A Brief Introduction to Micro Sensors
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Lecture - 04
Basic Mechanics - Part 01

Today, we will discuss more on the mechanics part.

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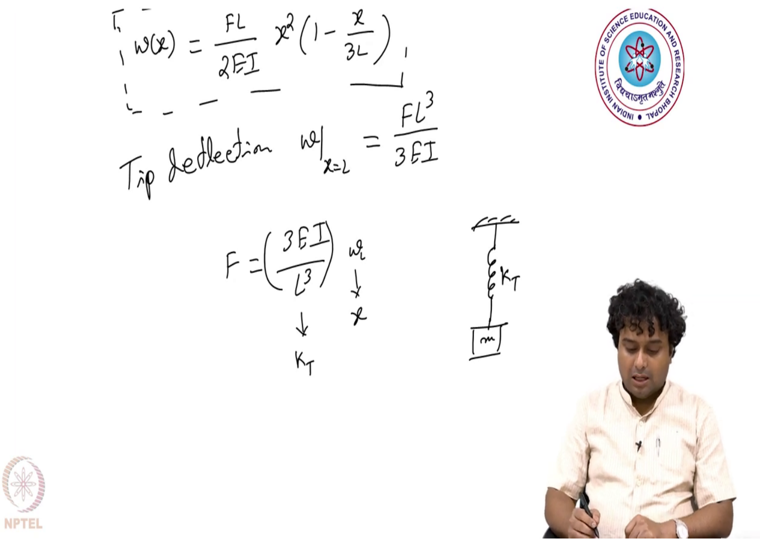

So, in the last class, while we are discussing about this transfers force, we saw that we had this term called inertia, right this I. Now, this I is not mass inertia. This is actually area moment of inertia, ok. Now, what do you mean by that. So, I will just little bit explain on the area moment of inertia.

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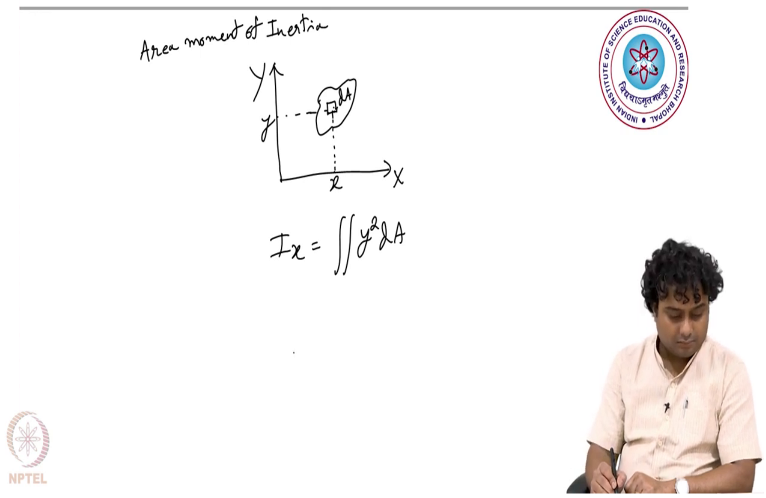

So, let us assume that this object or this area is at a distance x comma y in this rectangular coordinates system. Now, area moment of inertia is defined as like what is the moment? Moment is that how far is the area from some axis. And let say if we are calculating the moment of inertia with respect to the X axis then from X axis it is at a distance of y. So, I x is defined as area integral of y square, ok.

Now, why I am discussing this because you need to understand that the area moment of inertia depends on the direction, right; so, while I am calculating the moment of inertia with respect to this axis direct the X axis then it is y square d A. Whereas, if you calculate the moment of inertia with respect to Y axis then it will be x square double integration of x square d A, ok.

