A Brief Introduction to Micro Sensors
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Lecture – 09
Coupled electromechanics

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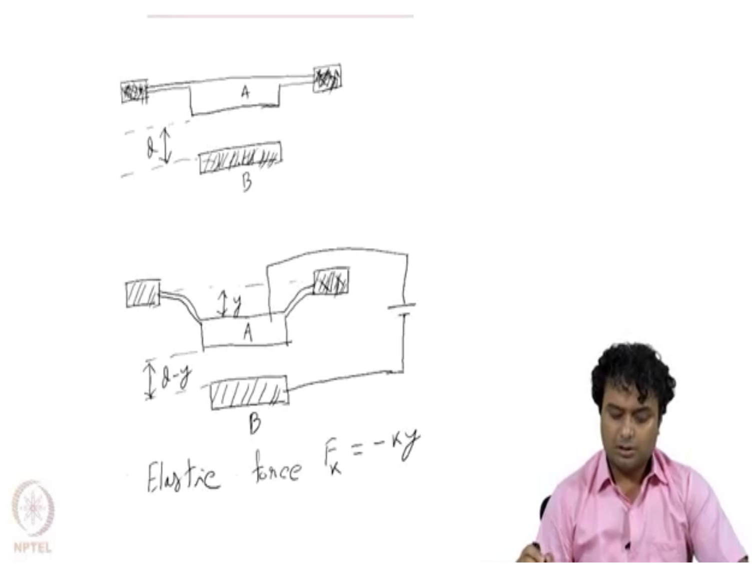

So, in the last class, we were discussing about the system which is a coupled system. Coupled system means that it is also attracted by the electric field as well as the mechanical force or the electrostatic force is trying to pull the A plate up. And in that case, we have already seen that.

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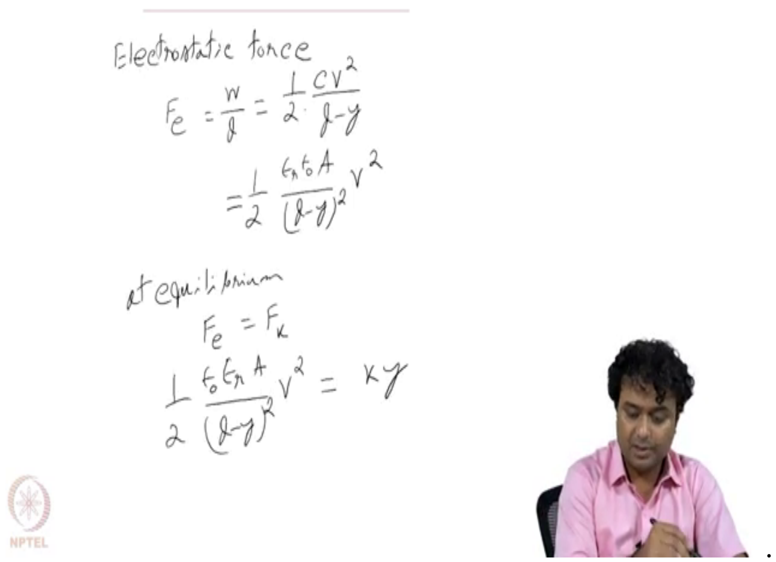

This is the electrostatic force right. Half into epsilon r epsilon naught A divided by d minus y whole square into V square. And the elastic force or the mechanical force is k y as we know, correct. And at equilibrium these two forces will be same.

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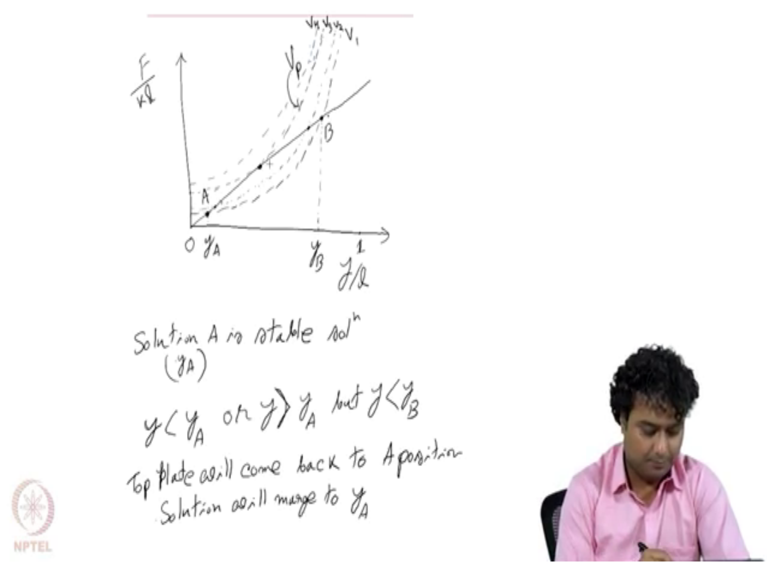

And then, we draw this the force diagram where we can where we qualitatively explain where we qualitatively explain that, as we are going from as we are going from y equal to 0 towards y equals to d right. Then the electrostatic force is increasing like that, like parabolically whereas, the elastic force or the mechanical force is increasing linearly.

