Image Signal Processing

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Lecture 20

Depth of field, Linearity

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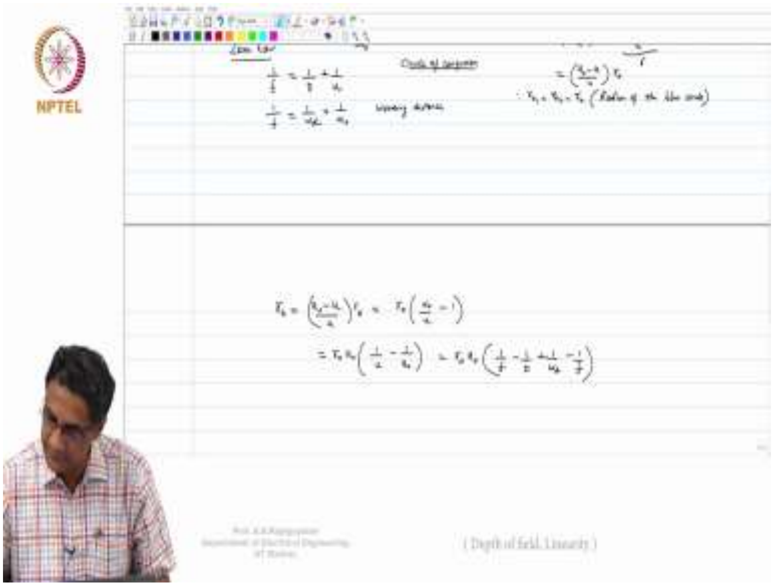

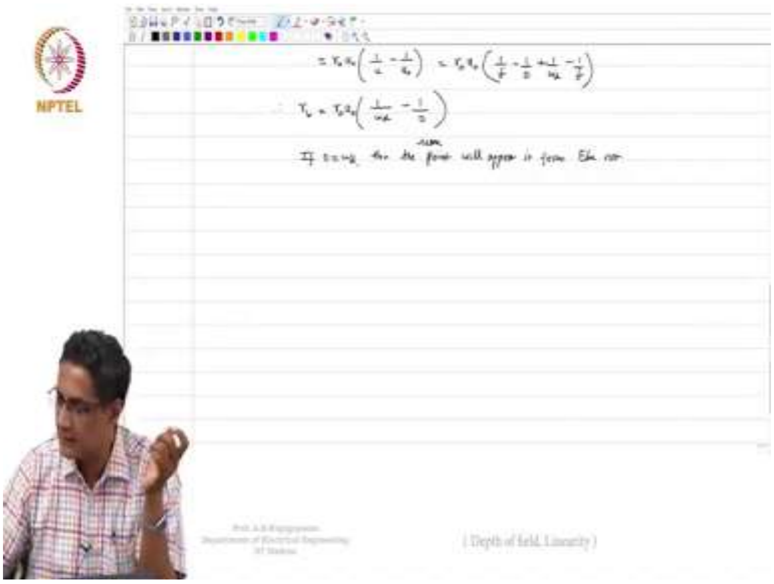

So, let us just look at this rb. What is it? Is equal to u naught minus u by u into r naught, naught minus u by u into r naught. So, let us just, let us actually write this as r naught into u naught by u minus 1. And, pull out let us pull out r naught. Let us pull out u naught. So we will get 1 by u minus 1 by u naught. Correct?

And now let us substitute for 1 by u from the earlier equation. I have written here something. So, go back here I had written this, so we can actually use that. So, by the Lenz law if you look at 1 by u that is 1 by s minus 1 by D, minus 1 by u naught. So minus 1 by u naught is plus omega 1 plus 1 by omega d minus 1 by f. And therefore, oops. And therefore, rb is equal to r naught u naught, so 1 by f will cancel off and we will get 1 by omega d minus 1 by D. Okay? And it actually makes sense.

So, what this means is that if your D that means the point is such that D is equal to omega d, then rb equal to 0, right? If D equals omega d then there is no blur. Then there is then the point appears in focus. On other words, there is no blurring. Then the point, point means the seen point, okay, will appear in focus in focus else not else not.

