A Brief Introduction to Micro Sensors
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Lecture – 07
Electrostatics

Hi. So, we know that MEMS means Micro Electro Mechanical Systems and in last few modules, we discussed about that mechanical part or the basics of some mechanical formulas or some basic of mechanical concepts, which we need to use. And now, in this model we will switch to electronics or like more precisely Electrostatic part and then after giving you some basics of electrostatics, we will discuss that how a coupled electromechanical system works ok.

So, while you have a electrical force also and while you have a mechanical force also acting on a body. So, let us go through quickly and so, I will not go into very much details, just a few basic concepts of electrostatics.

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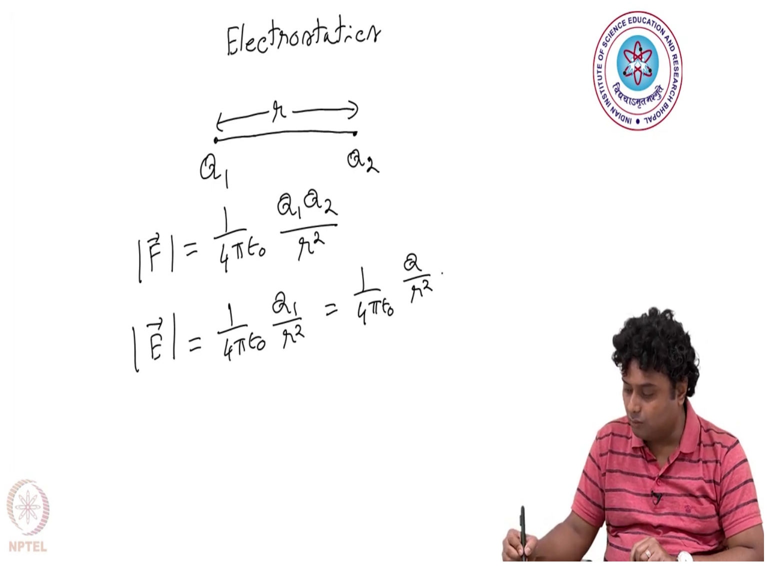

So, most of you are already aware of this part, actually in during your 12th standard or some basics undergraduate courses on an electrical field or electronics field ok. So, first we have discussing about Coulombs law. I am not going to write down the full law, but you can see here is I will just brush up the formulas once. So, let us say this is two charges; Q 1 and Q 2 right I am not writing any sign for them and they are list apart by a distance r, then what is the magnitude of this force like, the electrical electric field or force on.

So, now what is the force that is exactly one body, on charge on another that is 1 by 4 pi epsilon naught into Q 1 Q 2 by r square and this is you are already aware of right and not here we have mentioned that the magnitude of the force ok. I am not writing any reaction just for simplicity and then this is the force. Now, what is the field? Force power unit charge. So, field let us say electric field on one charge due to the another let us say on a Q 2, because of the Q 1 charge is 1 by 4 pi epsilon naught into Q 1 by r square, because force by Q 2 great.

So, this is the electric field or in general you can write for a charge Q 1 by 4 pi epsilon naught into Q by r square. Let us assume the charges are Q. Now, this is also you are aware of that work done is equal to our energy is equal to force into displacement right.

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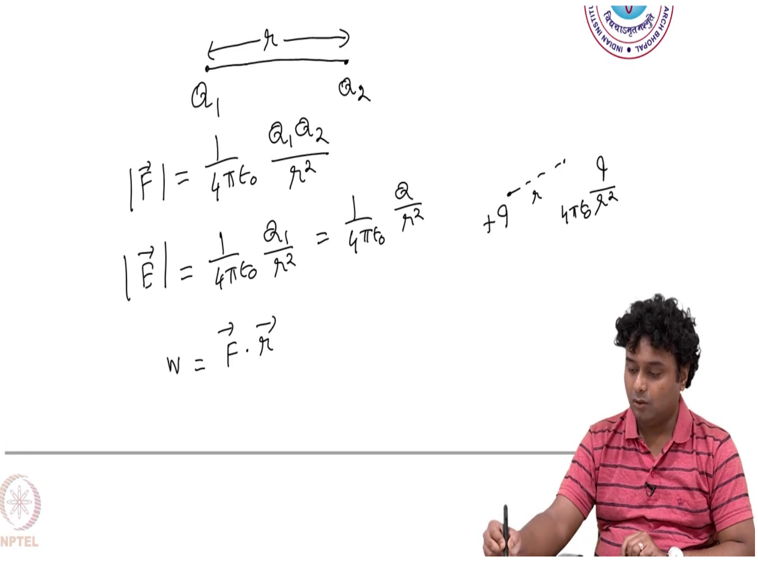

