A Brief Introduction to Micro Sensors
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Lecture – 08
Electrostatic force

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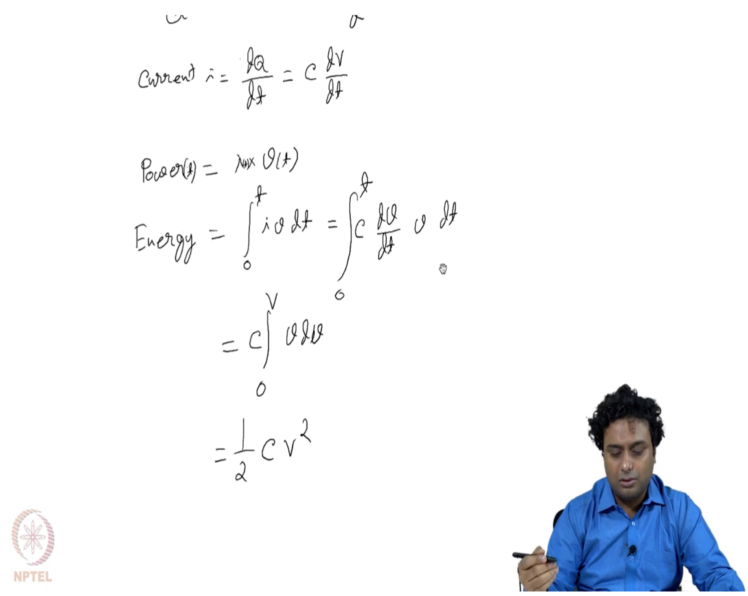

Hi. So, in this lecture we will go a little deeper in electrostatics. In the last lecture we have seen that for a parallel plate capacitor, the electric energy the energy stored was half C V square right.

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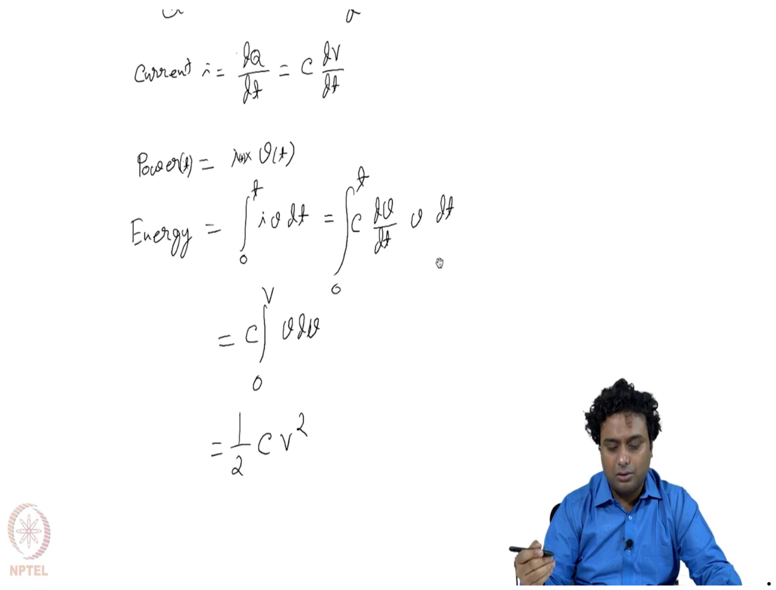

Now, we go to the a like little more description on this right. So, there you can see that these are let us say 2 parallel plate capacitor. And, let us say the area this is equals to A ok. And, then I am applying potential V and I am applying potential V. And, then this is the side view, or the cross sectional view and see this is the electric fields ok. And so, let us say plus Q charge is a at the top plate and minus Q charge is at the bottom plate. So, according to potential is V.

So, the according to the formula we know that C or the capacitance is equal to epsilon A by d. Now, what is this epsilon, this epsilon is actually epsilon naught into epsilon r, A into d, A by d ok. And, this is dielectric constant right, the electric permeability right. Epsilon r and for if it was like a just the vacuum or the air, then we would have written only the epsilon naught.

