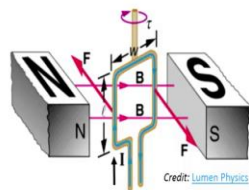
This also ensures the wires coming out don't twist around each other and snap

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## Torque on a loop

Torque  $|\vec{\tau}| = F * w * \sin(\delta)$ ;  
Where  $\delta$ , the torque angle =  $90^\circ - \theta$



This is a PMDC motor involving split rings and sliding carbon brushes for commutation

$$|\vec{\tau}| = BIl * w * \sin(\delta);$$

$$|\vec{\tau}| = BIA * \sin(\delta); \text{ where } A = l * w$$

$$|\vec{\tau}| = BAN * I \sin(\delta)$$

Where N is the number of turns

$$|\vec{\tau}| = \psi * I * \sin(\delta)$$

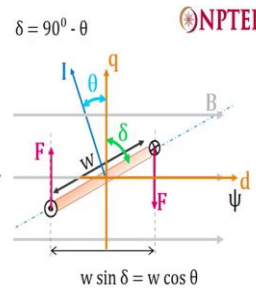
$$= \psi * I * \cos(\theta)$$

Where  $\psi = BAN$   
[Phase advance angle,  $\theta = 90^\circ - \delta$ ]

$$\vec{\tau} = \vec{I} \times \vec{\psi}$$

$$|\vec{\tau}| = I * \psi * \sin(\delta)$$

$$= I * \psi * \cos(\theta)$$



**How to increase torque?**

- Use Stronger Magnet (B)
- Increase the current (I)
- Make loops of larger area (A)
- Increase the turns (N)

Now, we will look at something called commutation, which looked at the act of torque production, we will try to follow it through an entire loop and see what happens. There is a picture similar to what we saw in the previous slide. As the torque direction is like this because the coil is going to be rotating in that direction. As it starts rotating, it will come to this position where the forces are as far apart as they can be. So this is the position, where I will get maximum torque.

The maximum torque is produced when the torque angle delta is 90 degrees because Psi into I into sin delta, sin delta is maximum when delta is 90 degrees. And that is when the current vector, which I showed you in the previous diagram, is actually perpendicular to the coil. That means theta is 0. I can quickly take you back to that. When delta is

at 90 degrees, theta is 0, current is perpendicular to the direction of the magnetic field. That is when I get maximum torque.

Now, the magnitude of that torque is  $B A w$ ,  $B l w$ ,  $B I l w$ , which is a maximum torque for one turn of the loop. The term  $N$  has been ignored in this drawing. What will happen further? Another quarter turn, 90 more degrees, and delta becomes 0. So sin of delta is also 0.

The forces are aligned in the same line in opposite direction and I have 0 torque. But in reaching this position, already the coil has acquired a certain speed, so it will overshoot. Although there is no further torque available, it is going to overshoot. And when it overshoots, it is even worse.

Now, the forces are trying to bring it back. They are not allowing it to move forward. You are having a negative torque. So this is terrible because all that I have built is not a motor but something which will just go like this, to and fro, and finally, it will stop. And when it stops, it will be always in this position. So this is not a good motor to build. How can we overcome this?

So normally, what is done is when I reach this position, I will quickly flip the direction of the current. This dot will become a cross and the cross will become dot. If I do that, what I am doing is I am reversing the direction of the force, the forces instead of being in that direction will be here and this force will be like this. So the positive torque will continue to get maintained because of reversal of the current. This reversal of current is called commutation.

And in the kind of motor that you, that I showed you in the previous slide, again let me refer back to the picture. This kind of motor is called a PMDC motor. The magnets are static and the loop is rotating. So the stator has the magnets and the rotor has the windings.

In this kind of a magnet what is called a PMDC motor, commutation is achieved by using what are called split rings, so there are two halves and a sliding contactor made of graphite, it is called a carbon brush, will jump from one to the other. There is a risk of sparking because contact is being made and unmade in a short interval of time. And all this movement will also cause friction, wearing, all these losses.