Then, this rb is right you do not have to worry about rb being sort of right a negative number right. That can also be a negative but the interpretation is that it is not like rb is negative no. It just means that whether the, whether this blur circle whether it is there.

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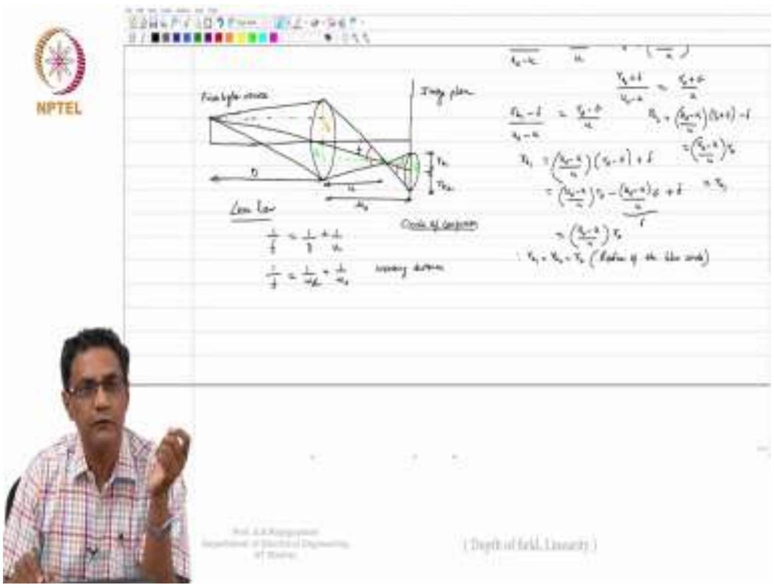

So, in this diagram right whether it is whether this this point of focus or whether it is in the front of the image or on the back. If it is negative, it means that the point of focus is at the back, okay? So, in that sense that is we would not really worry about the sign of rb. For us all that matters is the fact that if you are not keeping an object at the at this wd which is the working distance for that lens, then right, it will come blurred.

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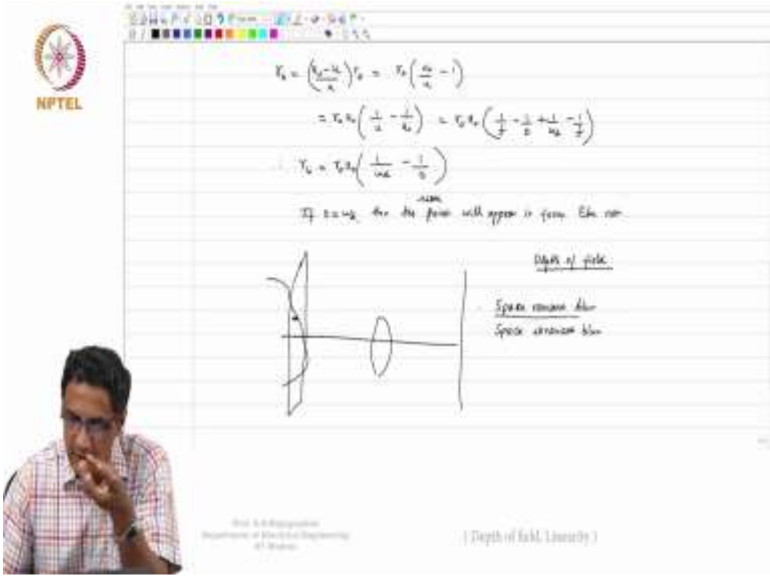

So, what does then mean is that, right, imagine that imagine if you had really, you know, really a 3d scene, right. And now if I had a lens, right, which we know is trying to image it and then I have an image plane. Now clearly right, I know it is impossible for me to satisfy Lenz law for all the points because a Lenz law will only say that for this focal length, and for this whatever it is aperture and all of that aperture radius that should be omega d.

Now, maybe right one of these points is actually satisfying this. Let us say a point here, it is actually satisfying that the Lenz law. So that will come in focus. But then the other guy is not at omega d right. Something is in the front, something is something is closer, something is farther and you know, each one of them will start to introduce its own its own circle of blur. And which will then whose radius will also start to change now, right? Because they are all not at the same sort of right at a distance.