And, then we know that energy is equals to half of C into V square. Now, we know that what is force?

Force is actually d F like if the work done is W, then d W d r or the energy first derivative of energy with respect to the distance or the displacement ok. So, the electrostatic force between these two plates will be this is F normal; that means, the normal force, this electric force ok. This is the electric field direction and also this let us say this is the electric fold, electric force direction also ok.

The normal direction from one plate to the another is equals to W by d and what is W? W is the energy stored. So, that is half of C V square divided by d ok. Now, what is C F N equals to half of, C is equal to epsilon naught epsilon r A by d. So, this will become d square and on top V square ok. Now, one important point to note here is see this epsilon naught is very small almost 10 to the power minus 8 or in that order, into minus 8 to minus 12 order, in different like in the system.

Now, these force is usually that is why this force is very small, because d in macro scale d is also large, let us see if it is like 1 centimeter or so, and then area and a voltage everything is in that level then, everything is in that scale, then because of the epsilon naught this is very like the normal force is very small and the electrostatic force between these 2 plates.

But, as we do miniature direction then this d can become smaller and smaller. So, if we have 2 plates in a separation of let us say few nanometers, then in that case I am getting a 10 to the power minus 9 at the bottom ok. That 10 to the power minus 9 takes care of that epsilon naught and because of that the force can be actually very high. So, we will see the we will see that with some example later on. So, now this is the normal force let us go to the next case.

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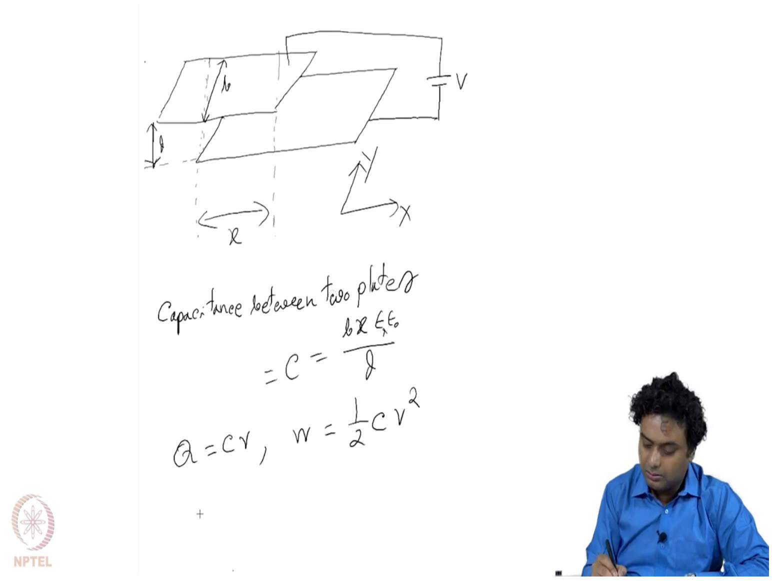

And, next case is like what is the transverse force? So, let us say I have 2 plates like this. So the here these two plates are of exactly same geometry means, both are having the same breadth length, width thickness etcetera. And, they are separated by some gap d. The difference from the last case is now they are not exactly on each other. They are they have a overlapping area of x that is the bottom plate let us say both the plates where at the same place initially then the bottom place is moved away little bit so, if so, it is something like this.

So, this is the top plate and this is the bottom plate and then I am moving the bottom plate. So, you can see it is like this. So, the overlapping area where this is these two plates can form a parallel plate capacitor is small and it is smaller than it is actual area and that is d into x right, d into x ok. This is the axis is placed because the axis system also.

So, this is X this is Y. Now, the capacitance is equals to C is equals to b x epsilon naught divided by d. Capacitance between these two plates is b x epsilon r epsilon naught divided by d. Now, Q equals to C into V and W is equals to half of C V square right.

