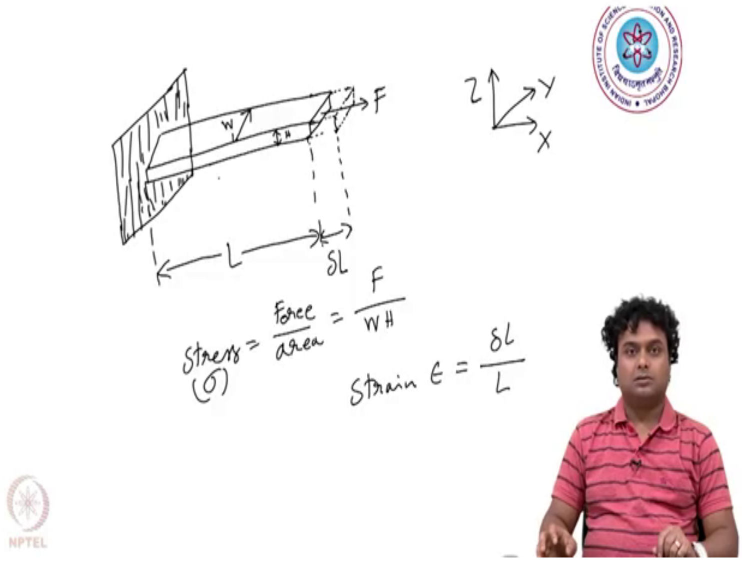
A Brief Introduction to Micro Sensors
Dr. Santanu Talukder
Department of Electrical Engineering and Computer Science
Indian Institute of Science Education and Research, Bhopal

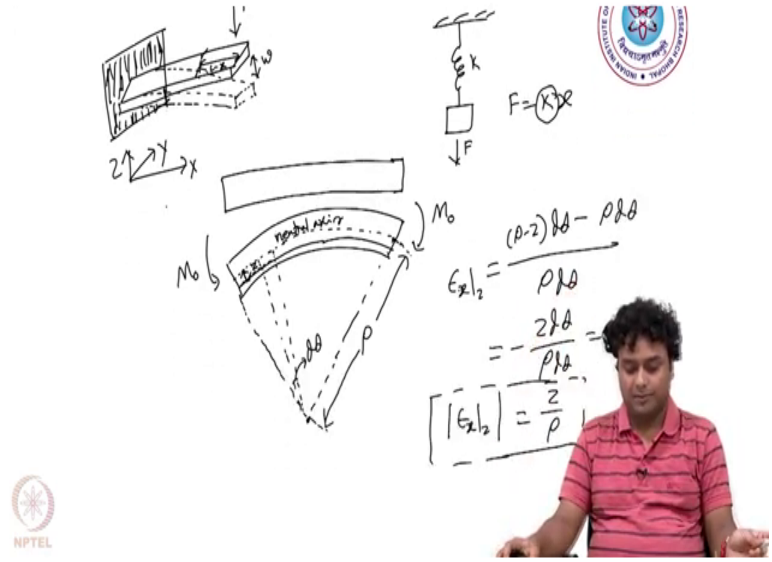
Lecture - 06
Basic Mechanics - Part 03

(Refer Slide Time: 00:26)