Now because of this right what happens is some sometimes-right people say that this notion of linearity and shift in especially the shift invariance part, right. Sometimes you see some papers saying that you know a lens is not shift invariant but that is a kind of a subtle point. That is actually not true, okay?

What is true is that a lens is actually shift invariant okay. It is a linear and shift invariant system but it becomes more tricky as opposed to 1D case because you have to understand what does it mean do we actually analyse a system for its shift invariance.

Now, the point right at this point of time, at least you should be able to digest the fact that why is it that you see images being formed and lens the way they are formed. If some things are in focus, some things are out of focus that is happening because of this. It is impossible to bring everything into focus.

But there is an associated notion of depth of focus. So, there is a notion of depth of focus or depth of the field in fact; it is a more common name. So, this depth of field changes for every lens okay depending upon its aperture size, depending upon its focal length, depending upon its circle of confusion that you admit because it also depends on what you admit as being in focus. So, you know there are so, so there 3 or 4 things on which the depth of field of which depth of field is a function.

Now, you can have lenses for whom the depth, so, what this actually means is that, you know what is the range of depth? That means what is the you know so for example right if 2 points are away by a certain sort of a distance from the lens, will you already start to see blur? Or they can go this much farther off and then maybe you will start to see that these two are not at the do not have the same level of blur?

Or is it that even if they are microns away, your lens can still trap the fact that this cannot be brought into focus or they both cannot be simultaneously in focus. So, this depends on the lens. So, for example, a microscopic lens in which if you just keep 2 layers if you show because it is microns away, it can still kind of find the fact that they are not at the same depth simply because this aperture, this lens will reveal it.

Because when you see the image, it look like this guy is in focus and this guy is blurred, or if you are focusing on this guy, this guy you will turn blurred, okay? Whereas you may have lenses for which you can go meters apart and still be able to see it all in focus. And then there could be situations where, let us say, well, there is something which is really far, you know, where there is a big depth difference and then you see a difference in blur, okay.

So, this depth of field, so, it is not true that all lenses will have the same depth of field. So, what we are saying is, it depends upon each lens. But the image formation process is still fundamental. That is all this only. But then whether you see this blurring effect or not, at what kind of depth differences you see an appreciable change in blur depends upon the depth of field of each lens.

Now this could be used here, like I said yesterday it could be a nuisance or it could make nuisance for you. So, some people actually realize the fact that if you are seeing a difference in blur that means what? That means is the lens is telling you that look, this this world is actually is not plane. Now, this is actually a 3-D way, what you are looking at. It is giving you that information. What you want to do with it is your outlook. Is it not?

See when I am capturing an image and suppose I see that this image has gotten a different blur there, a different blur here then it should actually wake up my senses and tell me that oh, the lens is actually telling me that the world is 3D, the one that you are looking at. If I see that all of this, all of it is in focus, the neither I can conclude that this depth of field is probably very large. That is why irrespective of where the objects are, everything is coming in focus.

But if I know the depth of field for that lens, and I still see that everything is coming in focus then it means that probably the scene is a planar scene which is why I can bring all points into focus. Because you know, right imagine instead of a 3D world, if I had just a planar scene like that, again front-parallel, every point is at the same depth d.

So, either all of them are blurred by the same amount because they will all have the same blur radius. So, it is all blur but the same amount or I can actually I can move this. Well if I cannot move the lens, then maybe if I have a way to adjust my focal length or my u naught, then I can always bring them into focus, bring the entire scene to focus right.

But in general, it is not true. Why? Because the scene you need not be constrained to be a plane. It could be inclined in which case again you will see a different kind of blur because it because then your depth will start to change for each point. Or the scene is completely a full blown 3D. Again, the same thing, right. You will get an image that will start to. So, a nature of blur like that is called a space variant blur. Okay, a 3D scene introduces what is called a space variant blur.

And if you have a planar scene that is front-parallel, you can it is possible to arrive at what is called, well why is it possible? You will get it is possible to bring the scene into focus but you will definitely get space invariant blur. We call it a blur because it is like something not being in focus, it is spreading.