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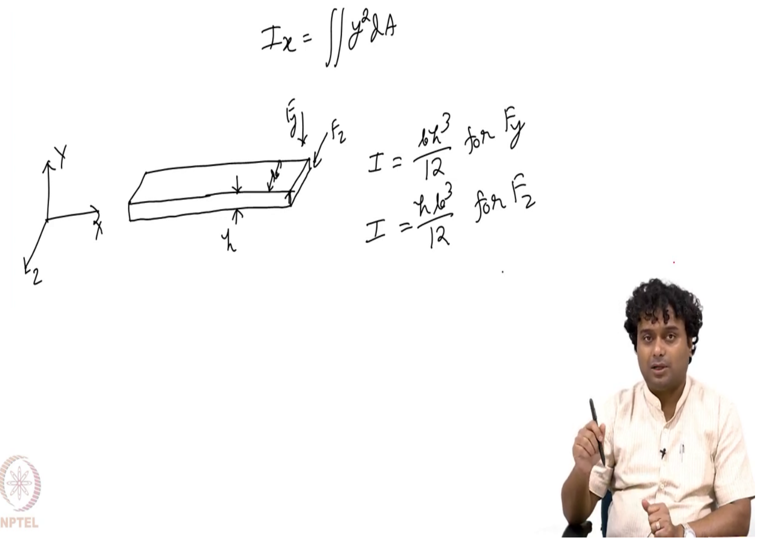

Now, let us consider the beam geometry we have. If we are considering the force; first of all for this kind of geometry, the inertia I can be written as b h cube by 12. Where b is the width or breadth and h is the height. And we are not going into the derivation of this expression, because this is not part of this course, ok so that you can find out from simple mathematics or mechanics.

Now, what I want to discuss here that if the force is in Y direction; if the force is in Y direction then this width is considered as width ok. And this is the height, but if the force is in Z direction like in the along the Z axis then it is perpendicular to this height means, the force is working perpendicular to the height.

So, in that case; moment of inertia will be actually h b cube by 12, ok. So, this is for F y whereas, this is for F z, ok. So, you need to consider that many a time for this kind of simple cantilever beams, we directly used that area moment of inertia is equal to b h cube by 12 or w h cube by 12, whatever you call the width, ok. But, there what we need to understand that the, it is not the width or the breadth is not only the based on its geometric it is also which direction the force is working. So, accordingly you have to put the formula like which is your breadth and which is your height.

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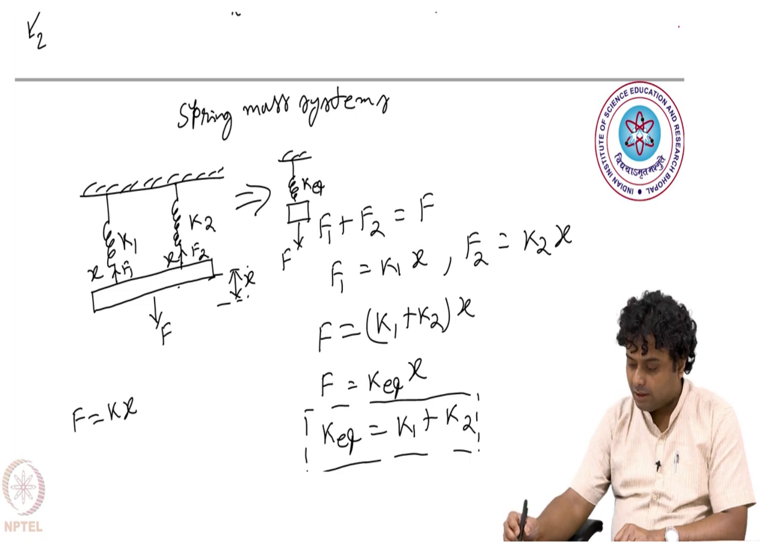

Now, we will see spring mass system. We have seen that; let say we have spring and mass connected to that, then you can simply write that if the displacement is let us say x and force acting is F, the spring constant is K then f is equal to K x. Now, and we are going to model our systems like this spring mass system. So, that we can directly use the K value and accordingly we can solve any problem.