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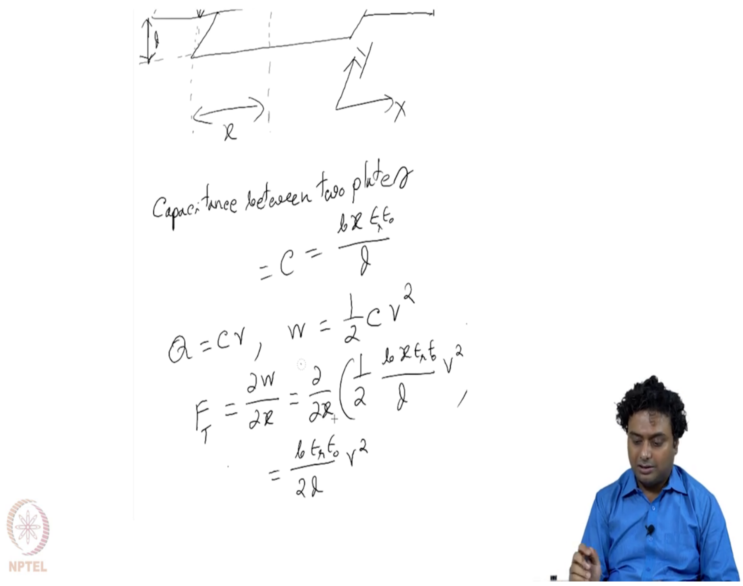

And, a tangential force because or the force in the x direction. So, that is the direction of movement also. So, tangential force F transverse or the transverse directional force F transverse is equals to delta w by delta x, let us write it as del del x into half of CV square. So, what is half CV square half of d x epsilon r epsilon naught divided by d divided by d into V square right ok. So, if we do del del x then we get b of 2 d b of epsilon r, epsilon naught into V square.

So, now here you notice that the displacement is x right. So, as I am moving the plate my transverse force is generated, but it is independent of the movement, because transverse force whatever be the displacement, the transverse force ultimately is a constant with respect to x. So, the transverse force is same wherever how much is the value of x irrespective of that.

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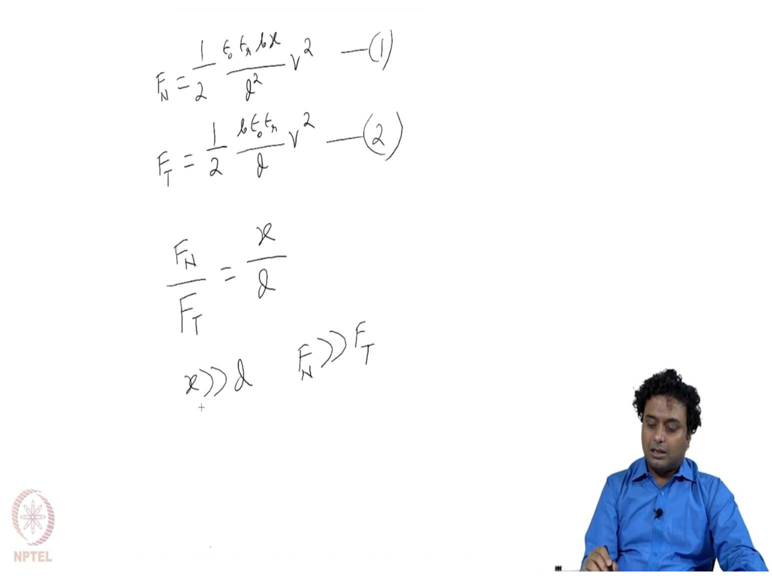

So, let us write down both the normal force and transverse force again ok. So, we got that normal force F N is equals to half of epsilon naught epsilon r b x by d square and at the top it is V square. And, transverse force is equals to half of b into epsilon naught epsilon, epsilon, epsilon naught epsilon r, then below it is d into V square ok. Now, first of all you can see that the normal force is dependent on the distance between the plate right, that is distance between the plate that is V square.

So, as we move the plate, normally or like in vertical direction the force changes right, because the d will change, but the transverse force is because of the horizontal movement of the plate right, but this is not changing with respect to the displacement of the plate ok. So, this is 1.2 note and from these 2 equation, let us say we call it 1 and 2.