Okay now, so, because of the fact that right this introduces a space variant blur, people generally hurriedly conclude that the lens is space variant. Because you know the scene is 3D and you see, you have a space variant blur and therefore it is the space variant system and all. It is not correct.

Now we want to, we want to understand that I can simply say that assume the lens is shift invariant and move on. But I generally do not do that, because I think when we want to understand image formation it is always good to because when you understand this and then somebody talks about a code and aperture anything else then somebody talks to you little, you will at least be able to appreciate what is going on, okay.

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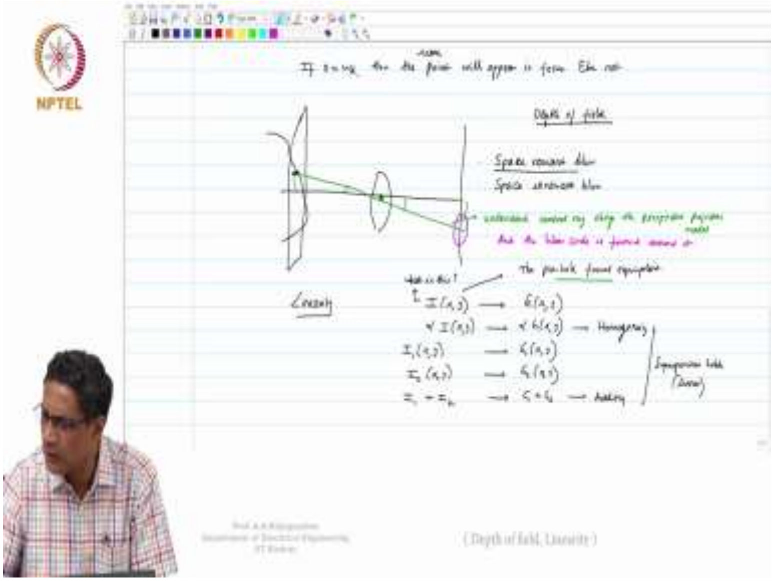

Now, let us get a look at this notion of linearity first. Okay, now when you are when you talk about linearity, right, what do you actually mean? You are kind of thinking right in your mind that if I input an image I of x, y. Okay, now for the time being I will simply take sort of a continuous case. It does not matter alright; it is a discrete or continuous. Let us say continuous.

I of x, y let us say gives you let us say this imaging system, gives you G of x, y. Then by linearity means what? It should satisfy the superposition principle. That means that if I do alpha times I of x, y, it should give me alpha times G of x, y correct. Actually alpha can be real, alpha can be complex, in general, when you (())(11:05) alpha can be any constant, real or actually imaginary.

Anyway, let us not worry about the imaginary constant; how you looking at image. Then, I1 okay now if I1 of x, y produces G1 of x, y. I2 of x, y produces G2 of x, y then, I1 plus I2 all the x y and all is still there should produce G1 plus G2 right. So, one of them is called actually homogeneity. It as you all know this called the homogeneity property. The other is called additivity. This part is what we call this additivity.

So, when these two together they are satisfied and we call then we say that superposition holds. Then we say superposition holds and which means that you actually a linear system.

Now the slight complexity that arises here is what do we mean by an input image? Okay, I that is the way we interpret when you have a linear system, what you usually apply in input, you watch for the output. That is the way we interpret right? Now, when I write here I of x, y it is not even clear to me what is the input image that I am trying to talk about. So, what is this guy first?

Output probably is here what I going to get out of the lens. What is this now? So, the way we interpret this I of x, y is that this is the pinhole focused equivalent. So it is saying that if I gave an image that is completely in focus, how does this lens; so it is saying that lens was not there. I would have seen actually, right, like I told you I am introducing the lens. Without the lens, what do I see?

I see like a pinhole through which my rays are coming. And then the only problem is my intensity. Let us just assume that (some calibration problem) okay, let us just assume that if there was an aperture only a pinhole, then I would have seen a focused image, okay. Except that I said it might be dark but let us assume that I can expose it long enough so that I get an intensity that would be the same as basic whatever I could gather through a lens in terms of accumulating all the rays and so on. So, these two let us just assume that we are able to do that.