Now, what is the advantage of using this kind of system, that we will understand that, we if we have multiple beams which are connected to each other, then we can consider that the whether the beams are parallel or in parallel or like in series and accordingly. We can write an equivalent stiffness for the system.

So, before we go into all those things, let us first see that what happens if two springs are connected to in a mass in series or in parallel, ok. So, now, see these two strings of having of stiffness constant K 1 and K 2 are connected in parallel to each other to the same mass.

Now, if I apply some force F then it is going to deflect by same amount for both the cases, let us say that is x. So, K 1 and K 2 spring both will deflect by x, right but, the force will be different in these two case right. So, let us assume that the reaction force here is F 1 and reaction force here is F 2. So, we can write that F 1 plus F 2 equals to F, right.

Again, for both the cases deflection is x only now, F 1 is equal to then K 1 x and F 2 is equal to K 2 x, right. Now, the total force is equal to then K 1 plus K 2 into x, ok. Now, if you consider this system as a single spring mass system then, you can model it like that right. So, this is let us say K equivalent and then you are applying the same force. So, this for so for this equivalent system, then we can write that, this is equals to K equivalent into x. So, K equivalent is equal to K 1 plus K 2.

So, if two springs are in parallel, we can directly get the equivalent spring constant simplify just adding their stiffness constant, ok. So, this is one important relation and now, let us go to the next case where two springs are connected to one single mass in series.

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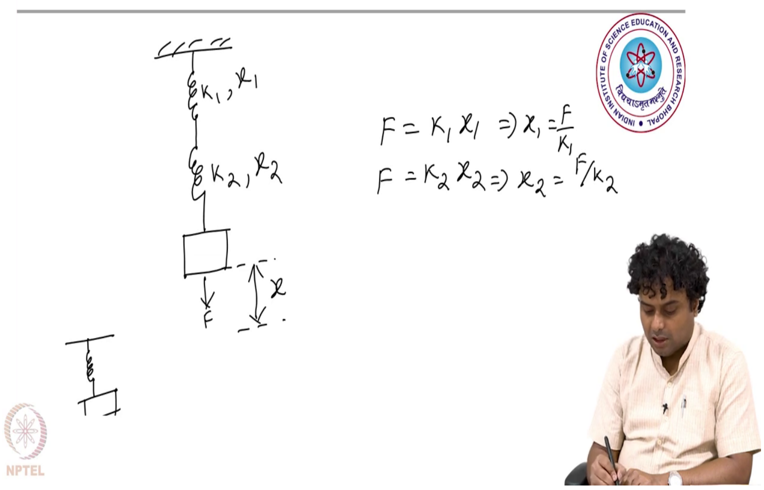

Let us say, the mass move by x because of the force F. Now, these springs will have different deflection and the other spring will have other deflection. So, let us say let us assume that K 1 will deflect by x 1 and K 2 will deflect by x 2; the force for both the cases are same, because they are in series. So, we can write that F is equal to K 1 x 1 and F is equal to K 2 x 2. So, x 1 equals to F by K 1 and x 2 is equal to F by K 2, ok.

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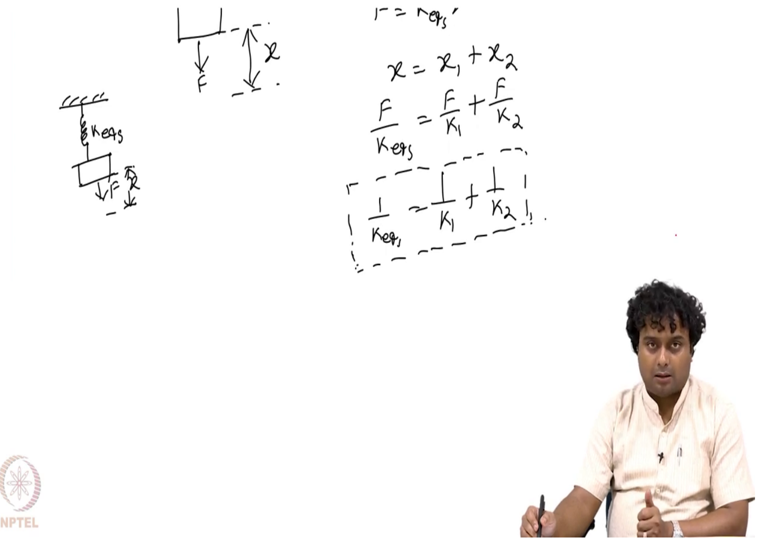