So, here if I have a charge Q at this point right let us say it is plus Q and then we have electric field at some point, which is at a distance r is equal to Q by r square into the consents are their 4 pi epsilon naught ok. Now, at various different at different distance is like if this r changes, then this electric field also changes right. So, while we are calculating work done to bring a charge from infinity to at some point r ok; so, let us say we have bringing another charge like let us say 1 Coulomb of charge from infinity to some point.

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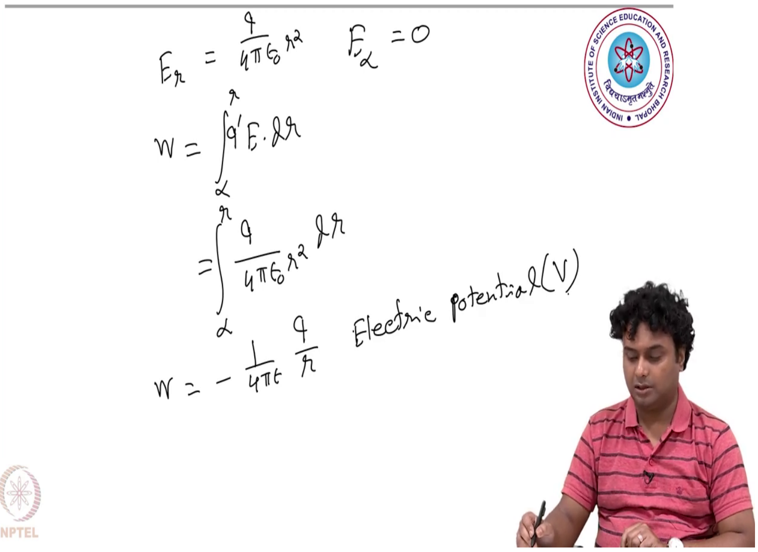

So, what is the electric field at r is equal Q by 4 pi epsilon naught r square? What is a electric field that infinity? Is equal to 0 correct, we can calculate it as r square r so, in a denominator there will be infinity so, it will be 0. To calculate the total work done what do we need to do? We need to integrate it. So, W is equal to E dot d r right, where r is equal to infinity to sum r. Now, the point is let us say if we consider the charge of Q which a like another charge if it is, 1 Coulomb charge, then it is E dot dr, if it is like some charge of Q prime, then I need to write Q prime E dr right.

Now, this you have already done, if you put the electric field then let us assume this is 1 coulomb charge only, then it is Q by 4 pi epsilon naught r square dr into infinity 2 r and then if you do the integration, you will get W is equal to minus 1 by 4 pi epsilon naught to Q by r and this is called potential right, electric potential due to the charge Q at a distance r, we will call it V electric potential V.

Now, another important concept you need to understand is this electric field or potential, whatever we are calculating is for open space or like a free space right, where there is no material, but if we have a material like a some metal or some dielectric or any kind of material. Then depending on the material properties there will be some kind of polarization. Polarization means that the electrons and protons inside that material will arrange themselves by some kind of arrangement by some due to the external electric field and, because of that the electric field inside the material will not be same as the external electric field ok.