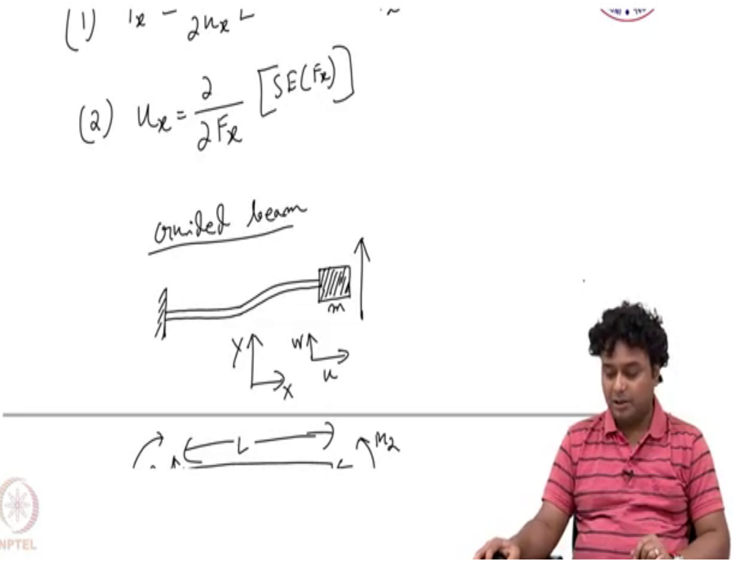
So, till the last lecture, we have already discussed the main concepts of mechanics and in that; we have mainly discussed about three specific arrangements of beams. The first one is as you can see here that the axial stress that you have a cantilever like this and then you are applying the force in this direction right you are pulling it, pure axial stress.

(Refer Slide Time: 00:56)



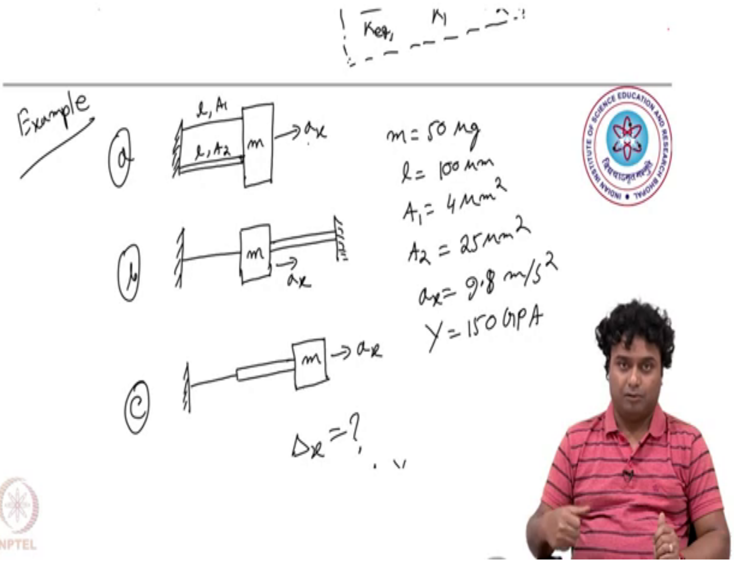
And, then we discussed about transverse stress right where I am applying the force in this direction from the top correct, from the top I am applying the force.

(Refer Slide Time: 01:06)



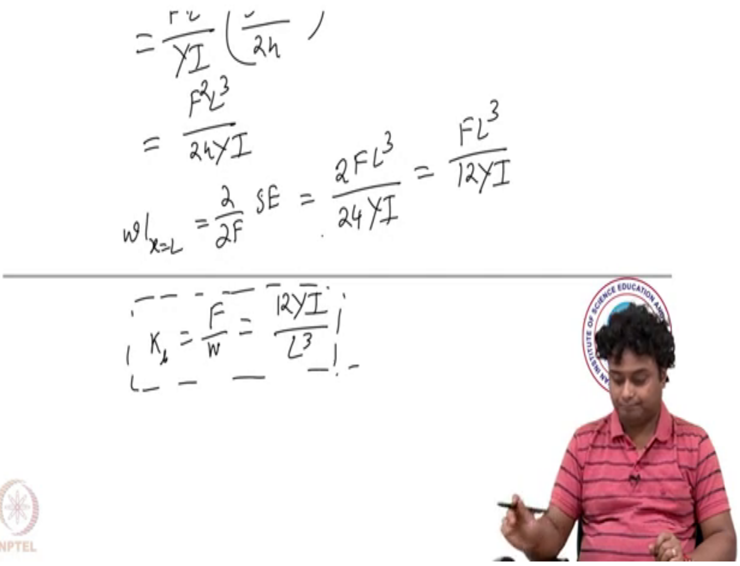
And then, the last one is that guided beam structure where, we are fixed that one in and another one is also fixed, but this fixed thing is or fixed reference frame is moving like this right. So, the beam can bend or deflect, but not at this point it cannot have a slew it is like this. And, all of these three cases axial stress, transverse stress and also the guided beam all of these cases we have calculated the spring constant. And what we do with the spring constant? Then, we saw that one example in one of the previous classes.