Now, if the system is modeled as an equivalent system we have only one spring constant right, and then one single mass also will have a force of F. And then this, let us assume this is K equivalent s. Last one let us assume this is K equivalent p or parallel and this is s for series. So, then you can write that F is equal to it will have the same total deflection right x same total deflection x. So, F is equal to K equivalent for series into x.

Now, what is x? x equal to x equal to x 1 plus x 2 and from there we can write that x equal to F by K equivalent series equals to F by K 1 plus F by K 2 or 1by K equivalent series equals to 1 by K 1 plus 1 by K 2. So, this is another important relation and for most of the problems, like complicated problems what we will try to see that if two beams are connected in series or in parallel. And accordingly, we will first find the individual spring constant for those beams and just can apply the series or parallel spring formula and calculate the combined Cygnus constant.

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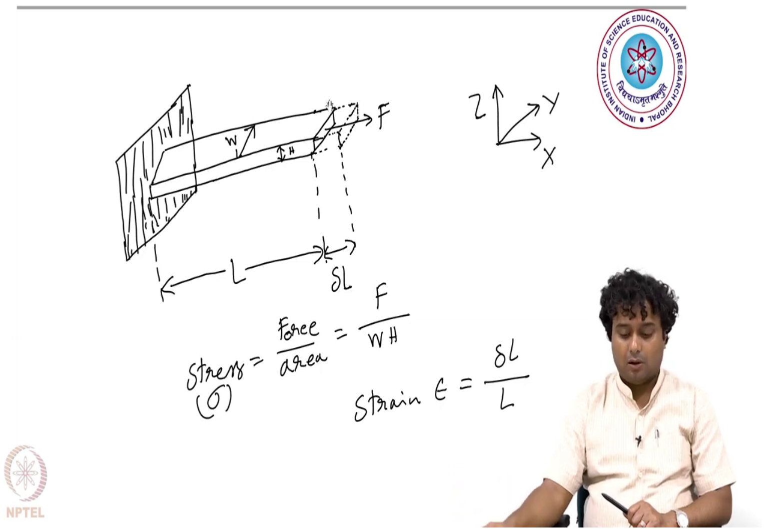

Now, we will discuss different constants. And constants means like for a beam structure and applied forces what kind of constant are applicable for that particular beam; means, what motions are allowed and what motions are not allowed. Like for this structure you can see that for the left hand side there is no motions are allowed like X direction Y direction Z direction no motion are allowed as well as rotations are also not allowed, right.

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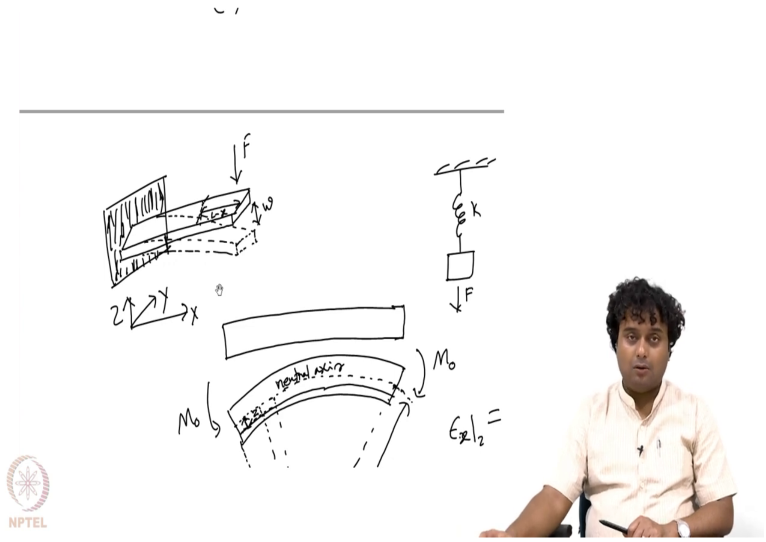

