Image Signal Processing
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Lecture No. 38
1D Unitary Transforms- Introduction

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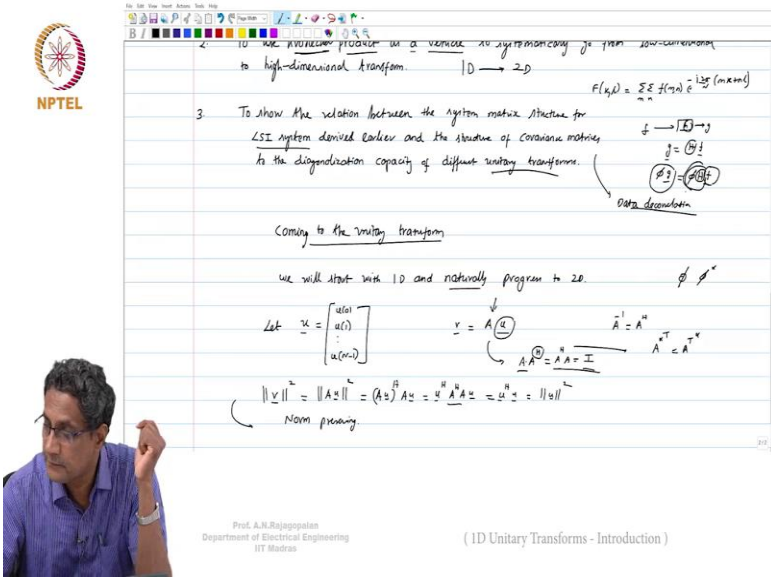

Now, coming to coming to the to the to the unitary transform itself. So, for example so we will start with 1D. We will start with 1D and naturally move towards 2D naturally naturally progress to the 2D. So, in a sense we will we will kind of naturally show how this extends to. I mean how this can itself can itself be kind of used to us to really a construct a 2D basis.

Let let u be a sequence which is like again in I mean all of this time and frequency all this is when there is there is not anything sacred about it we can always go kind of back and forth. So, u u0 except that real and complex if you have to worry about, yes then you should worry about but otherwise there isn't anything very sacred.

So, so we say that so we say that you know, we are applying a unitary transform, if, if let is say so, so when we do v is equal to a times u we we say that we have applied a unitary transform on u provided A satisfies a condition that A A Hermitian is equal to A

Hermitian A equal to is equal to is equal to identity. So, which basically means that A inverse is in fact you know a Hermitian itself. So as an example, if you look at a DFT matrix phi what is its inverse? What is a inverse of a DFT matrix?

Student: (())(02:13)

Professor: No, if I give you a DFT matrix phi, what is its inverse? Just simply phi star, simply phi star is its inverse. Phi start. You can write it down. 2 by 2you would not know because it is all real one what is a 2 by 2 will be like 1 by root 2, 1 by root, 1 by root 2 minus 1 by root 2. But if you like a 3 by 3, phi transpose is phi by the way. So, only phi star.

So, the point is so, so so there are several advantages to using a kind of unitary transform because it leads to leads to NSA least an orthonormal basis, with which with which you can you can expand your sequences and in fact all your DFT is the DCT is at all they are all of this nature.

They all have a basis which is orthonormal. So, the fact that A Hermitian is identity this this reveals the fact that you have bases that are orthonormal. Now, how it is and so on, so the structure that we will actually see. But, but then first of all let us kind of look at the fact that that, so one of the things that you will notice is that if we take norm v let us say let me just do square it and it should be like norm A u square, it should be like A u and write, one more thing.

We will always end up using using this say Hermitian. So Hermitian, as you all know, is simply A complex conjugate transpose which is also the same as A transpose and then you can take the complex conjugate. The order is you can do it in either way, so either take the transpose and then take the conjugate or take the can.

Student: (())(4:04)

Professor: So, this so the point is when you do this A Hermitian it what it well anyway wait a minute when we kind of try to do it at that time, you will see. Just just wait for 5 minutes. We will we will kind of do it.

Student: (())(4:20)

Professor: I mean, so it is basically a basis for your for reconstructing your sequences your U. So, you want to be able to express u in terms of a linear combination of basis and those basis come from A in fact, they come from A Hermitian they come as columns of A Hermitian and those columns are all going to say or orthonormal with respect to one another.