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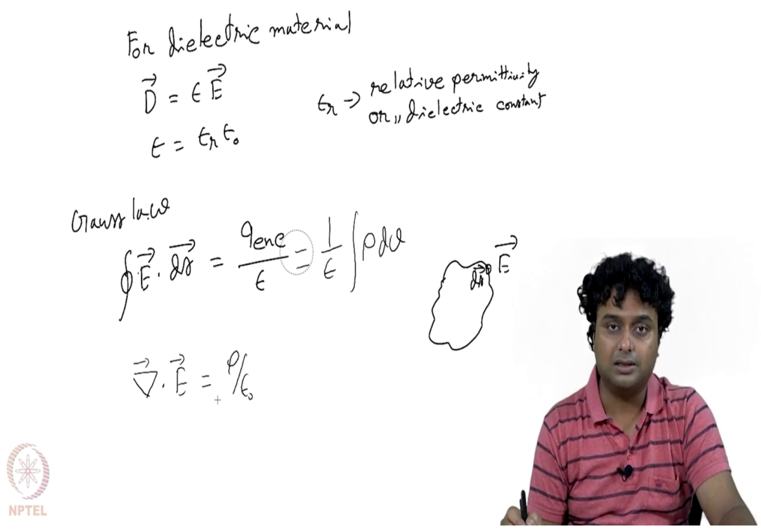

So, for dielectric material, we consider the electric field inside is D, some displacement field, then it comes as epsilon into the electric field create outside or the external applied electrical field that is E ok. So, and this epsilon is equals to epsilon r into epsilon naught. So, and this epsilon r is equal to relative permittivity, relative dielectric constant ok. So, this is just you need to remember that while we are applying some electric field, if we have a specific material, then the field inside the material may not be same as in the air or in the free space ok.

Now, we will discuss another very important concept of electrostatic, it is Gauss law. Let us assume, we have some charge distribution, and because of that we have this material and they are the electric field at any surface point is E and this surface is let us say ds ok. So, this surface is ds and the electric field this point is E. Now, this surface is also, you know that the surface is also a vector right. So, this has its direction perpendicular to the surface.

Now, according to the Gauss law, surface integral is E dot ds is equals to the charge enclosed Q Q enclosed divided by epsilon. So, as I was saying that at every point in this or every small elemental area, we have an electric field and that electric field if we take a dot product of that like the surface vector and the electric field then E dot ds is the like flux, through that through that small area and then if we integrate about the flux, then we get the charge enclosed divided by epsilon. So, this is Gauss law.

Again, you see that the charge enclosed is equivalent to is equals to we can also write that Q enclosed is what the total charge enclosed and if the charge density is rho, then the Q enclosed is the integration over rho d V right, where d V is that small volume element. So, here we can write 1 by epsilon is equal to into rho d V. This equation of Gauss law is a like basics of electrostatics.

I am not going into the details of that, but we can use this expression while whenever, it will be required ok. And another thing is from the vector calculus we can write this expression in another like this formula in another expression right and that is divergence of E is equals to rho by epsilon, epsilon naught for the free space. So, it is also comes from electrostatics or vector calculus.

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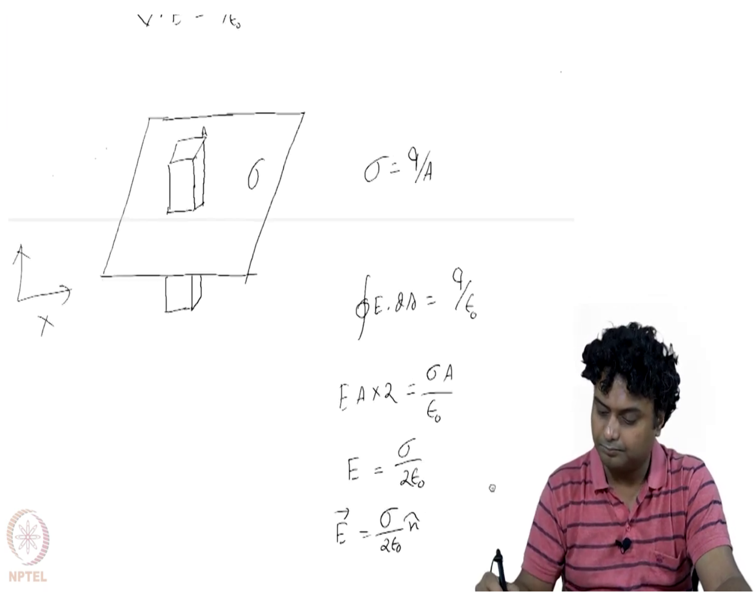

