

**Fundamentals of Electric Vehicles
Technology and Economics
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Lecture 11
Putting it all Together**

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Driving an ICE or Electric Vehicle

How much **Power** is required to drive a vehicle?

How much **Energy** is required to carry out a road-trip?

- What is the composite mass of the vehicle (including passenger and goods): **Gross Vehicle Weight (GVW)**
- What is the condition of the roads (**rolling resistance**)
- What is the aerodynamics of the vehicle (**Aerodynamic drag**)
- What is the incline that it needs to traverse? (**Gradient Resistance**)
- What are the velocities and accelerations at different points of time (**Drive Cycle**)
- What is the **maximum speed** and **maximum acceleration** of the vehicle?

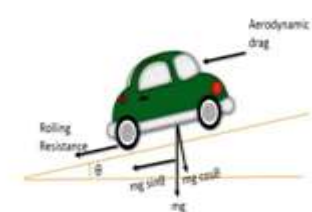
Last class we started with the vehicle dynamics and I had said that objective is to understand how much power a vehicle requires, how much energy does it require into a particular drive.

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NPTEL

What does tractive force overcome?

- Aerodynamic Drag
- Rolling Resistance
- Uphill Resistance
- Acceleration



Aerodynamic Drag = $\frac{1}{2} \rho C_D A v^2$

- v = velocity (m/sec)
- Air density @27°C = $\rho = 1.2$ (kg/m³)
- Vehicle Frontal Area or Projected Area = A (sq. m)
- Drag coefficient = C_D

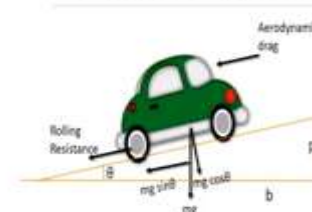
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And looking at that it was actually there are some very simple equations, it appear to be tough because lots of things are involved. But a very simple thing that I had talked about was that the forces that it actually encounters is aerodynamic drag which is a function of velocity square. I kept on emphasizing that and we will see that quite a bit as we go on.

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NPTEL

Forces acting on a vehicle in motion



Rolling Resistance = $m \cdot g \cdot \mu \cdot \cos \theta$

- Permissible load = m (kg)
- Weight = mg (newton or kg.m/s²), where g = 9.80665 m/s²
- μ = rolling coefficient

Uphill Resistance or Climbing Force = $mg \sin \theta$

- Maximum grade = $\theta^\circ = \theta \cdot \pi / 180$ radians

Grade/inclination:
Grade in % = $\frac{\text{Height of the grade}}{\text{Base of the grade}} \cdot 100 \% = \frac{p}{b} \cdot 100$
%

Grade in Degree = $\tan^{-1} \frac{p}{b}$


Tractive force created by power-train first overcomes these resistances and then provides acceleration

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And then it also encounters a rolling resistance which is mg mu depended on mass, gravity and mu, a constant called rolling coefficient. And we also said the third is the climbing force, the climbing force mg sin theta, mg sin theta this plays a important role. So these three forces are

always there in the vehicle whenever vehicle is moving. Ofcourse if there is no climb theta is zero, sin zero is zero, so the climbing force is zero but the two are always there.

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Thus Traction Force is given by

- $F_{\text{tra}} = \text{Acceleration Force} + \text{Aerodynamic Drag} + \text{Rolling Resistance} + \text{Climbing Force}$
- $F_{\text{tra}} = m \cdot a + \frac{1}{2} \cdot \rho \cdot C_d \cdot A \cdot v^2 + m \cdot g \cdot \mu + m \cdot g \cdot \sin \theta$, where a is the acceleration and is dv/dt

The energy consumed by vehicle in motion is the integration of Traction Power

- $\text{Energy} = \int P_{\text{tra}} dt$ in Watt-sec and is converted to kWh by dividing by 3.6
- Vehicle may have regeneration, which converts deceleration of vehicle while climbing down or otherwise applying brakes (using Regenerative Braking) into Regenerative Energy
- Thus net energy consumed is $R \cdot \text{Energy}$, where R is regeneration efficiency
- As Regeneration factor is typically 15% to 30%, R is $(1 - \text{RegenFactor})$ or typically 0.85 to 0.70

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Based on that we had actually computed what is the, we have added all this and sort of say the force traction is mass into acceleration plus all these three forces. And we had also talked about in most of the time the slope is very small so cos theta is 1 and sin theta can also be written as approximately theta. And we had computed all the equations for energy, power and torque. We had talked about that energy, power, force, energy, power and torque will play a important role.