So, there are multiple solutions like more than one solution. And here you can see that it is actually crossing like the elastic force and the electrostatic force is crossing over two points and we have seen that as we go more and more towards y direction like y equal to more and more at a towards the y equal to d direction then the electrostatic force is getting higher and higher.

So, that beam A like the plate A will be pulled in on the plate B and so, this will snap on that substrate. Also we have seen that this different also we have seen that these different lines are for different voltages right. And at some voltage where electrostatic force just touches over the mechanical force at this one, this is called pulled in voltage this is called pulled in voltage. Above that electrostatic attraction is always higher than the mechanical force.

So, there is no chance that the beam can be again pulled back right. So, this is called pull in voltage Vp. And this is the graph for Vp where it is exactly touching over the mechanical force right ok. Now, we will see this condition; the stability condition mathematically and then, we will try to derive the equation. And then we will try to derive the expression for pulled in voltage as well as the minimum gap which it can transverse without minimum gap which it can deflect without actually snapping on the substrate.

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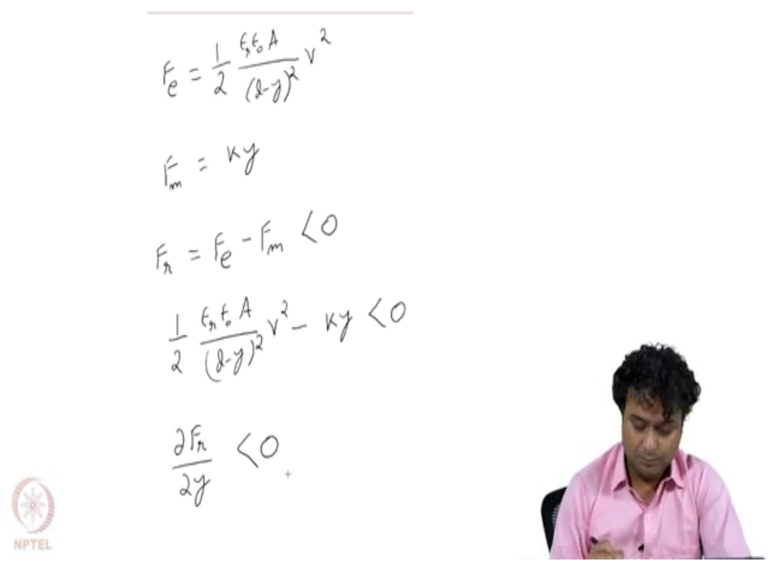

So, for that my electrostatic force Fe is equals to half into epsilon epsilon naught is called epsilon r epsilon naught A divided by d minus y whole square right into V square. And mechanical force let us say I call it F m equals to k y right. So, where from I am getting that? I have already derived this expression here right.

So, we know that electricity force is w by d it is which is actually half CV squared divided by d minus y, because the; if the y is the deflection, then the gap between the top plate and the bottom plate is now d minus y right. And then it you write C also in the form of epsilon A by d then you get d minus y your whole square ok.

Now, you see the first condition that the beam will be beam can be in that like in the stable condition. The first condition is that the resultant force let us say Fr is equal to F electric field or electrical force minus F of mechanical force will be less than 0, right.

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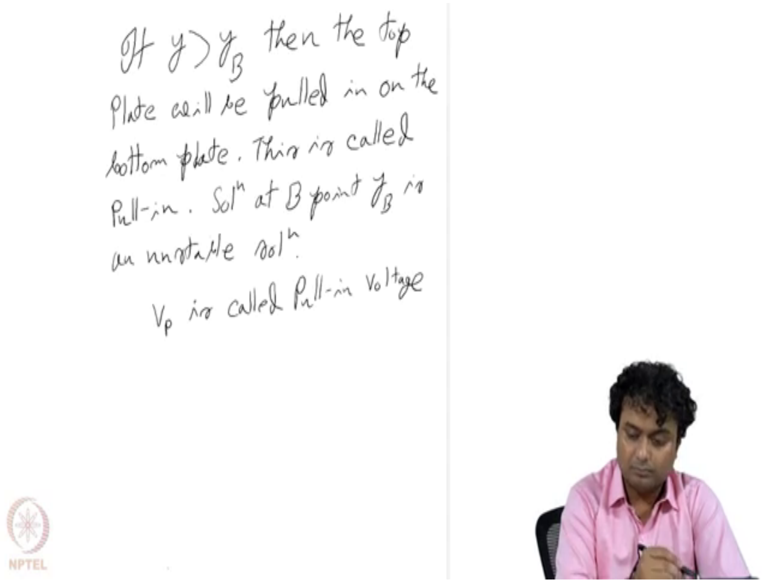

Because, we go to this previous graph and we see that while the electric force like for the top most graph right. For the top most curve, you see the electric force is always higher than the mechanical force. So, once it gets pulled in the top plate gets pulled in and it like gets attracted towards the plate B and it starts moving; there is no point where the mechanical force can be higher than the electrostatic force.