But, for this end it can have all that motions are free like X Y Z it can move as well as it can rotate. Whereas, for this particular case only we had movement in the X direction, but, for transfers case, you see movement was in or the deflection was in Y direction. Whereas, because of very small deflection, we have considered that there X direction has not in X direction it has not moved much, but technically it can move in X or Z direction also or even it can rotate.

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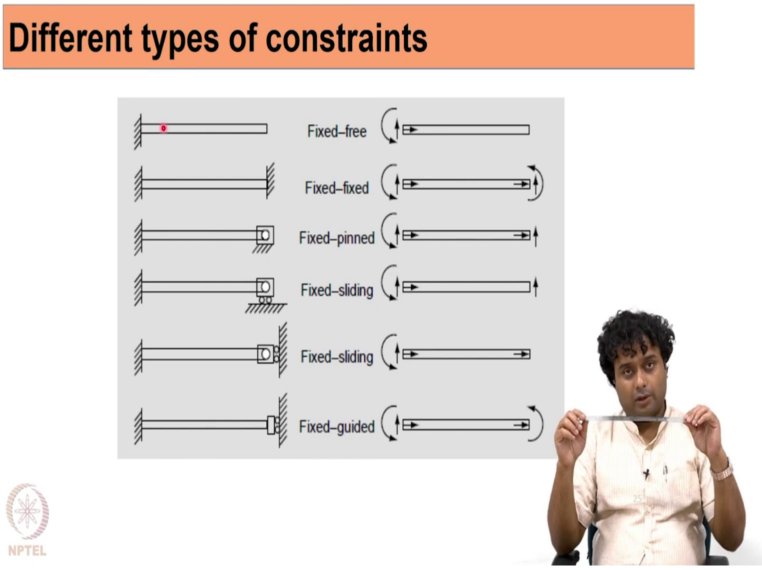

Now, I will see that how different constants work for different kind of geometry. Now, let us go through different kind of constraints one by one. Now first take the first case, like this simple cantilever beam where one side of the cantilever like the left side is fixed and the other side can move. And in this case see this side is totally fixed. So, it has reaction forces in X direction, Y direction as well as Z direction.

Let us say, we are not considering the Z direction in this plane or in the plane of paper it can rotate right. So, it has X direction force Y direction force and also the rotation, this. But, this end is totally free. So, there is no reaction forces either on horizontal direction or vertical direction or even rotation, no forces are there.

Now, if you consider a fixed beam, then what happens in a case of fixed beam. For fixed beam, both the ends are totally fixed. So, this end also it cannot move and this end also cannot move. So, the reaction forces will be in a horizontal direction, vertical direction as well as in the rotational direction, ok.

Now, the next case is fixed pinned. In fixed pinned case you see, pinned means that it can rotate across this pinned point. So, if it can rotate along this pinned point across this pin point, then there will be no movement at this end ok, but this is fixed. So, it cannot have a linear deflection. So, the horizontal and vertical reaction forces will be there while the motion is restricted then only reaction force is present, right.

So, this end will have horizontal and vertical reaction forces, but the moment will not be there because, it is able to rotate \ with respect to this pin point, ok. But at the other end is anyway totally fix so, it will have all the forces and also moment.

Now, the next case is fixed sliding; what does it mean, means it can rotate at this point as well as it can actually move in the X direction. If it can move in X direction then; that means, that the motion at X direction is not restricted. So, the reaction force will not be there, also it can rotate along this pin point. So, the moment also will not be there. So, the only reaction force will happen will be present in this fixed sliding case is vertical direction reaction moment.