Now, we will take a specific a specific case of surface charge distribution ok. So, surface charge distribution, let us say, I assume and I assume and infinitesimally like infinitely large surface and where the charge density is sigma. So, surface charge density is sigma; that means, that sigma is equal to Q by A right, where A is the area and then icon I assume a Gaussian pillbox. So, what is a Gaussian pillbox? Gaussian pillbox is this kind of parallel pipe you can assume and this is going, let us say in to the surface and then coming out of the surface ok. So, this parallel pipe is ultimately including some amount of surface charge.

Now, we will use Gauss law. According to Gauss law we know that integration over surface indignation over E dot ds is equal to Q by epsilon naught is where Q is the enclosed charge. Now, let us assume that here, in this point the electric field is E and then for both the sides, it will be the same constant electric field why, because this is infinitely large surface charge distribution.

So, even if we move in the x y direction where this is let us say x and this is let us say y direction, then it does not change the distribution with respect to any of those points right, in a plane it will be the same electric field. So, what we can write this e is can be constant for now, according to this Gaussian pillbox let us say Gaussian pillbox have a area of A ok, let us say the area is A cross sectional area.

Now, we can write that the total flux out of this, out of this area is E into A into 2 why? Because there will be flux out of this the top side and also from the bottom side correct. So, we get E into A that is 2 A into A into 2 is equal Q by epsilon naught now, what is Q? Q is equal to charge enclosed by this pillbox and this pillbox has an area of A. So, Q is equal to sigma into A, because sigma is my charge density. So, into area is my total charge by epsilon naught.

So, E I can writer as sigma by 2 epsilon naught right and if we are the vector sign then E vector is equal to C E vector is equal to sigma by 2 epsilon naught n cap or n cap is the surface normal vector ok.

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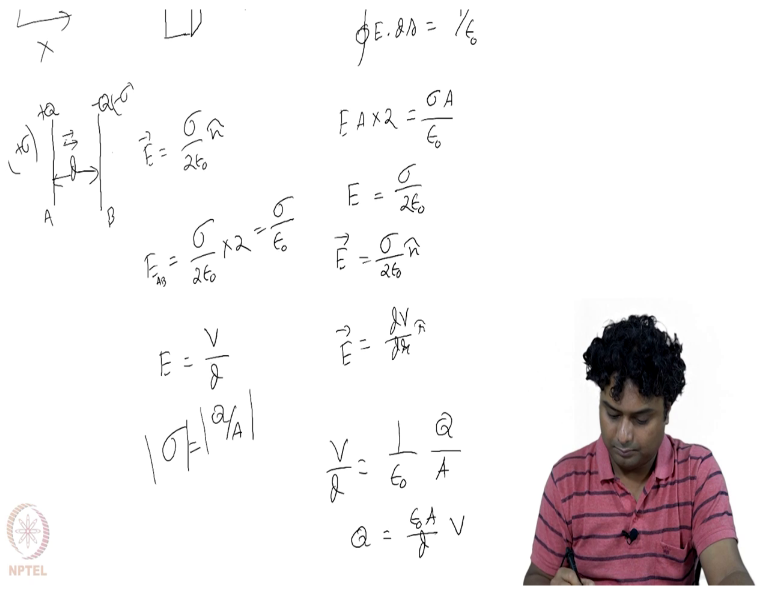

Now, let us consider two parallel plates separated by a distance d and two plates are having charged plus Q and minus Q ok. So, let us consider two parallel plates and they are separated by a distance d and each of the plate is having some charge Q 1 side, it is positive A plus Q and another side, it is minus Q.

Now, what I need to do is calculate the capacitance of this arrangement. So, if we consider as we just know discuss that for infinitely long a large plate, we can consider we can we calculated that that electric field at any point or at any distance is electric field is equal to E equals 2 sigma by 2 epsilon naught n cap right. Now, here for plus Q, let us say this is the same then we can consider that that charge distribution here, it is a plus sigma and here it is minus sigma.