So, A star is equal to AT star. So, that is what of this Hermitian rate that people write it as h on top. And so if you see here, it is the Au Hermitian Au. Or which in turn which in turn, is Au it is u Hermitian, A Hermition Au and then A Hermitian A as we know is identity because that is what we assume because is a unitary transform.

So, those u Hermitian u and therefore this is norm u square. So, these unitary transforms are norm preserving, they are kind of norm preserving and if you if you look at the structure of A itself you can kind of think of it as you can think of.

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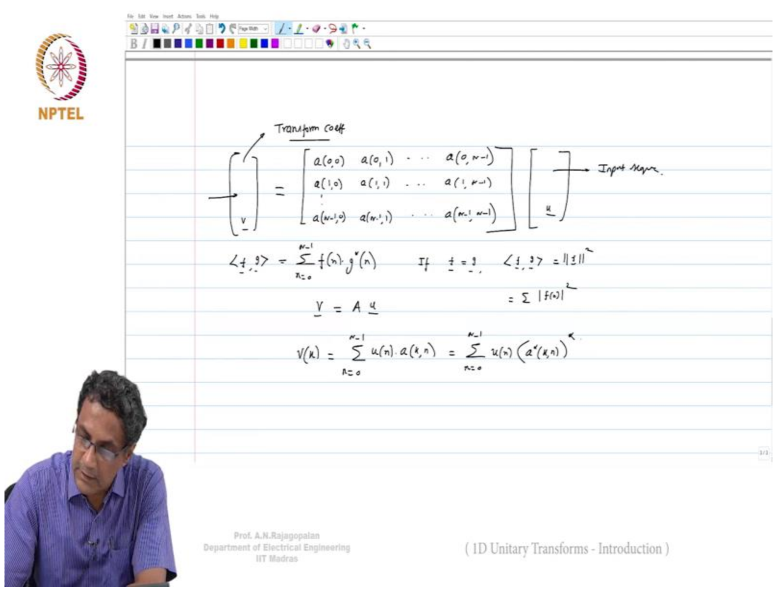

Let us say that suppose suppose we take a u what was that length I took. Did I take the length as something they say what is the length of u? So, so so this is like, this is like length n. So, so, let us kind of look at, look at A which which should then have to be n cross n matrix, so let us look at this guy.

So let us say it has entries A00 is A01, all the way up to a 0 and minus 1, A, then A10, A11 all the way up to A1 comma n minus 1 and then dot dot dot, all the way down to A n minus 1 comma 0, which is the last row A n minus 1 comma 1, all the way up to a n minus 1 comma n minus 1. This is how your write A will look. And this A, is actually acting on you.

Now prior to sort of interpreting what is the basis then and you know, what are the transform questions and all, prior to that we will kind to look at when we take let us say, the inner product of the some two vectors, we have f and then we have g, then what do we what do we get all I typically write this as they both of course have to be at the same length. Let us say let us say let us say n equal to 0 to n minus 1. How do we kind of see write this inner product? When you take inner a product?

Student: (())(7:18)

Professor: Go ahead, if fm fn, fn into g star fn g star fn. Now, of course, we also kind of see we also could have noticed that if f equal to g, then this inner product will give u norm square of whatever f or g because they are both the same norm square f and f equal to g otherwise.

So, so if f equal to g then this becomes modulus of fn squared, which is but which is but they say norm square of f. Now, so now so now the point is this so, so the way we can motivate this is, so we said V is equal to A times u. So, we are kind of multiplying this guy with some with our u, this is your u and then on the on the left right we have your V.

So, those are we so if you look at let us say, so so so this vector has what are called the transform coefficients. This is your this is your input sequence. Now, now if you look at look at a kth entry here, so for example, V0 is simply the first row multiplying this column and V1 is the second row multiplying this column and so on therefore, Vk is simply summation k un a k n going from 0 to n minus 1.

We get so now what we do is instead of writing it like this we will in turn write this as n equal to 0 to n minus 1 un a star k n the whole star means the same thing.