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Aerodynamic Drag (contd.)

For e-rickshaw at 25 kmph [$C_D = 0.44$ and $A = 1.6$]

- $F_D = 0.5 * 1.2 * 0.44 * 1.6 * (25/3.6)^2 = 20.37N$
- Power required to overcome drag = $F_D * v$ (Watts) = $20.37 * (25/3.6) = 141.4 W$

For a car (Limo) at 50 kmph, [$C_D = 0.35$ and $A = 2.5$]

- $F_D = 0.5 * 1.2 * 0.35 * 2.5 * (50/3.6)^2 = 101.27N$
- At 70 kmph it is 198.5N, whereas at 90 kmph Force is 328N
- Power required to overcome drag at 50 kmph = $101.3 * (50/3.6)$ or 1.4 kW
- Power for drag at 70 kmph = $198.5 * (70/3.6)$ or 3.9 kW and at 90 kmph = 8.2 kW

Aerodynamic drag increases as **square of velocity** and Power increases as **cube of velocity**

Typical values of Rolling Resistance

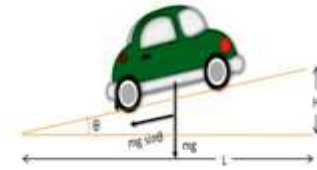
- | | |
|-------------------------------------|-------------|
| • Car tyre on smooth tarmac road: | 0.01 |
| • Car tyre on concrete road : | 0.011 |
| • Car tyre on a rolled gravel road: | 0.02 |
| • Tar macadam road | 0.025 |
| • Unpaved road | 0.05 |
| • Bad earth tracks | 0.16 |
| • Loose sand | 0.15-0.3 |
| • Truck tyre (concrete/ asphalt) | 0.006-0.01 |
| • Wheel on iron rail | 0.001-0.002 |

Force due to rolling resistance is a function of velocity **only at high speed**

Gradient resistance

$$F_g = mg \sin\theta$$

- For small θ , one can approximate $\sin\theta$ as (H/L)



Let gradient be 5° or 0.0873 radians

- 2-wheeler (wt=180kg): $F_g = 153.9$ N
- 3-wheeler (wt=680kg): $F_g = 581.4$ N
- 4-wheeler (wt=1200kg): $F_g = 1026$ N

If gradient is **12 degrees**, Force increases to

- $F_g = 367.1$ N for 2W, $F_g = 1387$ N for e-rick and $F_g = 2447.5$ for 4W
- Assuming wheel radius of 0.28m for 2W, 0.2m for e-rick and 0.31m for 4W,
- Torque required is 102 Nm for 2W, 277Nm for e-rick and 759 Nm for 4W

- As seen later designing motor will be tough
- Gear will help, but still...

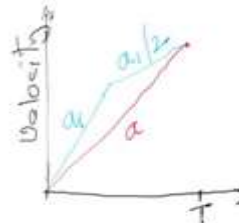
And then we give examples for 2-wheelers, for 3-wheelers we went into what is rolling resistance, what is aerodynamic resistance, what is the gradient force and gradient torque and gradient power. We looked at each of these things and we had done this.

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Power required for acceleration (pick-up)

What is the Force required to reach maximum speed v_f in T seconds?