So, in that case what will happen that it will always snap in. So, the there it should be less than 0, like the electric electrostatic force minus the mechanical force it should be less than 0. So, the mechanical force should be higher then only it can pull the beam up. So, we can write them that half of epsilon r epsilon naught A divided by d minus y whole square V square minus or it is mechanical force ky minus k y is less than 0 right ok.

Now another important point is here that if you see from the graph, you see from the graph, then what is the stability condition? What is the stability condition? At this point see initially at the starting let us say take the let us name this graph. So, let us say this is V 1 then V 2 V 3 which is also my Vp and then V 4. So, for V 1 case let us say for V 1 case; what we see, that initially the make a electrostatic force is higher than the mechanical force right. And as the electrostatic force start pulling in start pulling the plate A then it starts to move towards the plate B.

So, y start y starts to increase right, y starts to increase. And so, y moves in this direction and as it move then the mechanical force start increasing and at this point A, electrostatic force and the mechanical force is same right and after that if you it deflects more, then what happens? Then the mechanical force is higher right. Mechanical force is higher and electrostatic force is actually lesser.

So, if I draw that resultant graph if I draw the resultant graph then it will be something like that resultant force will be like electrostatic minus the mechanical force. It will be initially positive, it will be initially positive while y equal to 0 then it is slowly then it will slowly go down right it will slowly decrease and then it will cross over 0 right, it will crossover 0 and then it will go towards negative Because, in this point the mechanical force is actually higher right.

It will go toward negative and then it will reach a minima and then above that it will again start moving upward right. It will start again moving upward, because after that the electrostatic force is increasing even in higher rate than the mechanical force right. And in that case it will start again moving up and accordingly and it will reach the B, the B point where the both the forces will be same and after that it is like the electrostatic force is always will be higher than that mechanical force.

So, the resultant force in this region actually, the resultant force is actually going down and it will go towards negative also and after that; it will reach a minima and it will again move start moving up and while it reaches the minima, that is the pulling point right. That is the pulling point where it reaches the minima.

Now, while the resultant force is going down; that means what? That means, the slope is negative or delta Fr by delta y is less than 0. So, this is my Fr electrostatic force minus mechanical force and what I am saying that the resultant force will even go down right as we an increasing the y. And that is the region where the where we can tell or where we can assume the beam is in stable condition right. So, as the Fr is going down with y delta Fr by delta y also less than 0; that means, it is negative slope right. While y increasing; resulting force is decreasing.

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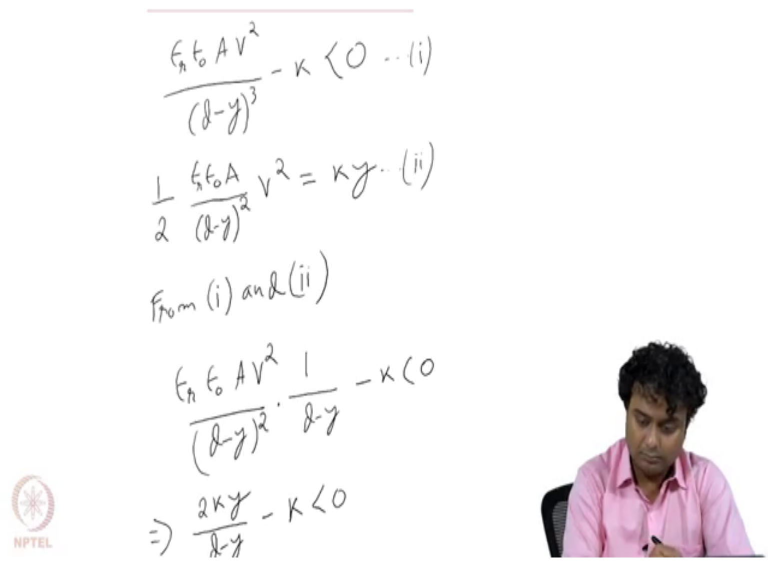

Now, what is delta Fr minus what is delta Fr by delta y? That is epsilon r epsilon naught A V square divided by d minus y whole cube right minus K less than 0. So, that you can actually this is the Fr expression right. This is the Fr expression and in this expression I am doing just the first derivative. So, if I do then we get a for this d minus y whole square these two comes and then I get d minus y whole cube below and there also a minus comes, but there is also minus y. So, that minus minus becomes plus. So, ultimately, I get epsilon naught epsilon r epsilon naught A divided by d minus y whole cube into v square right from this term this is just K.

Now, what we know from pulling point? From pulling point I know that the electrostatic force and the mechanical forces are mechanical force are both are same, exactly same; that means, my so here that means, my half of epsilon epsilon naught A by d minus y whole square into V square is equals to K y ok.