Now, because of that plate a let us say plate A and B, I named that 2 plates, because of the plate a the electric field at any point is at any point in between is sigma by 2 epsilon naught n cap and that is directing in this way right, because of the plus Q charge. Whereas, for the minus Q also, it is of the same magnitude, but deduction is same, because this is negative. So, the n cap also will be directing towards the surface right.

So, the equivalent electric field inside this will be E equal to sigma by 2 epsilon naught into 2, because both the fields will add up correct say, E A B. So, that is sigma by epsilon naught. See these two plates are in parallel and the field inside the plate is uniform electric field ok. Now, as it is uniform electric field if the potential generated in between them and the potential difference in between them is B, then we can write that E equals to V by d right. Anyway, you know that E equals to any electric field, we can write as E equals to dV dr right.

And some vector will come less r cap and you can write that the first derivative of the distance, first derivative of the potential which is (Refer Time: 17:53) distance is the electric field and here, the electric field is anyways a uniform. So, we can directly write that electric field is equal to V by d and another thing is that sigma is equal to Q by A right like; mod of sigma is equal to mode of Q by A. So, if we put this put this expressions in the electric field term, then we get that E equals to V by d is equals to 1 by epsilon naught is equal to Q by A or Q is equal to epsilon naught A by d into V.

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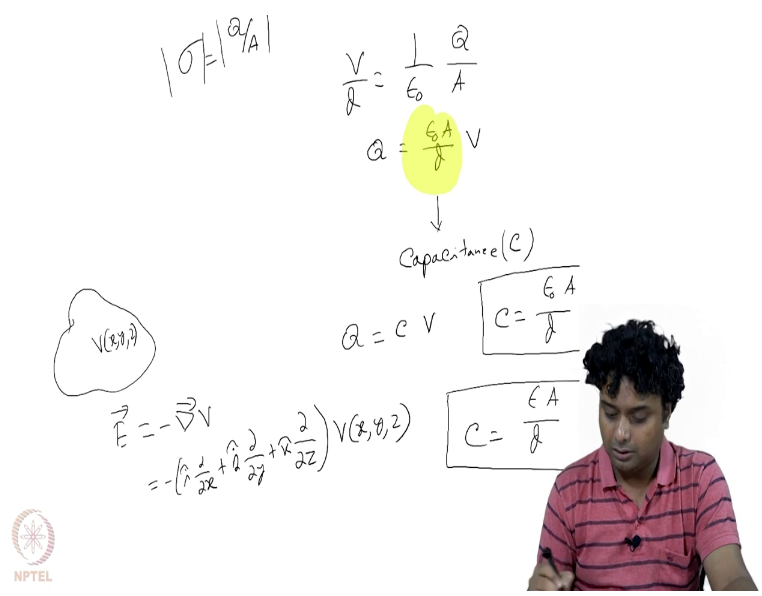

Now, this epsilon naught A by d, this term is known as capacitance C, capacitance C and if we put this c then we will get that Q equals to C into V, where C is equal to epsilon naught A by d. So, this is the expression for the parallel plate capacitor and then if we have a say here, in between in between the parallel plates, if we have any dielectric material of let us say some dielectric constant epsilon, then this expression will come as C equals to epsilon A by d ok.

As I was saying, because while you have while you have a dielectric constant or while you have a dielectric material then in that case the electric field E will not be same, rather it will epsilon into E. And, then in that case it will be replaced by epsilon epsilon naught will be replaced by epsilon, where epsilon is the free space. If dielectric constant of free space permittivity into electric permittivity epsilon naught; so this parallel plate capacitor equation will need in this course every very frequently.

So, now we will discuss a little more about the electrostatics which may be required for this particular course. So, as I was saying that electric field is equal to first derivative of the potential right dV dr. Now, if we have a 3 dimensional distribution of the potential like this that at every point xyz the potential V is different like it is a function of xyz, then in general form we can write that E equals to minus off grade V right, where V is the scalar.