- Depends on how acceleration takes place
- Assuming constant **linear acceleration**
 - Acceleration $a = v_f$ (meter/sec)/ T (sec) = v_f/T
 - F_a (Newtons) = m (kg)* a (meter/sec²)
 - Acceleration distance $s = 0.5*a*T^2$
 - Work during acceleration $W = F_a * s = 0.5*m*a*v_f^2$
 - Avg. Acceleration Power = $W/T = 0.5*m*a*v_f^2/T$
 - Peak power required = $m*a*v_f^2/T$



We finally also went into what is the power that you require to accelerate and we talked about there are two ways that you can accelerate; one is the constant acceleration you reach a certain velocity in some time t that is called pickup. Another is you accelerate in the beginning more and accelerate less later on. The main advantage when you accelerate more in the beginning that is a time velocity is less so the power consumed is less.

Power consumed is this velocity multiplied by the force acceleration force so though acceleration force is slightly higher the velocity is less. So the power consumed is less here. Ofcourse this will be higher than the power consumed due to this. But later on as the acceleration increases acceleration increases with the velocity becomes large your acceleration should sorry acceleration decreases because acceleration decreases your power consumption will become less.

And we had computed and we had given an assignment that power (consume), peak power consume if I follow this approach is only two third of the peak power that is used if you follow this approach. We will get into more of this later on but for the time being I think we will just kind of leave it and it is the next section that I said I am going to talk about putting it all together.

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2.4 Putting it up all together

Comparing Force and Power: 2-wheeler



When you put it all together how does it look like? And I think this is the first curve that I am going to talk about here also you see that on this screen the 4 curves that you see, the as a function of the velocity in kilometer per hour. I have also taught you how to convert the kilometer per hour into meters per second into rpm. So we have looked at all of those things but this are the four forces which are being plotted as a function of velocity.

Look at the rolling resistance. Rolling resistance is actually pretty much not a function of velocity, a very-very mild function of velocity and the force due to rolling resistance on a reasonable tarmac road is actually small. If you saw the force required for, this is for a 2-wheeler it is about starts with about 25 newton and actually as velocity goes up it can go up to 30 newton, 35 newton's not much more than that, that is not a major force.

The drag; drag starts at a very low value and it is actually a function of v square so this is basically a v square curve, this is a v square curve. So as the velocity increases, it increases fairly large so up to if I look at up to 50-60 kilometer it is still low. At 60 kilometer this may be your 60 to 70 newton. But as it goes to higher velocity like 80 kilometers or 90 kilometers, the force required due to drag increases significantly.

And we will see the implication of that when we go forward, we will see that the drag force will dominate at higher velocity. At low velocity not too much ofcourse rolling resistance will be higher and then it will become equal to rolling resistance, even if it is become little higher than

rolling resistance still it is reasonable. Till around 50 to 60 kilometer it is not very large, above 60 kilometer it becomes large.

And you will see actually that this drag is not a function of the mass of the vehicle. If you remember if it was $\frac{1}{2} \rho c_d$ into area multiplied by velocity square, now c_d and area will change but from vehicle to vehicle this is it is not going to make a major difference. So this curve that you are getting for drag is more or less going to be like this for a 2-wheeler, for a 3-wheeler, for a 4-wheeler, for a truck it is going to be like this.

So the minute you go anywhere above 60 kilometer per hour this can start dominating at 40-50 it is higher than rolling this but not that bad. If you go to 80-90 it becomes very bad and I have not even tried to draw it for 120 kilometer, 150 kilometer per hour. This is the kind of speed for example rs available on highways in Europe, the drag is very large. And you have to worry about the drag quite a bit.

But at low velocity is not very significant. There are two other forces which are very significant one is the gravity, sorry not gravity it is a force due to the slope. Now this is again constant it does not matter what the velocity is, the force is constant. And even for small slope it can be substantially high. I have actually taken gradient at 5 degrees, even at 5 degrees this can be substantially large.

And in fact it dominates if I look at the gradient force I can forget about the rolling resistance and the force due to either the F_d . I can actually forget about the both this drag and the rolling resistance till very high velocity this is very-very dominant. Ofcourse this happens only in the slope, if there is no slope this is not there. But even a 5 degree slope can become a problem and if you have higher slope like 8 degrees or 10 degrees it can become very-very large.