So, divide one by 2 then we can get that F N by F transverse is equals to x by d right, x by d. Now, in this usually, what is x is the displacement right, how much is we are shifting one plate from another. So, usually for micro scale systems x are very very greater than d ok, x are very very greater than d, because the gap between 2 plates we keep it very small.

So, in that case we can write that F N is very very greater than F T or the normal force is much higher than the transverse force. And, 1.2 note here is this x or the lateral movement is actually we control by like our designed how we are making the device and how are we are going to move it etcetera. But, the transverse force while the gap between the 2 plates is gap between 2 layers. So, it is there are some micro fabrication or fabrication constants, because of that we cannot make it very very high ok.

We cannot make it sometimes very high; it will be in micrometer or in like 1 micrometer or even few nanometers or like in nanometer scale ok, few 100s of nanometer etcetera. So, usually the d which is our gap between 2 plates or actually gap between 2 layers as such of our device is very small.

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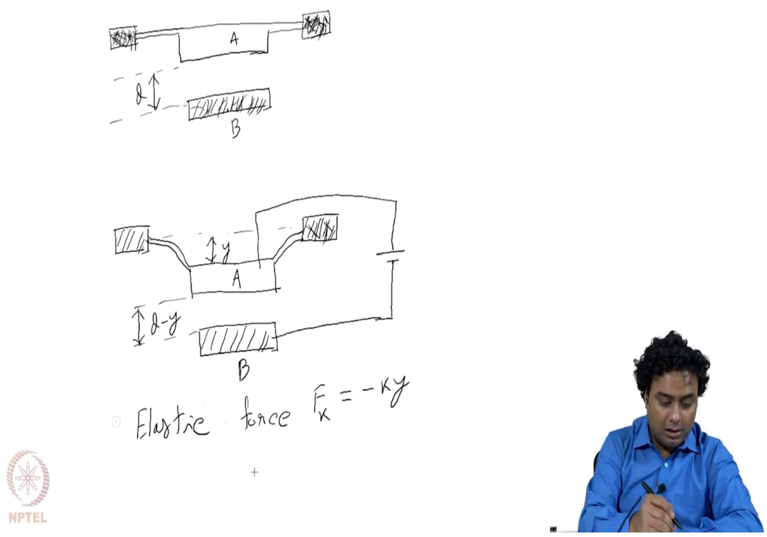

Now, once we know some basics of electrostatics we will try to couple the electrostatic force with the mechanical force, what we learnt in our previous modules? So, I am first drawing two different cases. So, this is a fixed plate let us say this call this A and B. So, A is A is a plate which can move ok. And, this is supported by 2 beams which are connected in the opposite direction by some fixed support as it is shown by this shaded area. And, this B plate is already fixed at the bottom ok.

At present there is no electrostatic force, which these are not electrically connected. So, there is no force and the beam is like let A is it is original position and let us say the gap is between gap between them is d, between these two plates as is d ok. Now, what do we do we connect electric potential to this. So, let us see what happens?.

So, these are the supports and the bottom plate anyway will not move, because it is fixed it cannot move, but the top plate will come down because of electrostatic attraction. So, this is just because of the drawing it became like that, but no beam actually bends like this. So, anyway the point is that the bottom that the top plate now come down or become even more closer to the so initially let us say it was here and then it comes down by some distance y.

So, now this gap which was earlier d it becomes d minus y right. So, now some electrical electric potential is connected, some electric potential is connected ok. So, these two are the fixed support, this one and this one are the fixed support. And this is a fixed plate which is electrically connected to the ground and the top plate is connected to some positive potential ok.

Now, these are there will be electrostatic attraction and because of that the top plate A comes even closer to B. Now, at this condition like while it is bent, what is the electrostatic force acting on that? Electrostatic force let us call it F k is equals to minus of k y. So we need to put direction in one side it should be positive and another side it should be negative.