(Refer Slide Time: 01:48)



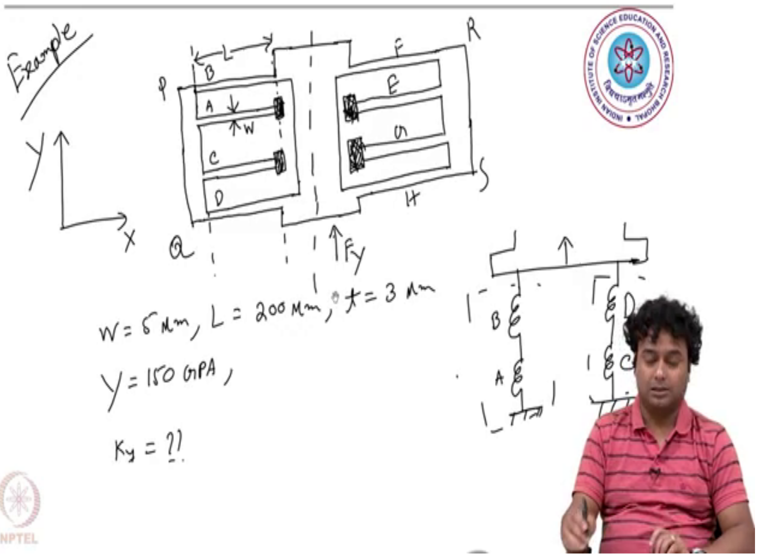
This example we show in one of the previous classes that once we know the axial like the stiffness constant for one particular beam and then whatever arrangement we have like with multiple beams just we can apply the parallel spring or the spring in series formula and can get the equivalent displacement or equivalent spring constant right. And, from that we can calculate back calculate the force.

(Refer Slide Time: 02:19)



And, last we found out like for the guided beam structure we found out this formula that K b is equal to 12 Y I by L cube. So that is the stiffness constant for a guided beam structure. And now, we will see one example where we will see that how we can apply this expression and can calculate the deflection for a particular proof mass or for a particular structure.

(Refer Slide Time: 02:49)



So, now we will see this example where we will try to explore how the guided beam structures actually work. So, for that I have shown this picture that this is the central mass with this is a proof this is called the proof mass ok. And, then this is connected to several beams in this arrangement. Now, see these beams I have named A B C D, this all these four beams are of same width, same length and thickness ok.

And across this dotted line or the central line along the Y axis both the sides are symmetrical, though according to my drawing it is not what you can assume it to be symmetrical and both the sides have same dimension same arrangements. Now, this shaded line black shaded lines are the one which are actually fixed ok.

So, this point is fixed, this point is fixed and on the other side this point and this point is fixed ok. So, these four beams at one end it is fixed and rest of the structure it not fixed ok. Rest of the structures are moving. Now, this is your slender beams whereas, these are thick beams right this side it is thick beam and the other side also it is thick beam.

List an name it something like. So, name if P Q R S this is let us say E F G H. So, as I was saying that A B C D or slender beams and slender beams means this will bend and P Q R S are like this stubby structures which you will; which you will not be able to bend or twist. Now, this proof mass is suspended it is like this proof mass is suspended and at the end this is connected to it; so, several licks right. So, this is the proof mass and then you have this arms connected and this arms are fixed somewhere. So, this proof mass is move can move in a Y direction as well as in the X direction.