Now, if I now, let us put this expression in the let us call it equation 1 and let us call it equation 2. So, from equation 2, if we put the epsilon naught epsilon epsilon r epsilon naught A divided by d minus y whole square into V square whole term as K y then we can write is so from 1 and 2 we can write so, what I am writing; epsilon r epsilon naught A V square divided by d minus y whole square. So, this is an equation 1 I am rewriting just in different form into d minus y right. So, d minus y whole cube I just separated d minus y square into d minus y minus K less than 0. Now this term is 2 K y right. So, 2 K y divided by d minus y minus K less than 0 right or 2 K y divided by d minus y d minus y less than K or 2 y less than d minus y or y.

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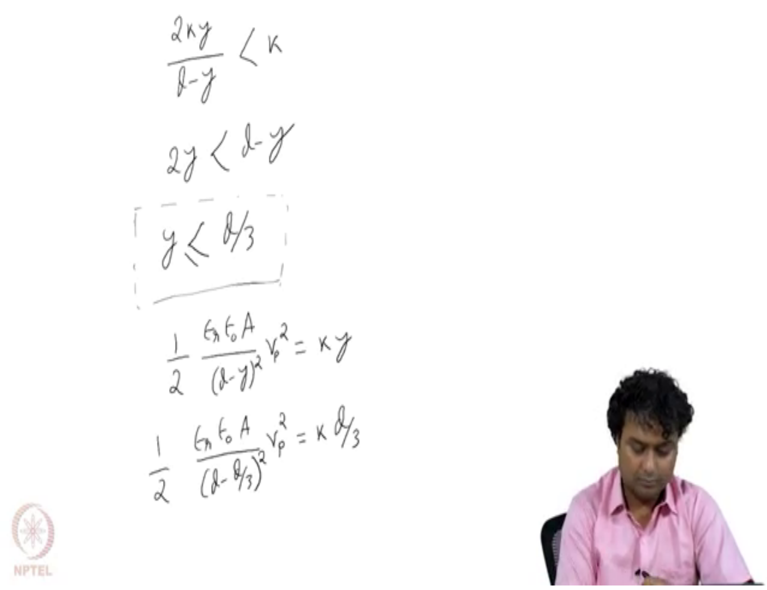

So, this is actually less or equal to d by 3 ok. So, it means that for a deflection of the one third of the gap the beam will be in stable condition and after that the beam will pulling on the on the bottom plate right. So, this is what we get and remember this d is that initial gap between the two plates ok. Now what is the pulling voltage? Pulling voltage; we can calculate like this. So, we know that; if the y is the deflection then the electrostatic force half epsilon r epsilon naught A divided by d minus y whole square into V square right this is the electrostatic force is equals to K y.

So, this is Vp, because at pulling voltage these two terms are same. And at pulling condition; what is the limiting condition for pulling? That is y equal to d by 3. So, let us put this y equal to d by 3. So, then we get epsilon r epsilon naught A divided by d minus d by three right whole square Vp square is equal to K into again d by 3 right and we will get that.

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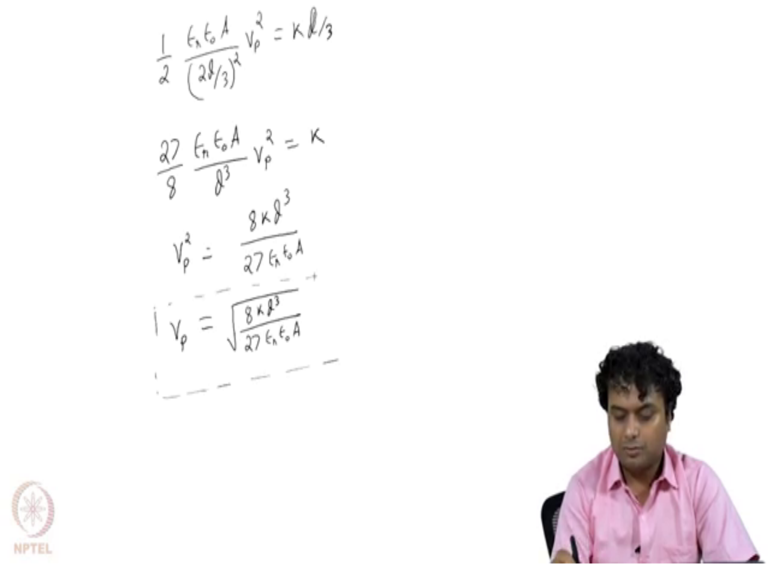

Half into epsilon r epsilon naught A divided by 2 d by 3 whole square right. 2 d by 3 whole square. 2d by 3 whole square into V p square is equal to Kd by 3 is equal to Kd by 3 ok. Now, we take this thing up. So, it becomes. So, 3 square, 3 square 9 that goes to the top this 3 also comes here. So, this becomes 27 right and then this is 2 square 4. So, 4 into 2 8 27 by 8 right into epsilon r epsilon naught A divided by let us say this is d square and we take this d also. So, d cube right into Vp square is equal to K right.

So, Vp square is equals to 8 K d cube divided by 27 epsilon r epsilon naught A ok. So, Vp is equal to square root of 8 K d cube divided by 27 epsilon naught fine. So, this is my pulling voltage in terms of the beam geometry.