But, this is the partial derivative like the del del x plus del del y plus del del z and with respect to it is like direction like, i cap into the del del x. So, these things you already know from the vector calculus j cap del del y. Thus, k cap del del z and then you have potential which is a function of xyz. So, this is one expression which we may require or while you are solving any problem, while you do not have a very simple case like point charges whether you have a distribution then calculating the electric field etcetera.

You need to apply accurately this general equation and in this equation if V equal grade V, in this equation if we apply and under first derivative right or another differential function and the delta function. Then we will get divergence of E is equal to minus of del square V right and divergence of E equals to earlier, itself we have seen from Gauss's law that is equal to rho by epsilon correct.

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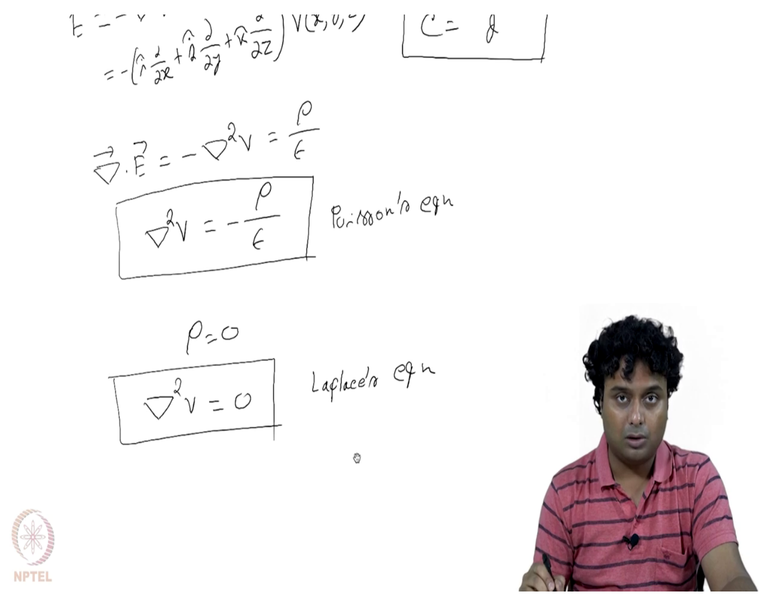

Now, del square V r Laplacian of V equals to minus rho by epsilon and this is known as Poisson equations, Poisson equation and if we have a charge distribution let us take a free space, where if we have a free space where rho is equal to 0, then you have Laplacian of V is equal to 0 and this is called Laplace's equation. So, this is Poisson equation and this is Laplace's equation.

Now, the point of discussing this is see one thing is that you know the potential the if the potential is given then from the you can calculate the electric field using this E equal to minus grade V. If you do not know the potential, if you are given the charge distribution there is you know the rho then you can calculate the potential from their Poisson equation right. And if you do not have any charge distribution, but if you are given just a boundary condition like the potential is mentioned has some point and, but the charge distribution is not right.

So, in that case also you can calculate the potential field like what is the potential or Vxyz at different point using this Laplacian equation and from that you can again use E equal to minus grad V to calculate the electric field at different points right. So, this is some basics of electric field like electrostatics and now move on, we will move on to the next part.

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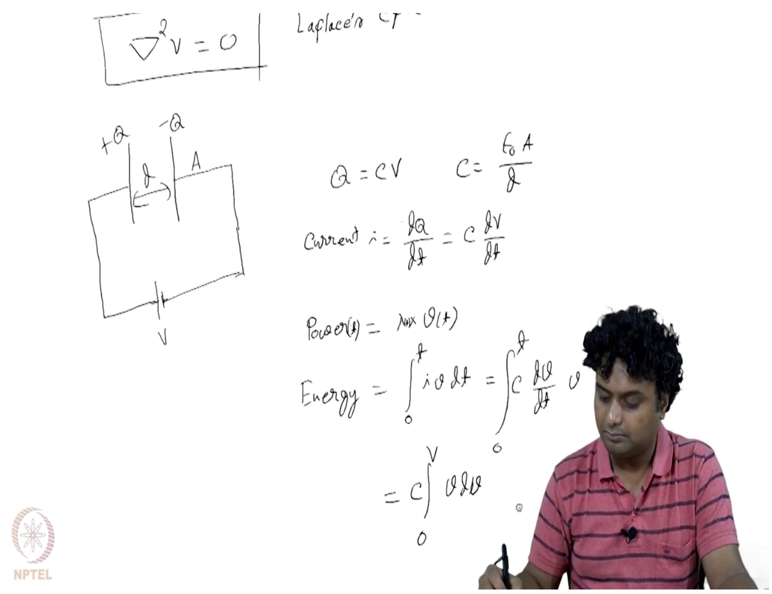