Accordingly this beams; accordingly these beams A B C D E F G H will bend ok. And what we need to find out what we need to found out find out is what is K y. What is K y? K y means the stiffness constant in y direction means; if this is the proof mass and I apply some force here in this direction then how much will be the deflection right. You know that if y is equal to k into let us say whatever is the deflection in y direction that deflection delta. So, we need to find out find out that K y ok.

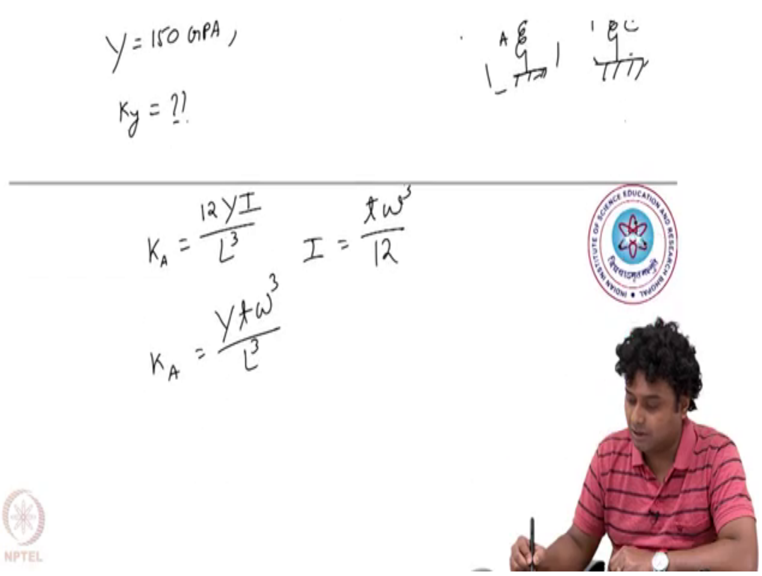
Now, how this is a guided beam as you can see here this point let us say if you take one particular beam like B then B beam is connected to the poof mass. So, one side it is connected it is like this one side is connected to these beam and these side also this side also it is as it is fixed to the proof mass it cannot have a slope there the slope is 0. On the other side it is connected to the p q like a stubby structure which also can move, but it cannot have a slope or bending at this point B.

So, ultimately if I consider the B beam across the proof mass and the P point then it will be like this somewhere. So, this is like guided movement ok. This is like guided movement. So, here the this proof mass is moving. So, let us say my this hand or the my right hand is the proof mass and the other side this is the stubby structure. So, though it is moving; this is also moving, but this is not moving as much as the proof mass is moving ok. So, but then both the side the slope is fixed and that is 0. So, it will be something like this, the moment will be like guided beam.

And similarly like B all the other beams like A. A also you see that; A is one side it is connected to p q which also will have a slope zero at that contact point and the other side it is already fixed. So, this side also will have even the deflection is also 0 and the slope is definitely zero. And C and D beam is also like A and B ok. And E F G H is just the symmetric part like the other part or a off a b c d.

So, all of these beams are under guided movement. So, now, we know that these are all guided structures. Now for the guided beam structures we have already calculated the stiffness constant and that stiffness constant is this term; that K b is equal to F by W into 12 Y I by L cube. So, now, we will put this we will use this here. So, guided beam structure for a single beam we know.

(Refer Slide Time: 08:14)



Let us say for a single beam K. K A is equal to 12 Y I by L cube. So, it is twelve yi by l cube. Now, y and l are given and I we need to calculate and for this kind of beam. So, Y and l are known and I need to calculate. We need to calculate the area like the inertia amount of inertia right and for this kind of geometry you have already seen that the I is equal to b h cube by twelve right and there h means h means the height or on which direction the force is applied just perpendicular to that like. So, if this is if this is a beam if I considered then if this is the force applying then h will be this thickness h will be this thickness right.