See, this y is the deflection right. And, we know that force is equal to k into x where k is the stiffness coefficient let us now, let us assume that this plate a whatever it is whichever beam it is connected to the effective stiffness constant is finally, k. So, the force is acting upward and that is minus k y sorry I wrote wrong here it is not electrostatic it is elastic force, that is the mechanical force that is the mechanical force right. So, it is elastic force. Now, we will see what is the electrostatic force?

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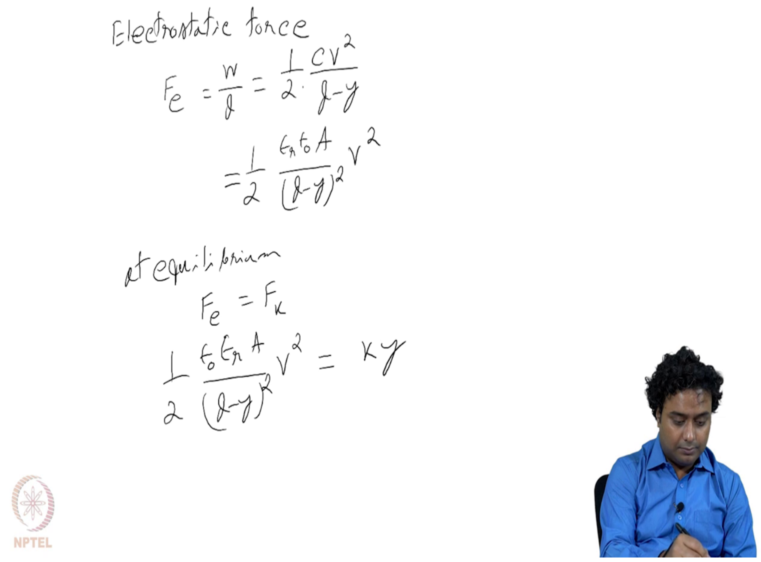

So, electrostatic force and as we already have seen that is our normal force right. Because, here the both the plates are exactly on each other and there is no transverse movement or horizontal movement, only the vertical movement. So, the force is w by d right. So, electrostatic force is F e equals to w by d is equals to half C V square divided by d minus y, because the gap is now actually not d, it is d minus y right.

And, C is also can we C is can C also can be written as half epsilon r epsilon naught A divided by C is now, also epsilon A by d that is d minus y. So, d minus y comes from the capacitance also. So, it becomes d minus y whole square into V square right. Now, these force that is the so, electrostatic force is acting downward, it is pulling the this A plate right whereas, the stiffness whereas the stiffness or the elastic force is pulling it upward.

So, at equilibrium both the force will be same so, force is equal to elastic force; electrostatic force is equals to elastic force. And, that gives us half of epsilon naught epsilon r A divided by d minus y whole square v square is equals to k y. So, from force balance we get that a half of half some constant into V square divided by d minus y whole square is equals to k y.

Now, if we take this d minus y whole square on the other side then we get a cubic equation of y that is y cube term will be there. And, by doing a solution or you can solve it by the usual mathematical techniques. And, then you will get 3 roots because this is a cubic equation. And, from that cubic equation you can find that which root is like, which root you can neglect or which is not a possible imaginary root and one will be like a proper root and accordingly we can get that what is the distance?.

But, to understand this phenomena more critically we need to understand, it the we need to understand this solution from graphical technique ok. So, that is what we are going to do next.

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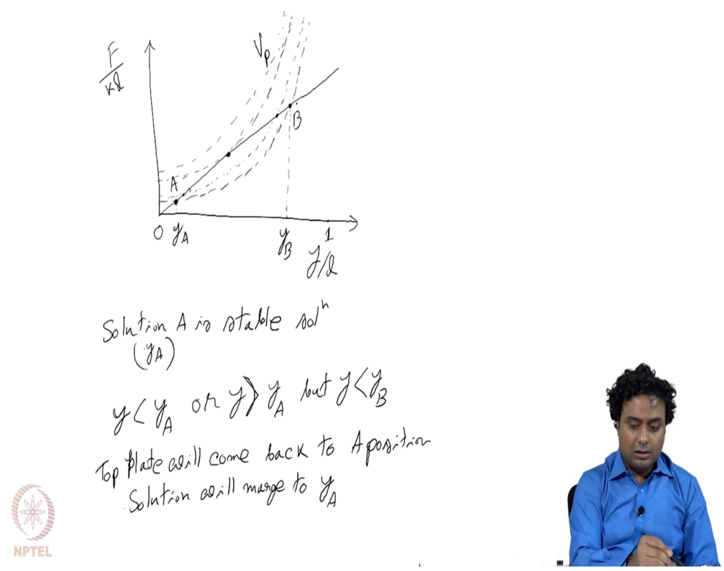