We will now, discuss one more concept on the a little bit capacitor that is let us say this is the capacitor as I was discussing. And, then you have a charge of plus Q and minus Q on this capacitor and this is the area of what the plate is A and the distance between them is d right. And, then if these plates like connected to a battery V in the potential difference when the potential difference between the plate size V right. So, then what we have seen that Q equals to CV, where C equals to epsilon naught A by d right.

Now, if this is Q then what is the current? Current i equals you know dQ dt right, time derivative of the charge, charge stored or charge flown and then we have C into dV dt, because C or the capacitance is constant and it is based on the geometry and the material right of the parallel plate system. Now, Q current i equal to C dividity. What is power? Power is equal to you know voltage into current that is power at any point t is equal to i into V that is i is current and V is voltage.

Now, this V and this i is not constant, because as the charge will flow, as the charge will flow the potential will also build up slowly let us say the capacitor initially it was not charged. So, capacitor was at 0 charge. So, the potential is also 0. Now, as we connected the battery, because of the battery now it is it size to stores start starts to store the charges and, because of that the potential starts from 0 and it build up till V what is the potential difference of the like the potential, but the battery can provide.

Now, this is the power. So, at any point t the power let us say if it is a function of time its i is also a function of time and potential is also a function of time, then what is the energy? Energy stored at some time d is sorry, at energy stored till some point t is i into V into dt integration over 0 to t right and that is C d V d t into V into d t 0 to t right and then we have let us see this t is a very large so, the capacitor has gone up to the voltage of the battery right. C you can take it out and V d V and V d V. This V is from 0 potential to the battery voltage and that is capital V.

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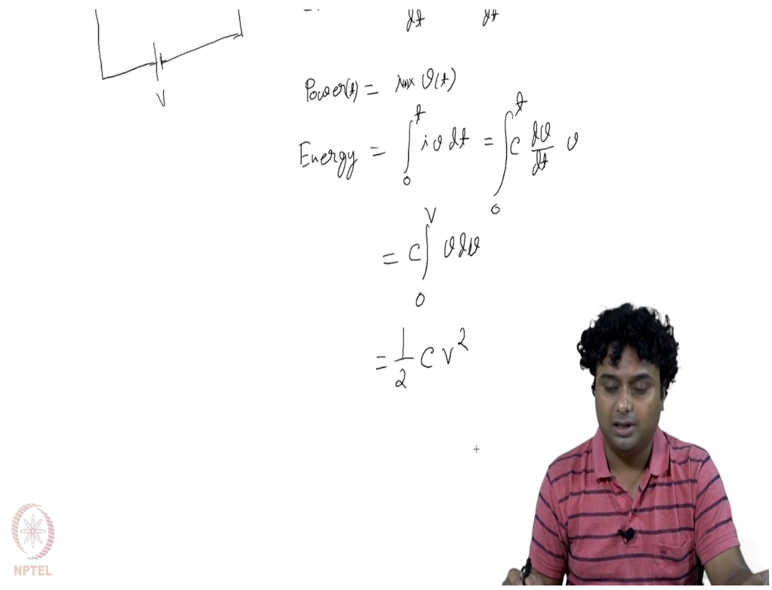

So, then if you integrate it then we get half of CV square right. So, the energy stored in a capacitor while it is just to the potential V is half CV square.