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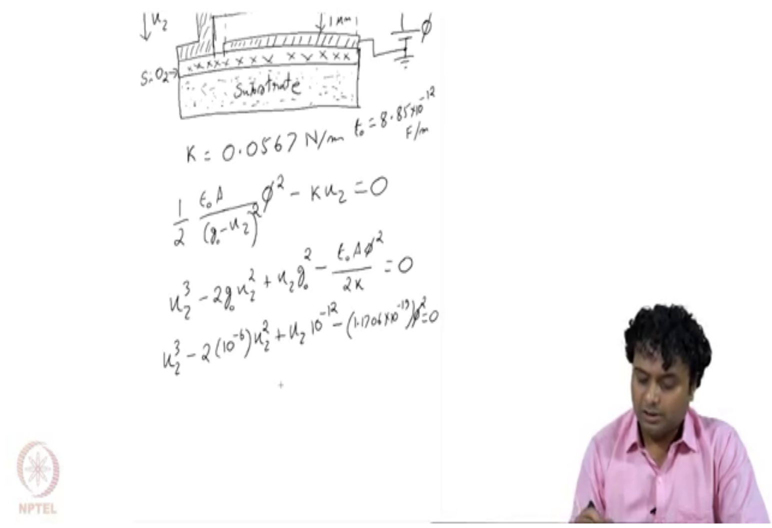

So to see now the pulling phenomena; I will describe this with the help of this example ok. So, there we have taken this kind of structure; where we have the substrate and on top of the like this substrate can be silicon or glass and then on top of that there is a there is an insulating layer; which is let us say SiO2 and on top of that there is a cantilever beam.

Now, this cantilever beam is like the metal electrode is like the plate A which we discussed earlier like the top plate. And at the bottom also there is a electrode there is an electrode which is also let us say made up of a silicone or polysilicon and then metal deposited on that.

So, these two top electrode and bottom electrode are electrically disconnected from the sample. Because, this is on a substrate this is on a SiO2 layer which is insulating. So, these are not connected electrically. The electrical voltage phi is applied from the is applied from a battery externally ok. And now, let us consider that the overlapping area of these two electrodes are the length is about 300 micron and the width is about 5 micron ok. And the gap between and the gap between plate A and plate B is 1 micron.

Now, we already know this expression; that is K for this cantilever beam, the stiffness constant of this cantilever beam which we can get from the mechanics. What we need to say? What do we need to find out? We need to find out the pulling voltage and also the how much gap it can deflect, how much it can deflect without actually pulled in on the substrate. From our earlier from our earlier analysis we know that the electrostatic force in this case will be half of epsilon r epsilon naught into A right. So, let us consider in between the these two plates there is air.

So, no epsilon r only epsilon naught and that becomes epsilon naught A divided by d minus y whole square, earlier we wrote right and here in this case it will be let us call it gap g 0 minus u z. So, I am considering the deflection, considering here; the deflection in this direction is u z ok, this positive u z. So, u at the while the beam is not deflected u z is 0 is and while the beam is completely on the plate like the bottom plate then u z is equal to g 0 into phi square right.

And at pulling condition the electrostatic force and the mechanical force will be same right minus K into u z is equals to 0 right and then we can simplify it to this expression u z cube minus 2 g 0 u z square plus u z g 0 square minus epsilon naught A phi square divided by 2 k equals to 0. And then we can write down all the values as we already know. So, it becomes u z cube minus 2 into g 0, g 0 is the gap the which is one micron.

So, that is 10 to the power minus 6 into u z is our unknown variable plus u z to g 0 square means, 10 to the power minus 12 minus. So, eps A we already know A is 300 into 300 micron into 5 micron that is 1500 micrometers square right. Epsilon naught also we know that is our a air permittivity. So, let me write down the epsilon naught value. Epsilon naught value is 8.85 into 10 to the power minus 12 farad per meter in SI unit. So, now, we put all those values K also we know right. So, if we put all those values then we get it will come as 1.1706 into 10 to the power minus 19 into phi square is equals to 0 ok.

Now, important point to note in this equation is u z is a like a it has cubic power right it is a power of 3. So, this is a cubic equation and; that means, it can have three roots correct and as all the other terms unknown, only the unknown value is the phi and putting different values of phi we can get different solution of u z right. So, let us put phi different values.