So, what I am going to do I am going to draw, force versus displacement for both the cases means elastic force as well as for the electrostatic force, and with respect to the gap between the 2 plates. So, I am just normalizing the force F by k d, where d is the initial displacement and then we have in the x axis y by d right. So, as it is y by d the maximum can be maximum y can be d. So, it is 0 to 1, the maximum possible range is 0 to 1 right. Because, maximum possible range is that y can go maximum up to d.

So, y by d maximum possible value is 1 right. And, we will see how the force goes right? Now, you see from our expression that elastic force is equals to k into y. So, elastic forces is directly, elastic force is directly related to the elastic force is directly related to the displacement by linear relation. So, we can draw this is the elastic force ok, this is the elastic force. And, it will go through 0 0, because at y equal to 0, F equal to 0 ok.

So elastic force is equal to F is equal to minus k y. So, at y equal to 0 F equal to 0 and this is directly related. Now, what is electrostatic force, electrostatic force is; electrostatic force is half into divided by like it some constant, divided by d minus y whole square. Now, d minus y whole square this is like a parabolic equation, like y square is equal to 4 a x y square is equal to 4 a x or you have a like one side y is y is having square and another side you have like power 1 and the other side you have power 2.

So, it is like y has the quadratic power whereas, force is only unity power right. So, this is parabolic equation and at y equal to 0 at y equal to 0 electrostatic force have some value, it is not 0 like elastic force, it has some value all right. So, and it is like minimum value as we as the y goes down the electrostatic force will increase more.

So, it will go something like this ok. Now, so this dotted line is my electrostatic force and the continuous straight line is my elastic force. And, there is a solution at this point right; there is a solution at this point. And, let us call it point a and there is another solution at B point right, where the elastic force and electrostatic force are same.

Now, the interesting point you should note is as initially the initially the gap is d right and there is no deflection. So, y equal to 0. So, the elastic force is 0, initially the gap is 0. So, the sorry the gap is d. So, the deflection or the displacement of the top plate is 0. So, the elastic force is 0, but there is certain value for electrostatic force. And, that will start; that will start pulling in the top plate or a plate.

Now, as it increases I mean the it is pulled down or the top plate is pulled down, the elastic force increases because deflection is increasing right. So, F is equal to k y and the deflection is increasing. So, the elastic force is increasing. So, at some point this elastic force, at some point this elastic force becomes equal to the electrostatic force right.

So, this is one of the solution, but the point to note here is that this is a stable solution why, because you see that, if it increased little bit more or if we have a perturbation from this a point little bit on the other side, or at this side at this side, the right hand side. Then the elastic force is still higher than the electrostatic force. So, the elastic force is still higher than the electrostatic force.

So, it will be able to pull it back, I mean the elastic force will be able to pull the top plate to it is original position right or pull it back to the A position, solution to the A position, but as it moves more and more or as it goes down more and more the top plate, then it reaches another point, which is another solution that is the B solution B point, but in this point if you note that the elastic force and the electrostatic force are same at this point right.

But, the interesting point is here now if it increases. If because of some perturbation the top plate goes down even little more, then what happens is then the elastic electrostatic force is more than the electrostatic force is more than the elastic force right. And, in that case; in that case what will happen, this will the top plate will get more and more pulled in towards the bottom plate.