Then again fixed sliding where the vertical direction is motion is allowed, but the horizontal or the X direction motion is not allowed. And in that case only the reaction force will be present at X direction.

Now, this is very important the last case that is fixed guided. Fixed guided means what? That it can slide in Y direction and there is no rotation allowed. So, it is like this that one end it is fixed and another end can move, but like this can move, but like this. So, there it cannot rotate, but it can move. It cannot rotate, but it can move you see the motion of this metal scale that here also it is totally fixed and this end the right; my right hand is actually moving, my right hand is actually moving, but this I am not allowed allowing it to rotate, ok.

So, this end will have the moment as reaction force and also the X direction because, X direction also no motion is allowed. Only the y direction motions we have allowed right and that too without any rotation. So, this is guided. So, this beam is guided in only in this direction (Refer Time: 17:18) direction. So, accordingly only the horizontal reaction force and moment will be present.

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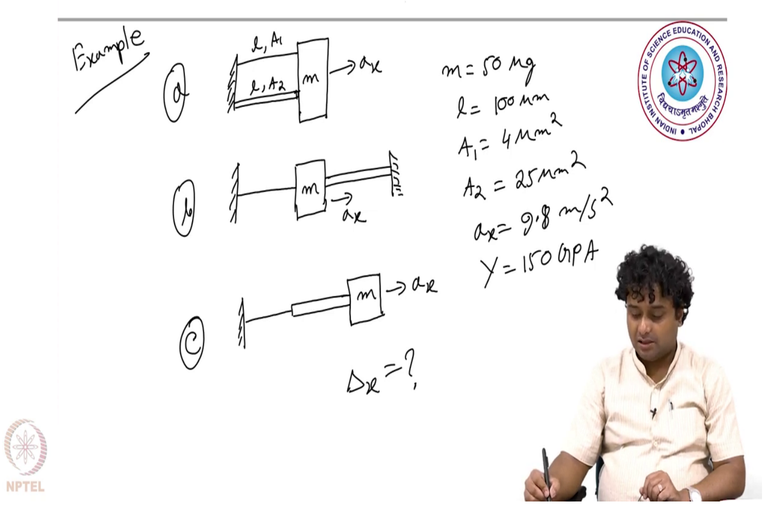

So, now we will see one example based on the concept whatever we have discussed. So, there are two beams, ok. There are two beams and these two beams the length of the beam is same, but the area of cross section is different, ok. So, let us assume this l, and l is the length and A 1 and A 2 is the area of cross section.

Now, this beam is these two beams are connected to this particular mass of m which is accelerated by some acceleration A x in three different cases. In the first case these two beams are connected like this, in the second case also this like these two beams are connected in the opposite direction and in the third case these two beams are connected like this right.

Now, m is equal to let us say 50 microgram l is equal to 100 micrometer A 1 is 4 micrometer square A 2 is 25 micrometers square and the acceleration which is applying on the mass is 9.8 meter per second square and Young's modulus constant Y or E whatever you call it is 1 150 giga Pascal. We need to calculate the equivalent stiffness constant and also the deflection of the mass. What is delta x for different cases? This case a, case b and case c, ok.

Now, the first thing you will see in this problem is in all the three cases the beams are both the beams are under axial stress. So, under axial stress, we can directly apply the stiffness constant of an cantilever beam which is under axial stress and that is what we found out that is W H E by L, right.