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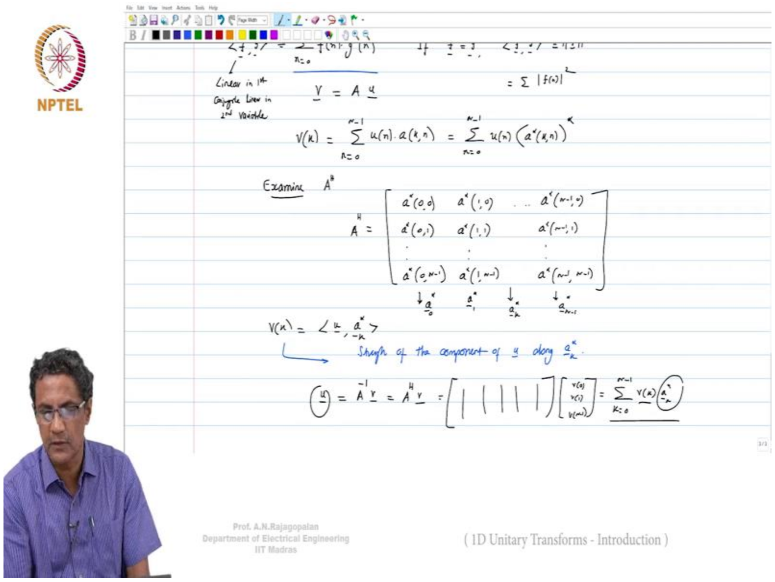

Now now now examine A A H which is the Hermitian so A Hermitian if you if you write it down, A Hermitian looks like this. So, it will be like a star 00, then what will be the next one? 01 all the way up to a star 0 comma n minus 1, then you have a star 10 a star 11 going all the way down to a star 1 comma n minus 1 and then you go on to the last column, which is a star n minus 10 and a star n minus 11 go down all the way to a star n minus 1 comma n minus 1. This is all your A Hermitian looks like.

Now, in this let us call so the first column they just they just indicate it as a 0 star. This is this is simply a vector, vector A 0 star. Let us call this guy is vector A one star. Similarly, some cases. It will be like a k star last one is of course a n minus 1 star. So, each is simply a column vector right column vector. So, the Kth column of A Hermitian is Ak star.

So, then write a clearly what we can do is we can write your V k which is the Kth transform coefficient we can say simply the t inner product of u and also and also that u will kind of see know that the inner product is something which is kind of linear in the first variable. So, this you know is linear in the first variable and it is it is a conjugate linear in the second.

So, alpha f comma g is alpha in a product f comma g but f comma alpha g is alpha star in the product f comma g. So, linear in first and conjugate linear in second second second variable conjugate linear in the second variable. These are things that you know so now, so so in this case what we can do is so we can say that this guy is but in a product of u with a k star because if you see, that will that will mean that mean not taking summation of whatever right summation use, you know un, and then all entries of ASR fk, and ASR fK already has this a star whatever k 0k 1, and so on.

And then you take the complex conjugate of that, because when you write the inner product that is how we write the product as you people told yourself like that is how it should be. And therefore, and therefore it you can look upon v k as inner product. So it is equal to saying that a transform coefficient simply indicates the strength or indicates the strength of the component of u along a k star. The strength of the component that u has along a k star, if it is zero, then it means it is orthogonal.

Perhaps, to that particular particular sort of you know to that particular a ak star. So, when you look at it you interpret this as the strength of the component of you along component of u along ak star
then it follows that now if I am if I if I want to get a right reconstruct my u.

So, for example, suppose I gather all my transactions want to go back. So, what do you do I will say use equal to of course A inverse v and this I can write because I know that A inverse exists and a inverse is but A Hermitian v correct

And you know and again out of the various ways by which okay you can interpret A Hermitian v so you have the one way to interpret this is you can you can you can get a look upon A Hermitian as as all these columns, columns, you know what are all ak star multiplying your V0, V1, all the way up to be n minus 1.

So, I would just leave it you leave it you to check that this is the same as what would be the simplest way to write this so that it makes no, it makes interpretability easier. This will be summation, let us say k equals zero to n minus 1, what would that be?

Student: (())(14:23).

Professor: vk, vk ak star, vk into ak star. So, so this product is the same as V0 into the first coloumn plus V1 into the second column, all the way up to plus V n minus 1 into the last column. So, so the way so so now so so this is again, something that you are all familiar with nothing new here.

So, it simply means that when you want to reconstruct u, then you find out what is the strength of what was the strength of u for that component, ak star, multiply it t weighted linearly, and then do this linear combination. And that and that kind of linear combination returns you exact returns you exactly no the (())(15:01) errors as long as you use all the components.

Now those all still still 1D. Next class, we will see how to kind of move towards. Now slowly we will move towards how to use all of this to go to a go to a 2D transform.