When there is a slope it is the force due to gradient which will dominate. We can by enlarge the others are play a minor role. The other major force is force due to acceleration. Now this is the acceleration well depends on how much you want to accelerate. Here I have assumed that in 20 second you want to get to the maximum speed and in this case the maximum speed I think we took as 2-wheeler 20 second pickup time to up to 50 kilometer.

And if you see the acceleration keeps on increasing depending on N velocity. If it is a 50 kilometer, the force is like this. If you want to get to 90 kilometer in 20 second the acceleration can become very, force can become very-very large. But you know what is the force that is always acting on a vehicle?

Even if it is traveling at a constant speed then there is no acceleration force, if it is not traveling on the gradient there is no force due to gradient. The force, the drag and the rolling resistance will always be there. On top of it, beyond drag and rolling resistance you need just to move drag and rolling resistance force.

Beyond that if there is a force it can help you accelerate. Beyond that if there is still force available you can ofcourse go up the gradient. So these two are always there when you are traveling on gradient and you do not travel on gradient all the time, you have to worry about only the gradient force.

And when you are accelerating and you are doing fast acceleration you have to worry about accelerating force. So these four forces now while this I have drawn for two wheeler the tendency will be same, in fact the drag force more or less will be same for 3-wheeler, for a 4-wheeler pretty much the same and rolling resistance will increase because the mass will increase then it will increase.

Gradient will further go up, acceleration can also go up, it depends on acceleration depends on pickup time that you want and what speed that do you want to reach, all these four. What about power? Power is force multiplied by velocity. So there is one more as velocity increases one more velocity term will come on every one of them so this is the power. If you see the rolling resistance was more or less flat, it goes up now it goes up as a velocity v it continuously increases as velocity v increases.

The gradient power, you now have to multiply this with velocity and the gradient power now also increases. Now what speed at which you are going up the gradient? Typically you do not go up to the gradient at high velocity. You go up at a low velocity remember in this computation we assumed that it will go up if a normal speed is 60 kilometer, it will go up at 20 kilometer one third of that.

So this curve has been computed assuming not this is not the value at 90 kilometer. We are assuming that you are actually traveling at 30 kilometers. At 60 kilometer we are assuming you are traveling at 20 kilometer that is a reason it does not go very as steeply, otherwise it would have gone up more steeply. But you are not expected to travel high speed on gradient term. You are supposed to slow down on gradient and that is a reason while it is a problematic thing it is not as problematic.

Look at the other two, look at the acceleration, ofcourse acceleration power now gets multiplied by velocity and it shoots up like anything. Look at the numbers, if I look at the numbers rolling resistance and gradient is all within thousand watts for a 2-wheeler but for acceleration it can go up to 3000 watts much higher. And look at the main culprit at high velocity the drag. Drag was any way up had a velocity square component.

Now in power it has a velocity cube component, so it is a cube curve and if you look at anything above like 75 kilometer or 80 kilometer it can become very-very large. At 80 kilometer it is 3000 and at 85 kilometer is 3500 watts and 90 kilometer it goes to above 4000 watts. Now this curve though it is for 2-wheeler will do the similar curve for other things is going to play a major role in designing of your motors and batteries.

I am going to keep this curve on one screen as it is, the reason is that I will need to refer to this and I am going to go ahead and start looking at it.

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2-wheeler

Power

- Gradient and Acceleration not required together
- Gradient never done at high speed: Climbing 5° slope at 15 kmph will require about **700 W**
- Acceleration (pick-up) power is small at 25 kmph, and only **1000 W** even at 50 kmph
- Rolling resistance on decent roads is small and higher than others only at very low speed
- Drag power is only **700W** even at 50 kmph, but can become **very high at higher speed**

Force related to Torque: Only gradient or acceleration torque matters at all speeds
- $T_{req} = 44.8 \text{ Nm}$ ($R_{wheel} = 0.28\text{m}$) at 60 kmph