And in this beam; in our case, in for this example force is applied in y direction and. So, this w or the which is actually the width that is the height here right. So, and the thickness is actually perpendicular to the height, but thickness is actually perpendicular to thus force. So, here I equals to t W cube by 12. And accordingly, we get that K A equal to Y T W cube divided by L cube both the 12 gets cancel out. So, Y T W cube by L cube and this is the stiffness constant for all the beams. Because, all the beams have the same dimension.

Now, what do we need to understand that how these beams are arranged which beams are in parallel which beams are in series and accordingly we can calculate the equivalent stiffness constant ok. Now, we need to very carefully understand that how these beams are connected see this is the proof mass this is the proof mass and this proof mass is connected to the to like a this B beam and D beam directly whereas, a and c is not connected to the proof mass directly right.

Now, let us assume if the proof mass move upwards by 5 micrometer, then not necessarily this p q also will move upwards by 5 micro meter because this is under not under the same force ok. Now, let us say because the proof mass is moving upwards we are pushing the proof mass in the upward direction for 5 micrometer then the beam B and D starts to bend upward and because of that it starts pulling up the P Q structure also.

And because of that, let us say the P Q structure moves by let us assume that 3 micrometer then if the P Q, P Q structure moves upward with 3 micrometer; that means, the B beam has bend both the sides has a bending or the separation of 2 micrometer. Because, for the B bending for the B for the b beam this point which is connected to the proof mass has move upwards by 5 micrometer whereas, the other part which is connected to the P structure has moved upwards by 3 micrometer.

So, it has a guided movement of about 2 micrometer whereas, as the p q structure is moving upwards by P Q structure is moving upwards by 3 micrometer then the A point A for the a beam this point which is connected to the P Q structure will move upward with three micrometer. But, the other side of the A beam which is fixed we will have no movement. So, in that case then the a move a structure has move upwards in a guided direction by 3 micrometer.

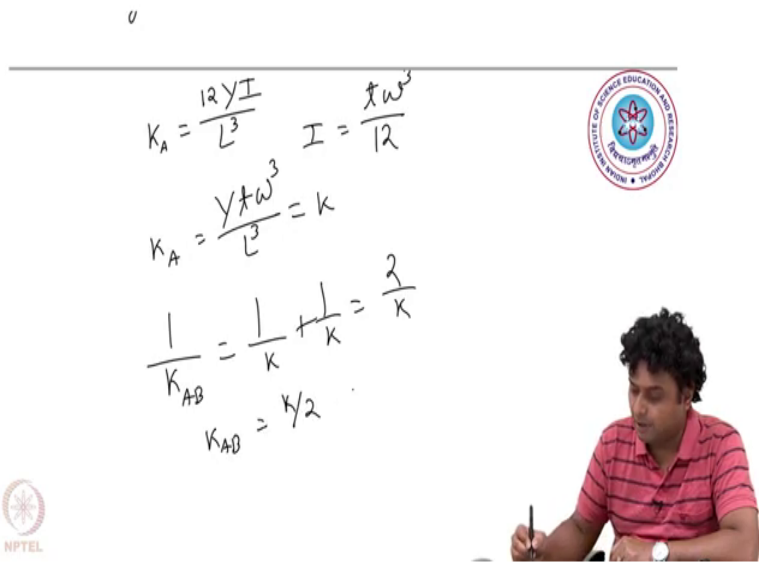
So, the point here is that A and B is under the same force, but the deflection is not same; that means, A and B are in series a connection this A and B are in series connection where the deflection may be different, but the force is same. Now, if I draw this structure then in a spring mass arrangement. So, this is A, this is A and this whole platform is moving this whole platform is moving upward or downward ok. So, this is the B side which is connected to the proof mass it is connected to the proof mass.