So, this is called pulling phenomena, this is called pulling phenomena, where the top plate will snap onto the; will snap onto the bottom plate ok. Because, elastic force or the mechanical force is not is not able to pull the top plate back to it is place. Because, you see the mechanical for the mechanical force, the force is increasing with F is equal to k y, that is linear. But, the electrostatic force; electrostatic force is increasing with this power of 2.

So, it is more and more it increases and then at some point, while it is very close the d becomes 0. So, the like theoretically or technically the elastic force becomes almost infinite right. So, it will not be able to like the mechanical force will not be able to pull it back right. So, what we can tell from here that the solution A is stable solution, stable solution. Let us call this solution x A and let us call this solution sorry this is this x is we have considered y so, let y. So, let us call this y A and this is y B, the solution A that is our x A stable solution and sorry.

So, solution A y A is a stable solution and for y less then y A or y greater than greater than y A, but y less than y B top plate we will come back to A position; to A position ok. And, solution will merge to y A right.

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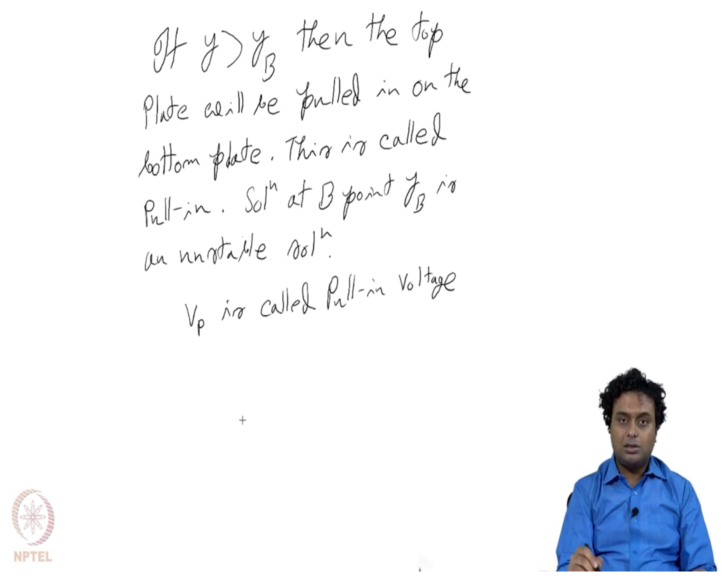

If y is greater than y B, then the top plate will be pulled in on the bottom plate or it will snap on the bottom plate. And this is called pull in this is called pull in ok, solution at B point that is y B is an unstable solution, unstable solution. Now, one important point is that ok, A solution is the stable solution and B solution is unstable, but while it starts moving it first, it will first cross A right because initially y equal to 0 and then as it gets pulled down, then it will go from 0 to towards one like y by d, if I take right.

So, it will first cross A point and then there is no reason for the plate to go to the B point. Now, what happens, what happens, if we increase the electric potential? So, that you can mathematically see that as we increase the electric potential this car will move more and more upward, this car will move more and more upward, because the constant will increase right and you will get something like this. So, the nature of the car will be same, but it will move upward right.

And, these two points, so, now the intersection is at this point A and B point is at this. And, at some point it will come at for some potential, where there will be a single touch between the mechanical force and the electrostatic force. Let us say something like this. And, this voltage where we get just the elastic force and electrostatic force is matching only at a single point this is called pulling voltage or V p.

Because, above that voltage any voltage there is no point where the mechanical force is higher than elastic force right, higher than the electrostatic force, because on above the pulling voltage, see the electrostatic force will go like this. So, at all the for all the this all the ys it is already above the; already above the mechanical force. So, it cannot be like mechanical force will not be able to pull in to restrict the pulling. But, if it is only below this point V p, then only it will be able to only it will be able to pull it back to it is original space original place that is upward.

So, then if it is only below this pulling voltage, then only these beams will be able to pull it back to it is original space or even to the less than another solution, which was the stable solution a point right. Otherwise, it will snap on the bottom plate. So, this voltage V p is called pull in voltage and the mathematical derivation about this we will do in the next class.