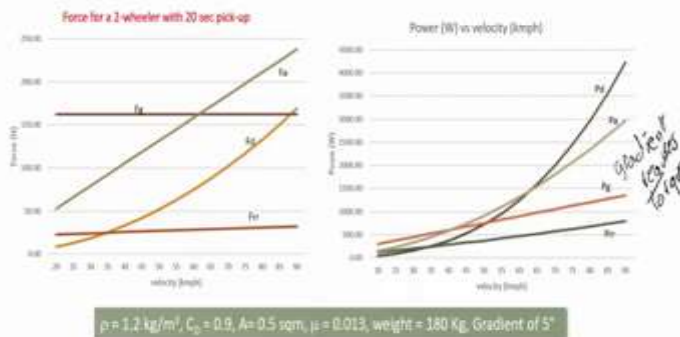
Speeds below 25 kmph

- **500 Watts** motor will be enough
- With **20 sec pick-up** to 50 kmph
 - Acceleration Power reqd. itself is **1 kW**
 - Drag is also considerable: Power reqd. **2 kW**

Power required is **6 kW** at 80kmph and **9kW** at 90 kmph

- For slower pick-up, a **5 kW** drive will just about be ok for up to 90 kmph

Comparing Force and Power: 2-wheeler



If I look at that this is the same 2-wheeler so I am actually using these things also, if I look at the power it is the gradient and acceleration that really matters, gradient and acceleration. Well for force also not just power, for force also it is the it should be actually not power it is a force.

If I look at the gradient and acceleration matters, fortunately gradient acceleration does not go on together, so it is either force or force due to the gradient or force due to acceleration. So if you do have for example design something which can handle approximately let us say slightly under 200 newtons, 170 newtons or so you can get good gradient travel or you can get a decent acceleration.

Now you will say what about the rolling resistance and drag? Well they will matter so they will also have to be taken into account to that extent your, you have to, you cannot run at that speed you have to go at probably lower speed. But if you see the force required, now the force is important because the torque is force multiplied by the radius. So if you look at it, it is a gradient torque which is not a function of velocity and its acceleration torque which is going to be the problematic thing.

Now as I pointed out gradient is never done at high speed, so for power curve you really do not have to worry about it at high speed but if you look at it even then at, if I travel at even 15 kilometer per hour even as low as 15 kilometer per hour, what is the total force required? I require about 500 watts total power required 500 watts for the gradient plus I will require some both rolling resistance and drag at 15 kilometer per hour.

At 15 kilometer per hour they are low but totally I will require about 6-700, if I have a 700 watts of power it is enough to travel at 15 kilometer per hour. But if I want to travel at higher slope you will have a problem. Acceleration; now look at acceleration, up to 25 kilometer per hour it is not that bad, up to 25 kilometer per hour it is not that bad. At even a 50 kilometer per hour it is only about 1000 watts.

Ofcourse to this I have to add the rolling resistance and the drag but up to around 50 kilometer per hour, even they are not too bad. So they are all around, so the power is approximately about 500 watt. So if I have little more than 25 kilometer power I can do it very easily, 50 kilometer per hour I will require slightly higher power but if I have about 1000 watts I should be able to handle acceleration plus 1000 watts I will be able to handle acceleration.

But I also need to handle the rolling resistance and gradient probably I will require 1500 watts. Rolling resistance on decent road is small if you look at it whether you talk about power; power requirement is not that large. Well at 50 kilometer it is around 400 watts, it goes up to 5-600 watts. Drag is only at 700 watts, 700 watt at 50 kilometer per hour. But at higher kilometer per hour drag can become the dominant.

So for 2-wheeler I have to worry about the gradient power always, but gradient power will not go along with acceleration so I only have to worry about gradient power. And for acceleration I have to worry about acceleration power and I have to add the rolling resistance and drag also.

Now force is related to torque. Torque I have to suddenly come to this curve, this curve when I talk about torque.

So if I look at the torques, if I look at the torque the gradient torque since the gradient force is very large torque will be very large and it is independent of speed it is flat. So torque required if I assume a 0.28 meter wheel radius my 2-wheeler typically has 0.28, it is about 45 newton meter that is a large torque. This is what is required for gradient and it is whether you travel at ofcourse it is always done at low speed.