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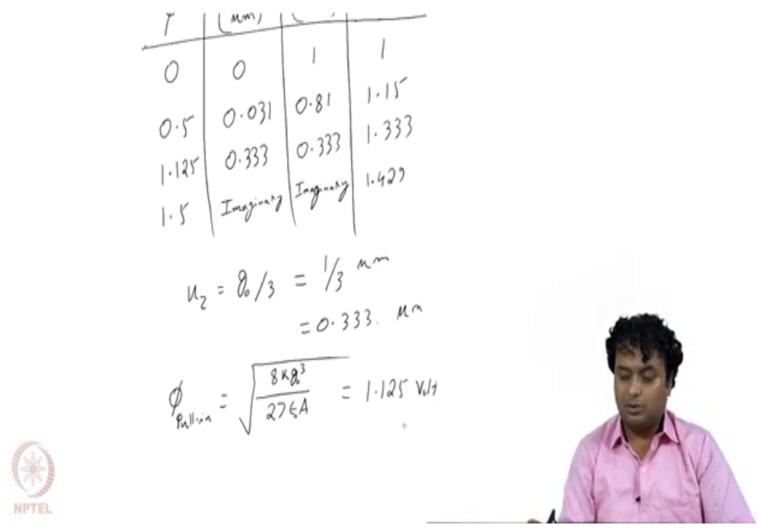

Let us say this is my phi and then I will have three roots right. So, this is let us say uz 1 then I have uz 2 and then I have uz 3 right. If phi is equal to 0.5 volt. The first case let us see that what happens if phi is equal to 0. Phi equal to 0 then 1 root will be 0 and other two root will be 1 1 right. And that you can see from this equation also, if you put phi equal to 0 then it becomes like u z cube minus 2 into 10 2 into 10 to power minus 6 u z square plus uz into 10 to the power minus 12 equal to 0. So, uz goes out. So, you get a uz equal to 0 1 root and uz minus like one micrometer as another root.

So, you get 0 1 1 and this is these all are in micrometer, all are in micrometer. So, this will be 0 0 for 0 voltage; 1 root will be 0 and other two roots will be 1. Now what happens for 0.5 volt. For 0.5 volt; the roots will be 0.031, 0.81 and 1.15. Now, see in this in this voltage combination; all the tree roots I have got. Now, here the 0.031 micrometer is possible, because this is below 1 micrometer. This is also a second root also is possible, but the third root is 1.15 micrometer.

Now as the gap itself is 1 micrometer 1.15 micro meter deflation is not possible. But, so, this is one of the impossible roots. But, mathematically you cannot mathematically you cannot actually avoid this root, but from physical interpretation you know that more than 1 micron deflection is, it is not possible because the gap itself is 1 micro right. But, while I am putting it into this equation; we are not considering that that is the maximum value of u z right.

So, you can get that root. And this is the root A like in this diagram, in this graph. Let us go back. Is root, the first root is like this A point and the second root is like B point right ok. Now, take another voltage. 1.125 volt 1.125 volt and in that case, you will see that the two roots will be 3 3 3 like this right. And the second root also you will be like 0.33 and the third root will be 1.333.

So, here at the third root is definitely not possible, but the first two roots are possible and this is the case where actually, y is equal to or the uz is equal to gap by 3. Because, gap is one micron right and in this case, we are getting both the roots as gap by 3 or g 0 by 3. So, this is, we go back to the. So, this is this case like V 3 out or my Vp. So, from now from this root itself we can say that; there is Vp is 1.15, but anyway we will actually derive that also, but this is this case right. Where the voltage is a Vp or equal to the pulling voltage and it is exactly touching at this point. So, both the roots are merging at 0.333 and then the third root is any way impossible root.

Now, take less and more voltage than that; then, if we take 1.5 volt we get and this is also imaginary 1.429. So, if we take more voltage than pulling voltage; then already, we know that there is no stable condition, means; there is no point where the electrostatic force and the mechanical force can be same. So, we get the imaginary root another root is more than 1 micron gap which is also not possible. So, there is no possible solution for the case where it is more than the pulling voltage while the gap is more than 0.333.

So, now, we will calculate that pulling voltage. And for pulling voltage you already know from this expression. Already know that; for y will be or the deflection will be less or equal to gap by gap by the 3 or one third of the gap right. And then; if we right here then we can write that.

At pulling condition u z is equal to g 0 by 3 or equal to 1 by 3 micron, it is say 0.333 micron right. And in that case, we know that; Vp or phi pulling is equals to the square root of 8 K g 0 cube divided by 27 epsilon A right. And here, if you put all the values because, epsilon naught we already know, and then A is 1500 micrometer square g 0 is 1 micron 1 micron and K is already given, K is given as 0.057 newton per meter and, if we put all those then you get phi pulling is equal to 5 volt, 1.125 volt.

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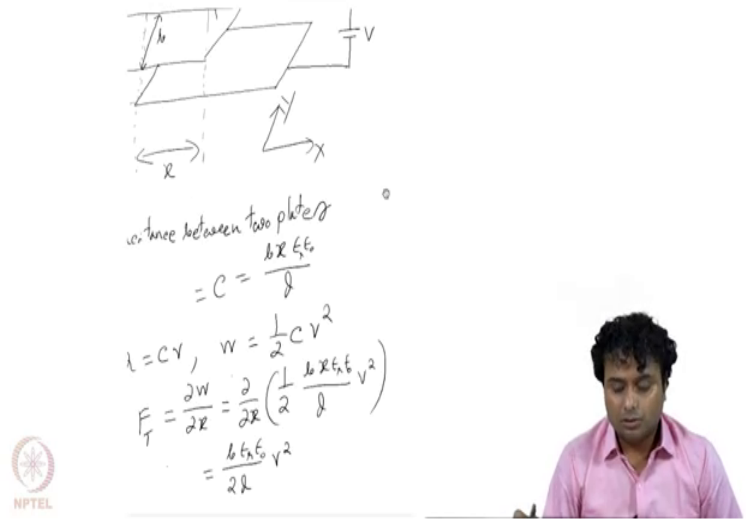

So, before going to the next topic; we will revisit once that transfers displacement case where we have seen that if the two plates are moved parallel to each other like this top plate and a bottom plate. So, one plate is fixed and another plate is moved in transverse direction then the overlapping area actually changes.