Then when you introduce a lens, something is going to happen to this I of x, y. It is going to change. Now, we want to understand. So, it is like saying that I have a point light source. Okay, because this let us start with actually a point light source because then we then we can have one point light source, add one more point light source. Then that plays both of them together right. That is the way it is working.

So, when you have one point light source right with respect to which okay you see a focus point and then the point is right when that when you introduce an image or introduce a lens and something will change, okay. Now, we tend to interpret this as a complete image which is also okay which means that you know, not just one kind of one sort of point light source there is a whole sort of thing, a collection of point light sources for which you get an I of x, y.

And then if you replace the aperture with a lens, it turns into some G of x, y. And now you are asking, if I increase my intensity will my output image that I get out of the lens will that also correspondingly increase? If I have two such scenes or two such points, where do you can think of; this is easier understood if we have tried to think of it as a point light source.

One point light source leading to one image, another leading to another. Can I keep them simultaneously when I get the summation of what I got (())(15:03) that is the way we look at it. And clearly, the linearity part is very straightforward. And one of the key things to remember is the blur circle. Let me write that here. One of the key things is that it is the central ray. Okay? That is where I told you that we borrow from the pinhole the analogies one analogy that you have already borrowed is this, the pinhole focus equivalents.

The pinhole still has a role. It is not like it has no there is no baring once you introduce a lens. When you think of a focused image in a lens, you can correspondingly think about, oh, that is that is what I would have obtained if I had a pinhole camera. So, what is lens is really? Lens is really a pinhole output. So that way there is still a relation between an analogy between what is going on with a lens and what you get with respective pinhole.

The other thing that you have to do to capture is the fact that this guy, this central rate which goes undeviated, goes undeviated central ray, obeys; what does it obeys? The perspective of projection model. See, for example, if you think of where it can arrive this is again deviate when and where this is, right, this angle is equal to this and whenever we take with perspective involved that kind of a PP model, Perspective Projection this guy obeys, okay because this goes totally undeviated.

And the third interesting point is the blur circle gets fall about this guy. So, it is like saying that so the principle ray or the central ray about which your circle gets formed right. And note that obeys the and the blur circle is formed around it, around it. This is after it see the image plane, the blur circle is getting formed around it.

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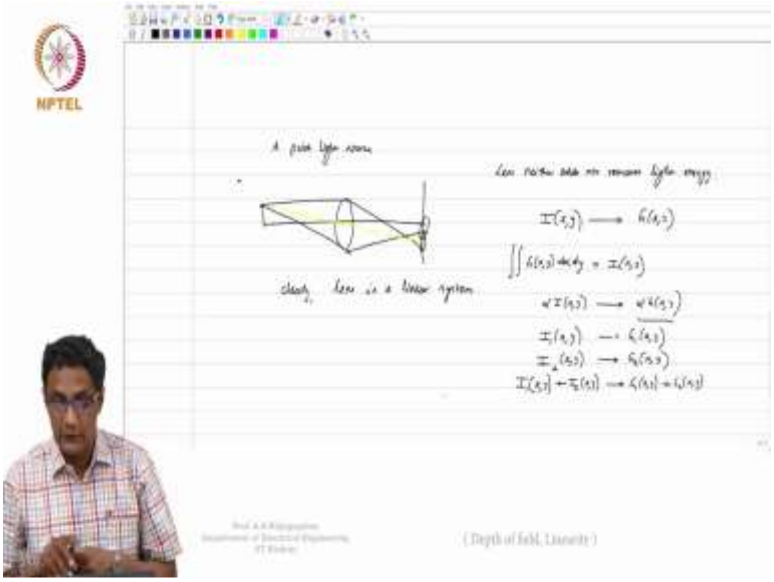

Now, the point is something which is kind of which will again go back in terms of pinhole. Now, we okay now try to go into shift invariant and I have not even talked about linearity. I am not finished with linearity yet. So, as you can see the linearity principle. Suppose you assume that I have actually a point light source, okay. Let me assume that, that I have one point light source; a point light source okay.

Go back here, now the lens, an image plane and let us say at this point light source and it goes and it blurs something and then what I see is that okay. What I see is kind of a blurred circle and ofcourse, when you have a central ray then it is still sitting there about which this whole thing is getting formed okay. That is a central way about which the circle is getting formed.