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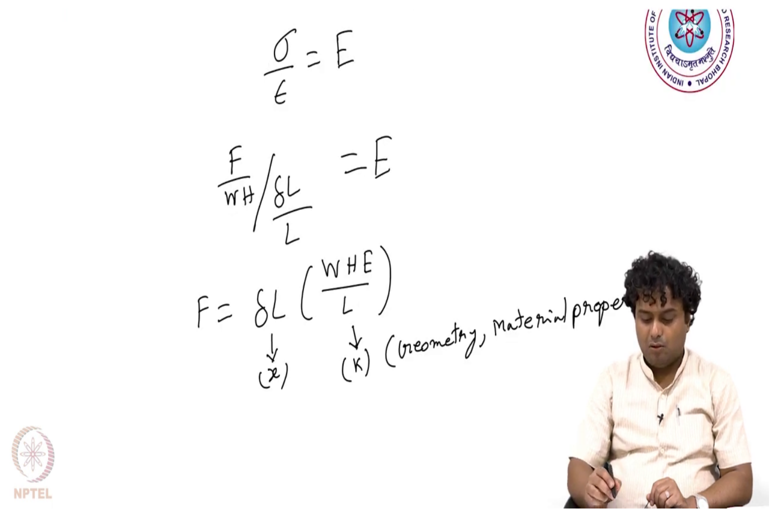

Here W into H is the area and E is the Young's modulus. So, now, let us say we will just then the formula for our case will be A into Y by L.

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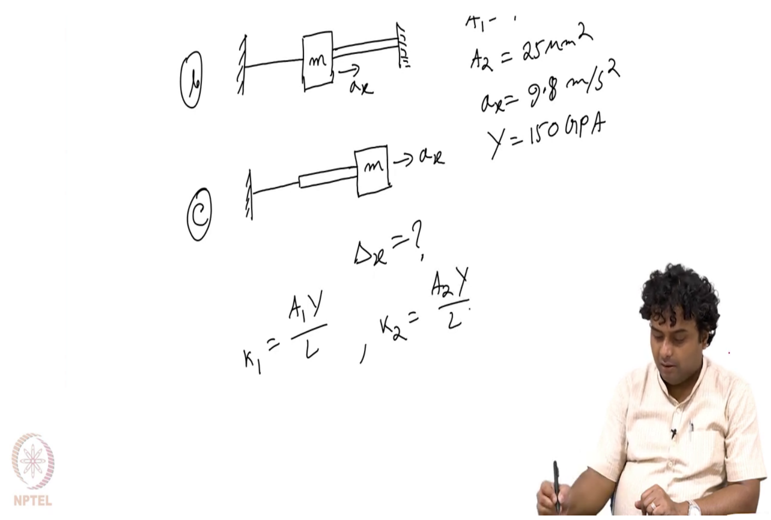

K 1, K 1 means the thinner beams and K 2 is the thicker beam let us say. K 1 is equal to A 1 Y by L and K 2 is equal to A 2 Y by L.

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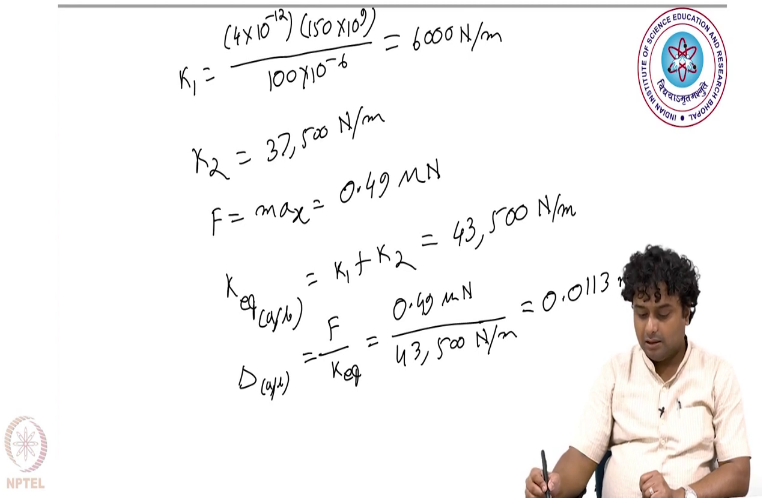

And if you put the values then you will get K 1 equals to 4 into 10 to the power minus 12 into Y is 150 giga Pascal so, that is into 150 into 10 to the power 9 right and then L 1 is 100 micron. So, that is 100 into minus 6 equals to 6000 Newton per meter. And similarly K 2 will be how much, K 2 will be just here it will be 25 and it will come as 37500 Newton per meter, ok.