And there we saw that the transfers force is equal to b into epsilon r epsilon naught divided by 2 d into V square. Where, this b is width of the plate and d is the distance between them and V is applied potential and epsilon r are the permittivity. Now these concept we need for our next case which is a comb actuator.

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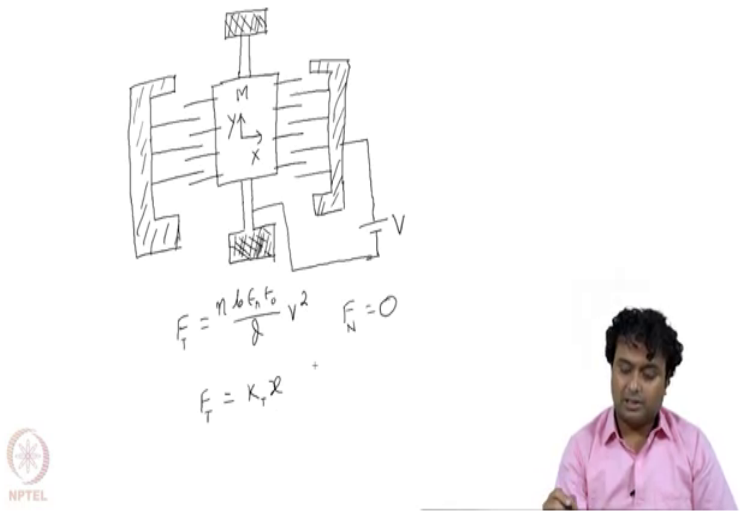

So, what is a comb actuator? comb actuator is a structure like this. So, we will draw the structure. So, this plate is fixed at this to support. So, this shaded region at the points or at the beam is fixed. Now, there are some comb kind of structures connected in the both side of that proof mass right and these are like fingers and then we have some static fingers also.

So, similar kind of structure is present on the other side also. So, this mass M can move in X and Y both the direction ok. And the voltage is applied between the static combs and the moving combs. So, this combs which are connected to the poof mass are the moving combs right, this combs and the other combs; which are connected to this static plate cannot move.

Now, if we apply a potential difference between the static comb and the moving comb. What will happen because of the electrostatic attraction, it will pull the it will pull the device or the actuators towards it right. And the force F will be equals to b into epsilon r epsilon naught divided by d V square and that is this is for one this is for only one pair right and there are let us say n number of combs.

So, it will be n b epsilon r epsilon naught divided d into V square and here the important thing is that this is V and shall important thing is; this b is the uh width of width each comb means, it is out of the plane out of the plane like the in the paper out of the out of the plane displacement here the beam the this b is the width of the beam means; if this is the comb, if this is the comb and if this is moving in between the other two combs like this, then this b is this width right. Which is perpendicular to the plane of the paper, because I have I am drawing it like this.

This is the plane of the paper and I am drawing on this right and this is the comb which is the static comb. And then this structure of the moving combs are coming towards it and going away from it like this right. So, this b is that width; which is actually, which is actually perpendicular to the plane of the paper right, this way. So, important point to note from this expression is that; the force is only dependent on that velocity square. This n is the number of comb this b is the like width of the beam which is also like beam geometry only. Epsilon is material property and then d is the gap between each comb and that is also geometry that also fixed and comes from the geometry.

So, there is no dependency on the uh; how much that beam has, how much the combs have moved right. So, we can put uh, we can decide the force we can decide the force, but by just changing the voltage or this V right. And again, if we know this these are the two fixed points and for these beams; if we know the stiffness constant then, for displacement let us say in X direction; we can write that this is so, this is transverse force transverse force is equal to transverse stiffness constant into this is the x which is the displacement.

So, accordingly, so whatever displacement or whatever movement we want for the proof mass; that we can that we can implement by just changing the voltage ok. And another point to note is that these combs the static combs and the move away combs are and in such that; there is no force no resultant force and in the y direction. So, in this direction no force, so that is F normal or the normal force is to 0.

So, there will be no movement in the y direction only the x direction movement will be there and that we can control by applying the external voltage and this is called comb actuator. This is a kind of device where we can actuate it by applying some electrical voltage according to our requirement.

