Image Signal Processing
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Lecture 56
Applications of SVD

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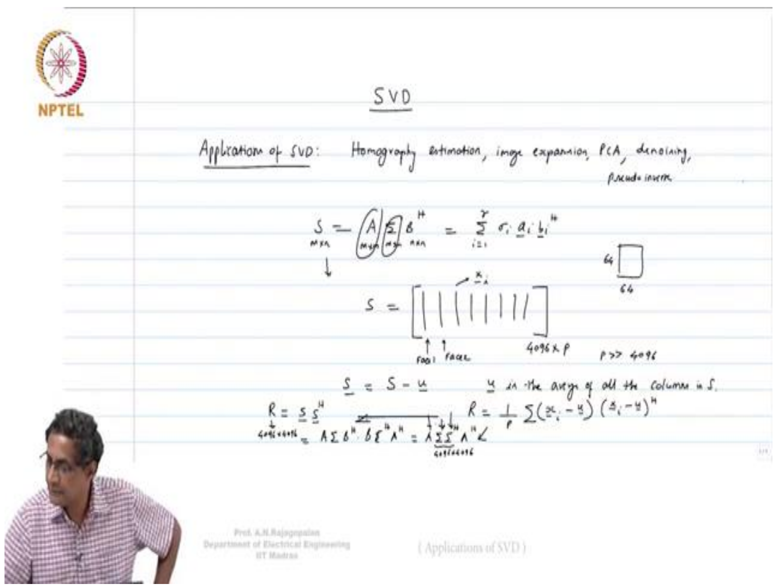

So, the last class we were actually looking at singular value decomposition, SVD and we were almost at the end of SVD. I just wanted to point out that there are certain applications of SVD some of which you have already seen and some of which I would like to point out now. So, the applications of SVD as I said as I said earlier too, it has a lot of applications left and right, it has been applied. One that you have already seen is in homography estimation.

So, the homography estimation part we have already seen that, homography estimation. Then other than, rather than homography estimation, where you have got to see other applications such as, well, in fact, you can even it is also used to compute the Eigen vectors when you do a PCA. For example, when I talk about PCA what I really mean is if you kind of look at S as some rectangular image and you expand it as a sigma b Hermitian.

Where, of course, we realize that A is m cross m, sigma is m cross n, and B is n cross n where A is individually unitary, B is individually unitary and sigma is a diagonal matrix, but it is rectangular. And one way to kind of look at it is we do some kind of image expansion where we said that this is also equal to summation sigma i, i bi, bi is Hermitian, where we said ai is ith column of a matrix, bi is the column of Matrix B, there is one way to get a look at it, where i goes from 1 to R, if R is the rank of rank of S.

So, you could have do a expansion over all the single using all the singular values. Now, this is one way to see, this is what we have already seen. Let me just double this PCA and rather say image expansion. But then it is seldom used for this. It is seldom used for image expansion, but we still want to point it out because of the fact that this is an image transformed chapter and therefore, makes sense to talk about image expansion.

Now, the other thing, when I talk about its use to its linkage with actually the PCA, it comes about like this. If instead of looking at this as a rectangular image, suppose you think about S as a data matrix. Suppose, we can go back to that example where we had to do something like a face recognition. Now, each of those faces at the time that we were talking about was like 64 across 64 which means if you stack it up as a vector, it comes out to be a dimension 4096.

Now, suppose you had several face images, and you stack all of them as columns here. So, it is like saying that each. So, there are 4096 rows because each example. So, each of these is a face, this is like face 1 and this is like face 2 and so on. So, suppose you had p number of such faces which could span different individuals and so on. And where of course, p expectedly m, n, o, p, surely, we need this to be much larger than 4096.

So, that means you have you have a 4096 dimensional vector, but then you have got lots of lots of examples, an example faces for you and suppose you stack all this up within this matrix S. And suppose you generate a new matrix S, suppose I call this S underscore and suppose we make this as S minus, let us say mu, where this mu is simply the average of all the columns in, is simply the average of all the columns in S.

What that actually means is that with respect to the face example, it would mean like the average face. But in general it could be anything this could correspond to faces, this would correspond to some other kind of kind of a data set, whatever it be. So, mu is average of all the columns in S. So if you do this, then you are subtracting you are doing some kind of a mean subtraction.

And suppose you did S, S transpose. I am not having to writing Hermitian, simply speaking it should be SS Hermitian or if you wish we can do it as SS Hermitian because that is what you will do in general. If this matrix had some complex entries and so on, then that would give you a covariance R, that would give you the covariance of this data.

So, in other words, what you would have otherwise estimated as R is equal to suppose you had p examples you would have estimated it as 1 by p. Now, let us say each of these examples, suppose I call them as xi, then what I would have done is xi minus this mean mu into xi minus mu Hermitian or transpose. Generally, it is a transpose but well we can use as a Hermitian.

Now if you were to, so if you were to do, so, for example, there is a column vector, that is a row vector, therefore, you get to actually matrix. The sum of all these matrices so as to get an idea of the covariance, the same effect that you would get by doing this operation is what you get by doing this, which is simply SS Hermitian. Now, if you substitute through singular value decomposition, we know that S by itself is a rectangular matrix can be kind of decomposed as A sigma B Hermitian then we know that this will turn out to be A sigma B Hermitian.