But this is the way it is done and if I even found a cleverer way of avoiding all this mechanical contactor and jumping, it will not work here because then the wires will simply come out and start twisting against each other and snap.

So just to isolate the wires from the rotating, the wire leads going to the plug from the rotating armature, I need to have this arrangement to separate them. So this is the overview of what is called commutation and why we need to flip the current.

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NPTEL

### Torque: a closer look

The torque due to the magnetic field is

$$F \cdot w \cdot \sin(\delta) = BAN \cdot I \cos(\theta)$$

$\psi$

Which component of the current produces torque?

- The parallel component is  $I_d = I \cos(\delta) = I \sin(\theta)$  **Direct Component**
- The perpendicular one is  $I_q = I \sin(\delta) = I \cos(\theta)$  **Quadrature Component**

$\delta = 90^\circ - \theta$

$w \sin \delta = w \cos \theta$

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Before we go into other architectures of the motor, this is the PMDC architecture. We are not going to be building PMDC motors but I am starting with PMDC because it is easier to understand and visualize. So before going into that, let us take closer look at the torque that we discussed so far.

The torque due to the magnetic field, we already know, depends upon the B, the A, the N, and the I. And from now onwards, I will start using theta rather than delta, and so, into cos theta. The question I am asking you is I can resolve the current in this diagram into two components,  $I \cos \theta$ , which is perpendicular to Psi and  $I \sin \theta$ , which is parallel to Psi. What is parallel to Psi, I am calling that as the direct component because it is directly aligned with the magnetic field, and what is perpendicular to Psi is called normally the quadrature component.

And I want you to tell me which of the two components is resulting in magnetic torque. Which is the one? You can look at the equation at the top and tell me, which is the component that is generating torque. Is it the direct component or the quadrature component? The direct component is  $I \sin \theta$ , the quadrature component is  $I \cos \theta$ .

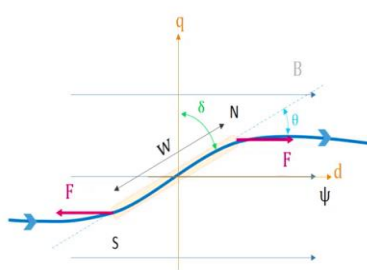
We have seen again and again that the perpendicular component, even in the previous slide is where we get the torque. So the simplest way of representing the torque due to the magnet in




the motor is  $\Psi$  into  $Iq$ . This is a very important relationship, please remember it, we will keep using it, again and again, many times in all the subsequent slides. And this is the magnetic torque. By magnetic torque, I mean the torque created by the magnet.

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## Torque: a closer look





Imagine it was just an iron loop – with no current flowing

? Will it rotate? Why?

The magnetic flux flows more through the iron – due to its lower reluctance. This 'induces' the iron to act as a magnet

The induced magnet aligns with the external field

This tendency to align with the magnetic field causes

Reluctance Torque =  $F * w * \cos(\delta)$

=  $F * w * \sin(\theta)$

In the aligned position, reluctance is minimum;  
Permeance or Inductance is maximum

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Now, the winding is usually not just in air, it is wound around a core. And often, that core is made of steel, some kind of a soft iron or something like that. Now, suppose I ignore the copper wire and only look at the steel inside it, that steel is also inside a magnetic field. Just as the copper wire is inside the magnetic field, the steel is also in the magnetic field. Does the steel experience any torque or is it only the copper wire that experiences a torque? That means, if I remove, if I strip the motor of all the winding wires, will it still rotate or not is the question?

If you look carefully, the magnetic flux is flowing from left to right, which means somewhere on the left, there is a north pole and somewhere on the right-hand side, there is a south pole. But along the way, there is the steel. The steel offers a path for the magnetic flux to flow, which has lower reluctance.

And, for example, if there is a stream of water and in between, I offer it a path which has very lower resistance, then more water will flow through that path than through the other path that is available. Same way, the magnetic lines will preferentially flow through the steel than through the air because air offers high reluctance.