So, now you see that A is connected one side to the face and another side to the proof mass and now it has the it has the same force, but deflection may be different. Now, the interesting part is that if you see the C D arrangement. C D arrangement is also connected to the proof mass and another side it is fixed. So, if I now draw the c d arrangement then the beam D is connected to the proof mass and beam C is connected to the fixed point; so, ultimately because it is connected to the same proof mass.

So, the deflection the combined deflection of A and B and combined deflection of C and D will be the same right like in this picture. And in that case what will happen that that this arrangement this A B arrangement and this C D arrangement is in parallel. In that case what we will find out first the stiffness constant for the one site of the bills that is A B C D.

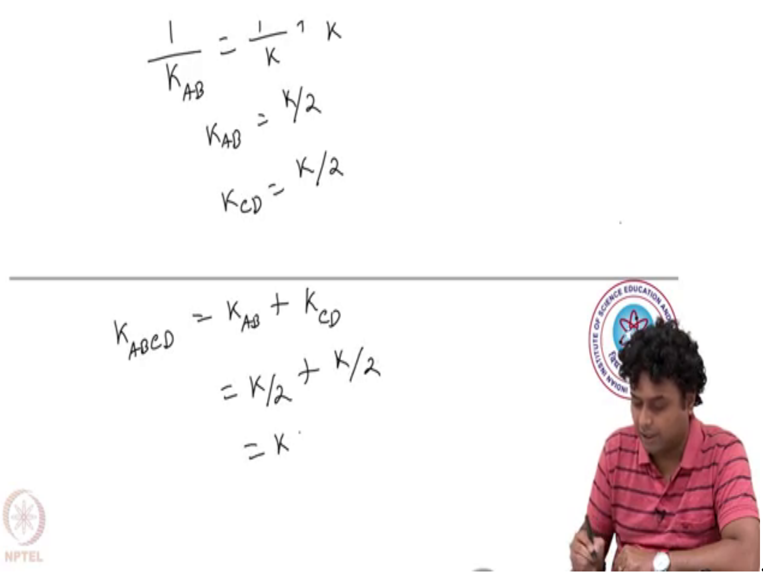
So, we know that this is the formula that K A is equal to Y t cube Y t W cube by L cube right. And from the series and parallel formulation of spring mass system we know that; for A and B for A and B which is in series then K A B.

(Refer Slide Time: 15:33)



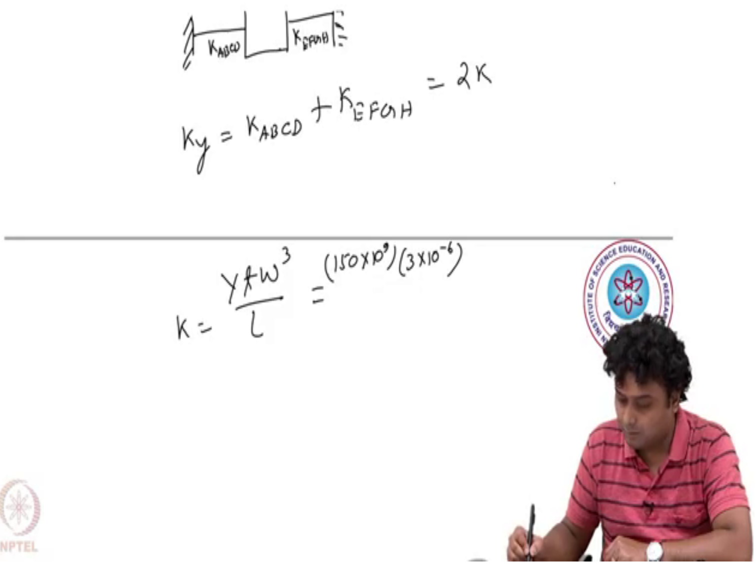
If we consider the equivalent stiffness constant to be K A B is will be 1 by K A B is equal to 1 by K A plus 1 by K e 1by K A plus 1 by K B and let us assume both to be K ok. So, 1 by K plus 1 by K, it is 2 by K. So, K A B equals to K by 2. Again A B and C D is in parallel.