S Hermitian as B, A B sigma Hermitian, A Hermitian and because B Hermitian, B's identity, you see you end up with A sigma, sigma Hermitian, A Hermitian. And of course, sigma sigma Hermitian as you can clearly see is 4000 so those 4096 by 4096. And R again of course is 4096 by 4096, if you take it to be the special case, then this matrix is a covariance and it is of size 4096 by 4096 for a SS Hermitian.

So, what it actually means is that, if you wanted to do an Eigen value or Eigen vector kind of a kind of a decomposition for this covariance matrix, then those Eigen vectors that you need to do this decomposition can come straight away by doing the singular value, by doing an SVD on S. So, you could take this data matrix do the SVD and then, by doing SVD you get this A which is what you need in order to write the covariance in an Eigen value, Eigen value, Eigen vector form, and the singular values that you get in sigma can then can and will also need to be used here.

So, if you just plug the ball here, you are an Eigen value Eigen vector say no decomposition has sort of ready. Now, you can see which ones to use, which one, how many significant Eigen vectors and so on. So, that sense it is very closely knit to the PCA also. And then, you know, there are of course, applications of like denoising, which is something that we will see later when we do transform domain filtering.

SVD can also be used for some kind of denoising, image denoising. And one more thing that I would like to point out is what is called pseudo inverse. So, like I said, it has several applications, I am just kind of mentioning a few here, I am just listing a few of them here. Now, by pseudo inverse what we really mean is this, a pseudo inverse.

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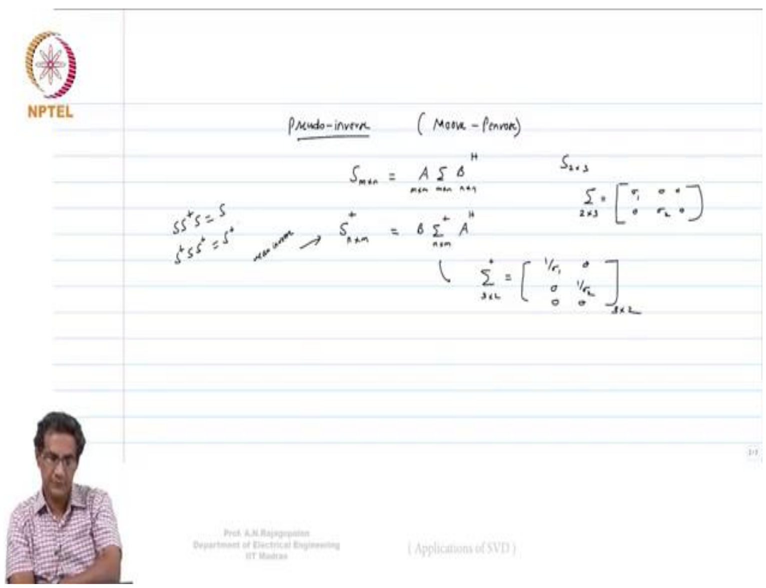

So, it basically means that generally you would think that, you would (())(7:43) talk about inverses of square meters provided it exists. Now, when you talk about pseudo inverse, you can even talk about inverse of a matrix which is just rectangular and so on. So what this means is so it is not really is in that sense it is kind of a weak inverse. It is not the true inverse that you have all learned.

It is called a pseudo inverse. It is also called the Moore Penrose pseudo inverse, Moore Penrose pseudo inverse and what it means is this. Suppose I have a matrix Smn. Let us say that I have done an expansion as A sigma B Hermitian. Now, of course, again this m cross m, n cross n, n cross n and the pseudo inverse is typically indicated by actually a plus on the top. This will have a dimension n cross m, and that is given as B, and sigma plus which will then become n cross m and A is A is a Hermitian where sigma plus.

So, if sigma let us say that suppose S was 2 cross 3 suppose as an example, then the sigma would, this matrix sigma which is actually 2 cross 3 would then look like this sigma 1 0 0, 0 sigma 2 0. Then the sigma plus would then look like, so the sigma plus will of course be 1 by n cross m, therefore, 1 by sigma 1 0 0, 1 by sigma 2 0 0. Because sigma plus should have a dimension n cross m which would be 3 cross 2 now.

So, your sigma plus which is now 3 cross 2, would look like this. So, in general so sigma plus will simply mean that whatever singular values which are non 0, you will just take the reciprocal. So, you will make it as 1 by sigma i. And which is what would then can lead to sigma plus and then you can use that to compute your S plus and this S plus is called the pseudo inverse and it satisfies, it is kind of a weak inverse, this guy will satisfy S plus S is equal to S or S plus S, S plus is equal to S plus and so on.

So in that sense it is kind of a weak inverse. But, and when the actual inverse exists, suppose the actual inverse exists then the pseudo inverse will then be equal to that. If in case the true inverse exists, then in that case it will be equal to the pseudo inverse, will then be equal to the true inverse itself. In other cases, you would get something like this. So this sort of ends our SVD. So, we have gone through SVD. We have gone through also the applications of SVD.