So though I put 60 kilometer per hour you will actually do it at 20 kilometer per hour. Speeds below 25 kilometer I have said 500 watts, if I look at speed below 25 kilometer there are and let us assume there is no gradient, there is a acceleration force is approximately 300, the drag and rolling resistance is even smaller. So if I have a 500 watt motor 25 kilometer I can drive quite well for the speed including some pickup.

So actually the 2-wheeler which are limited to 25 kilometer per hour 5, 700, 800 watt motor is very often put below a kilowatt 5 to 700 meters. But if I go to 50 kilometer per hour, situation changes at 50 kilometer per hour if I see the acceleration alone is 1 kilowatt, drag and other things are also considerable. So you will require about 2 kilowatt. If I look at 80 kilometer or 90 kilometer look at 80 or 90 kilometer the dominant power required is the drag, 80 or 90 drag becomes very large.

So that itself will be about at eighty kilometer it is, as I pointed out is three kilowatt at 90 kilometer is a four kilowatt. And then you have to add acceleration whatever acceleration that you want and rolling resistance. Loading resistance is still about just about 700-800 watts, so if you have a 6 kilowatt motor it will give you drag enough to go to 80 kilometer per hour and you will but you will require about 9 kilowatt motor at 90 kilometer per hour, this is what you can see 90 kilometer per hour.

If I go to 90 kilometer hour my drag is approximately 4.5 kilowatt, I will require acceleration approximately 3 kilowatt and I will still require rolling resistance approximately kilowatt so I suddenly require about 8-9 kilowatt at 90 kilowatt per hour and if I want little bit acceleration to go up to that, so 8-9 kilowatts I will require. So this is what a 2-wheeler and this is ofcourse slightly lower slightly low weight so 200 kilogram including passengers.

There are 250 and 300 kilowatt kilogram vehicles, appropriate force required will go up. But what I am pointing out the high the 80-90 kilometer 2-wheelers you have to worry about otherwise 500-800 watt. So suddenly a 500-800 watt engine motor will get you 25 kilometer but for a high end 90 kilometer you certainly require nine kilowatt. So you require two very distinct requirement.

Now look from India point of view. A lot of low end 2-wheelers are used; 25 kilometer, 30 kilometer I can probably use a one kilowatt motor appropriately what will happen your battery will also come down. And I can make a low cost vehicle if I want a 50 to 60000 rupees target vehicle tomorrow 65000 rupees I have to speed limit 30 kilometer 35 kilometer, I cannot go more than that.

If on the other hand I want a 90 kilometer per hour, I will require a motor which is 5-6 kilowatt with peak going to 8-9 kilowatt and that is a distinction that you see today in the market in India. You will see that there are vehicles by hero, amper which are 30 kilometer, 35 kilometer they are 50, 60000, 65000, they will have limited range battery but that is of entry level vehicles, 30 kilometer 35 kilometer.

If you go slightly higher price like 80000 you may be able to go to 45 kilometers per hour. But on the other hand you look at vehicle like ether that promises 90 kilometer per hour that will require a 6 kilowatt, 5 kilowatt 6 kilowatt motor with peak going to 9 kilowatt, 5 kilowatt going peak going to 9 kilowatt that is what is required. And comes out straight from this simple work the battery requirement will also go up because you will consume a lot of energy.

Remember power is high now when power is high at this point energy consumed also will be high. So you will require larger battery, cost will go up. So the vehicle starts at 120000 rupees in India. So do you understand why there are two distinct kind of vehicles and 2-wheelers, your battery changes, your motor, controller everything changes. We will come to the battery later more where we calculate the energy and the energy required during a drive, we will come to that.

But from force what I wanted to point out here is force and torque requirement. You will see I am not emphasizing it enough here but as we go on we will be emphasizing. Torque will play a very important role. Does it give me the torque required?

Ofcourse normally if I can make something which will give me a torque requirement for gradient it will give me for the acceleration also. But if I still want a very high pickup like in ether bike then gradient is not enough 5 degree, 6 degree gradient otherwise will give me a decent vehicle.