(Refer Slide Time: 16:03)



So, K A B C D, the whole left hand side of the structure is equal to K AB plus K C D sorry, K C D and K AB equal to K by 2. Similarly, K C D also will be equals to K by 2 and here then we can get that the K by 2 plus K by 2 is equals to K. Stiffness constant in one side is K. Now, we see that this is the stiffness forcing K is for one side right and for the other side; we have the exactly same structure. So, if E F G H all these beams will also have the same stiffness constant that is K. Now this is the proof mass.

(Refer Slide Time: 16:57)

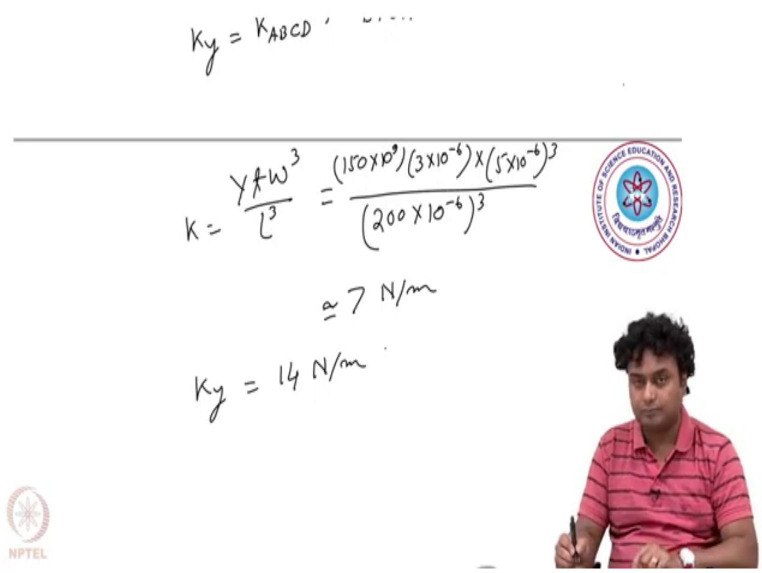


Now, this is the proof mass let us says and one side it is connected to this beam which is K A B C D and another side it is connected to the beam which is K E F G H right and both the side it is fixed to the other ends ok. And how this beams are connected already you see saw in the previous example that these are in parallel connection. Because, because it has the same deflection, but the force may be different.

However, for these case K A B C D and K E F G H both are k both the stiffness constant are k. So, the force and deflection both are same. So, the ultimately equivalent K Y for this kind of system is K E F G H is equal to 2 K. And if I now put all the known values of K, let us go back K is equal to Y t W cube by L cube. Now, if we put all the values of Y t W and l as we have already as we already know. So, then it will come as 150 into 10 to the power 9. This is the Young's modules.

Then a 3 into 10 to the power minus 6, this is the thickness. One thing here probably I have wrote these things as millimeter is not millimeter is a micrometer.

(Refer Slide Time: 19:14)



So, Y t into W cube, W is 5 micron 10 to the power minus 6 divided by L cube. L is 200 micron. So, is equals to 7 equivalent to 7 Newton per meter. And then K y or the stiffness consent is 14 Newton per meter. So, now it means that if for this beam or for this structure if I applied 14 Newton force in Y direction then the movement will be 1 meter or 14 micro Newton force then the movement in y direction will be 1 micrometer ok. So, we get the stiffness constant for this kind of beam and the advantage for this kind of structure is or this analysis is, is now this can be a sensor right and for this particular sensor, let us say if it is a force sensor then we can measure the deflection.

If we measure let us say the deflection for this particular proof mass is about a 0.5 0.5 micron or 500 nanometer in y direction. Then we know that the force has been applied is a 7 micro Newton right. So, this is how we can calculate and with that we will close this module.

Thank you.