Now, what you see? One of the things is a lens by itself right a lens is something which neither lens neither adds nor removes light energy; adds nor removes light energy because whatever enters it will completely focus. Somewhat will spread into a circle of confusion. And it by itself does not hold anything back. Whatever is coming in it just kind of throwing it all out.

And because of this, what you can imagine is that if this if instead of a lens we had a pinhole and suppose you saw some intensity that would have been a point in focus, no, it would have had some intensity, correct? Now, when you replace that aperture with a lens, what has happened? Now, that point has gotten spread now. But because of the fact that the lens itself has not added or done anything, what do you expect?

The sum of all the intensities within this blurred circle that should be the same as what do you see as a focus point, right? Because you cannot lose energy anywhere no. Correct? Either the lens is also bringing it into focus, in which case you would not see any change in intensity. You saw a point earlier; you are still seeing a point. They both will have the same intensity. Correct?

Now, if blurring happens then it means that the intensities are getting spread now. Which means that the intensity is may be lower because as you walk around, you may not get the same original intensity that you got for the focus point. But if you sum of all the intensities that are lying within this blur circle, they should all add up to the same focus point. This is fine, right? Okay, there is not anything unusual about that.

So, what this means is that if I of x y; if this is a point, a point light source if this gives you G of x, y which could be blurred, then double integral G of x y dx dy should be equal to I of x y. Correct? This is if you had a point light. So, I am not trying to write out a convolution model. I am saying there is almost a point light source. If it spreads, then because the lens is not going to take off anything, so you just add up all the intensities within the blur circle. It will add up to what you saw as a focus point.

Now, imagine that I actually increase my intensity here, my radians of that point and corresponding to that let us say the pinhole focused image went up by alpha. Correct? In the seen nice I made the point light source brighter or whatever dimmer. And correspondingly what happened? My point, the pinhole point, the focus point will also either go up or go down by some alpha.

Now, again if you introduce a lens what will happen? That intensity will spread and it is again true that you will get alpha G of x, y because if you integrate this, it should give you alpha I of x, y and it cannot, it follows. Now, instead of one light source suppose let us say I1 of x y gave you one blurred circle which is let us say G1 of x, y. And, then I introduced another point light source somewhere there. Okay?

This could be 3D world. You do not care about it. I am not assuming planar at all. Some other point lies somewhere here. So, this point was here, this point is here. And then for that point light source I write I2 of x, y is the intensity that you get if only that was present in the scene corresponding to that what you have what is called G2 of x, y. Correct?

And now, if you say that I apply both of them together. So, at one time, we were applying only one at a time and suppose I put them both together then we get what is called G1 of x y plus G2 of x y. I mean that is what you will see, right. You will see one blur circle. Then maybe something else which could be overlapping, need not be overlapping. But as far as the whole image plane is concerned, if I average if I add all the intensity together that should add up to those two points, the focus points that I had originally.

This is strictly speaking of situations where you can think of some situation where this can fail. Those are very extreme situations. Like for example, what is called a self-occlusion? It is like saying that but how do I show that? Anyway, I mean it is not really relevant and I do not want to take you off the track.

But it is like saying that if I had had an object like this, something like this if my 3D scene look like that and then if this whole point is not there and if I had only this, then all these rays, right can go through the lens. But then if I put this alone then again all these rays can go through the lens. Now if I put both simultaneously then this guy's rays get blocked by the guy in the front, right?

I mean see this; this ray cannot reach because there is some point sitting in front. It is blocking it, right? In such cases, this will fail. But let us not worry about such cases. Simple, let us just keep the math simple and the principles simple. But yeah, I can think of situation where in fact, if you watch through actually a keyhole. When you have a room and through the keyhole if you watch, you will see that the image formation does not follow this because you have what is called a partial.

So, the blur circle right, the blur circle gets truncated because not all rays are going to come. Some are getting knocked off, right and so on. So that is all you know that is all only for explaining those strange situation (())(24:20). Let us keep it simple. Okay, so what this means is that; so, you all agree that lens is linear, right? So, clearly right lens is a linear system. Okay I will stop here because the next notion is one of shift invariance and that will require some explanation. Okay.