So more magnetic lines will flow through the steel as a result of which, the steel has got magnetized. The steel is like a magnet, the point where all the magnetic lines are emerging is

like the north pole, it is marked here as N. And the point where all the magnetic lines are entering it is like a south pole. And this south pole will get attracted to the north pole here on the left by a force  $F$ . And this north pole will get attracted towards the south pole on the right, with an opposite force  $F$ . So there is a torque here.

And this torque is proportional to  $\sin \theta$ . The magnetic torque was proportional to  $\cos \theta$  and this torque, which I will call as the reluctance torque because it is produced by the reluctance of the steel, is proportional to  $\sin \theta$ . What the magnitude of the force is? We will see subsequently. But I want you to see that just the steel, because of the reluctance, can cause a torque.

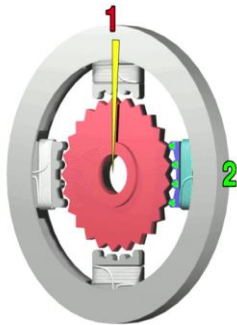
This is a very important point, which is somewhat subtle and not understood. And I want you to pay attention to that because this is what gives the electric vehicles that extra oomph when you are trying to deliver maximum possible torque.

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Assignment 6.3.1

NPTEL

## Magnet-free Stepper Motor



This motor has no magnets. There are 25 teeth on the rotor, 4 teeth on each pole of the stator

- ? How is torque produced?
- ? How can you increase the torque?
- ? How would you control the speed?

The motion of this motor is jumpy

- ? How many 'steps' make a full revolution?
- ? Can you halve the size of the steps to make the motion smoother? (Without changing the mechanical construction of the motor)

Credit: [en.wikipedia.org](https://en.wikipedia.org)

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And in order to get a better understanding of reluctance, I will introduce you to a toy motor that many of you may have used in some high school projects, et cetera. It is called a stepper motor, it has no magnets in it. And whenever I fire a coil, that coil has 4 teeth on the stator, and there is some tooth in the rotating rotor which is nearest to something else; whichever are the two most proximate teeth they will come in alignment.

This tendency to come in alignment is due to reluctance; to minimize the reluctance. And then if I fire some other coil, some other tooth pair will come in alignment and I can go on like that. So this is the way this thing moves. It is not a smooth movement, now this aligned,

then this is aligned, then something else is aligned, something else is aligned, and the cycle continues. That is why it is called a stepper motor.

So assume that this has 25 teeth on the rotor and 4 teeth on each of the poles of the stator. How is torque produced? Through reluctance, we have discussed it. The question that I want you to ponder over is how can you increase the torque? How will you increase the speed or control the speed?

My question is how will you do it without changing the mechanical construction of this. Sure, you can increase the number of poles, why will increasing the number of poles? Increase the torque. At any time you are firing one, right? And whatever is the nearest misaligned teeth, will come in alignment. By increasing the number of poles it is not obvious to me that I will increase the torque.

If you increase the misalignment angle, if I increase the misalignment angle, why will I get more torque? If I have more number of teeth then, per revolution the number of steps will become larger. If I have more number of steps then the jump in each step will be smaller, it will be smoother.

But without changing the mechanical construction, if I want to make it smoother what can I do? For example, if I can, halve the size of this step in some way, means, I have more number of steps per revolution then it will become smoother. It is easy to do that if I change the mechanical construction. But without doing that can I make it smoother?

It turns out there is a way to make it smoother by halving the size. Think about it and then you can extend that concept if you are imaginative enough to make it perfectly smooth if you want to. Absolutely no jumps, all that is possible. Whether one wants to do it is a different matter because it will require some investment in the electronics and controller but there is a way to do it. So think about this.

And all of this is in a motor which has no magnets at all. It is entirely reluctance torque. So by reflecting on this, you will get a strong intuition about what is reluctance torque.