And also we know the force is also same for all the cases because the same mass and same acceleration is equal to m into a x and equals to your m is 50 micro gram and g is 9.8 meter per second square 0.49 micro Newton.

Now, for these three different cases the beam arrangement is different. The first case the a case, see that both the beams are connected parallel, because, it is the same deflection which will happen for both the beams right. Because they are in parallel so, it cannot both the beams cannot have separate deflection. And for b case also it is though; it looks like series, but it is actually parallel because, in this case also both the beam is having the same deflection. So, this is very important point you need to understand that this is not only how it looks like. Since what you need to see that; whether the same force is same or the deflection is same.

As we discussed that see for the parallel case while two springs are in parallel then what happens is same force like sorry same deflection, but different forces are applied right, on the two difference. Whereas, if two springs are in series then you can see that it is like same force is applied as they are in series, but it is two different deflection x 1 and x 2, right.

And in this case, in this example now, for the case one like case a and case b, in both the cases; the deflection is same. So, in these case we can say that the beams are in parallel, right. So, we can write that K equivalent for a or b or both the case, it is K 1 plus K 2 and this is equals to 37500 plus 6000 43500 Newton per meter, ok.

So, now delta or deflection for both the case a comma b or a or b is equals to what is my total force, F divided by total spring constant K equivalent and that is equals to 0.49 micro Newton divided by 43500 Newton per meter.

So, you note another thing that this meter will go at the top or the numerator, right. So, and this should be like that is only because this is the deflection. So, deflection will should have a length unit like meter centimeter like that right, so this is correct 0.0113 nano meter.

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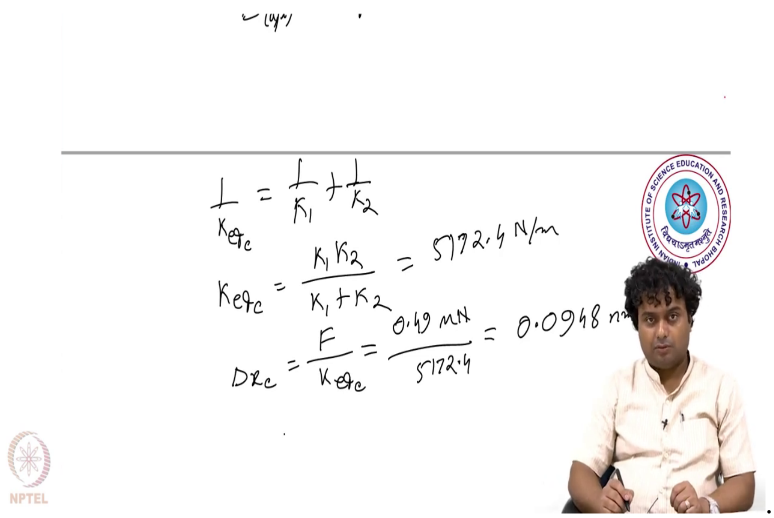

Now, for the c case, see this is in series, because it is the same force which is applied, but the deflection is different, right. So, for these case if the equivalent spring constant is K, K equivalent c, then we can write that K equivalent c equals to 1 by K 1 plus 1 by K 2. So, K equivalent c equals to K 1 K 2 divided by K 1 plus K 2 right and if you put the K 1 and K 2 value, then you will get 5172; 5172.4 Newton per meter. And then what is the deflection? Deflection is delta x c equals to forces again same divided by K equivalent c right force is equal to deflection into stiffness constant.

So, I we already got the equivalent stiffness constant for c case and then just force by the stiffness. So, that is that will give you, how much was the force, force was something about 0.49 micro Newton which is 0.0948 nano meter ok, so, we have now found the deflection for all the three cases.