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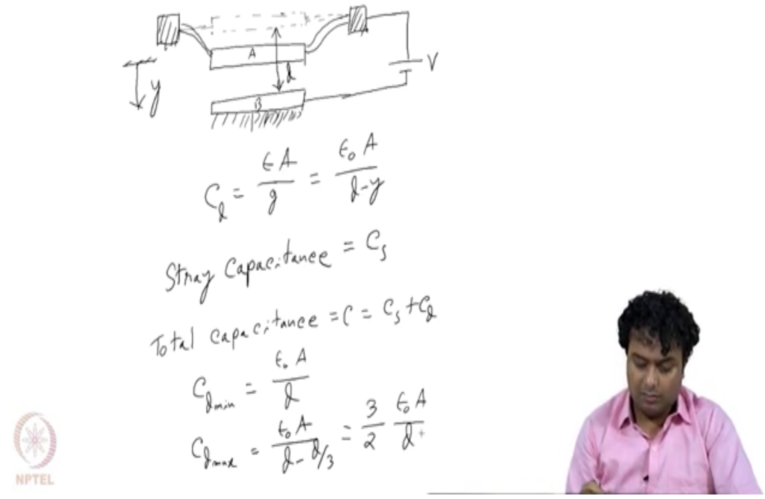

So, we have already seen this structure right or two plates are there the bottom plate is fixed and the top plate is moving and you can apply some potential V let us say that plate was initially here the actual distance between the plates while it is not deflected it is d. And then the deflection is taken in this direction it is y right deflection is y ok. Now, this kind of structure also can be used as variable capacitor. How?.

If we apply, if we apply different potential then the top electrode will be attracted towards the bottom electrode and accordingly with different kinds of potential; we can make it stable at different position as we have seen that for different kinds of voltages as we apply. We have seen that for different kinds of voltages we get different roots right we get different possible roots.

And there, we can actually make the cantilever or the top plates stable at different position. Now, if the if the cantilever is stable at some position then that if we measured the capacitance in between those plates then, it is different from its earlier position right. Because, capacitance c is equal to epsilon a by d like the parallel plate capacitor formula where a is the area of the two plates and d is the gap between the two plates right.

And here, C is equal to at any position where y is equal to the deflection is y. We can write it as let us assume only air is there epsilon A by d minus y right. Now, because of the external circuitry; there is some stray capacitance also. So, let us call that stray capacitance Cs. So, this is because of the external circuitry, because of the context and everything else ok. And the device capacitance, so this is the device capacitance Cd ok. So, this is the device capacitance Cd ok. So, this let us not call it d, because our main gap is d whatever is the gap between g whatever is the gap at that position while it is deflected by y. So, that is d minus y right ok.

Now, the total capacitance is equal to C equal to C s plus C of device; where stray capacitance per as the device capacitance. Now, this Cs is from the external circuitry. So, this cannot change anyway, but this device capacitance can change because of the different gap or different y. And now, what is the minimum value of device capacitance? Cd min is equals to epsilon naught A by d right because y can be 0, deflection can be 0; while it is not at all deflected. So, this is the Cd min right.

Well what is the Cd max? So, that we know from our pulling phenomena. Because, from our pulling phenomena; we know that, it can the gap can maximum can be d by 3 without snapping on the substrate right. Because after that if the gap is more than d by 3 like lesser than d by 3 then the top plate will snap onto the bottom plate right. So, Cd max will be epsilon naught A d minus y or maximum value of y and that is d by 3. So, that is 3 by 2 into epsilon naught A by d right.

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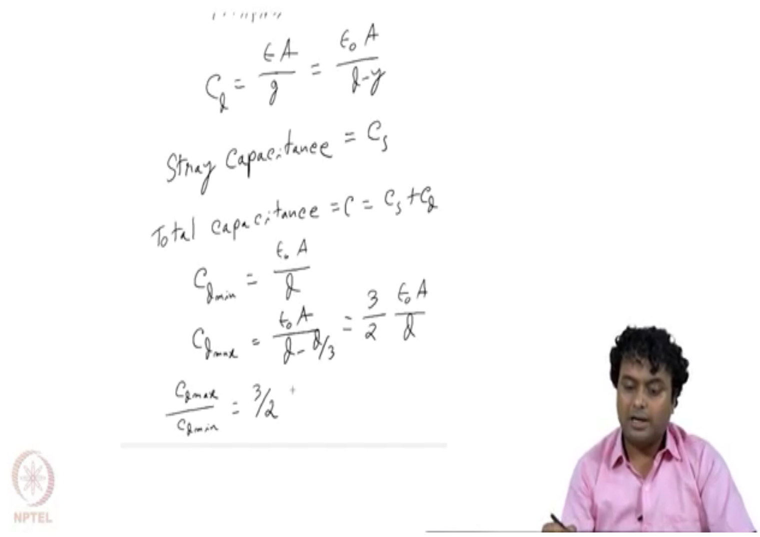

And from that, you can also write that; Cd max divided by Cd min is equals to 3 by 2 right. So, we can actually, we can actually apply different voltages and can make the top plates stable at different position and accordingly, can change the capacitance of these two parallel plate and this is called variable capacitor.

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