A Brief Introduction to Micro Sensors

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Lecture - 03
Some Sample Mechanics

Hello, in this lecture we are going to talk about Some Simple Mechanics. Now, as you know, this course is on MEMS. MEMS means micro electromechanical systems and this systems or devices have both the mechanical and electronics parts. Now, I know that many of the students, who are taking this course can be from non mechanical background and, but all of the whether you are from mechanical or non-mechanical background, you need to understand some basics of mechanical engineering or mechanics.

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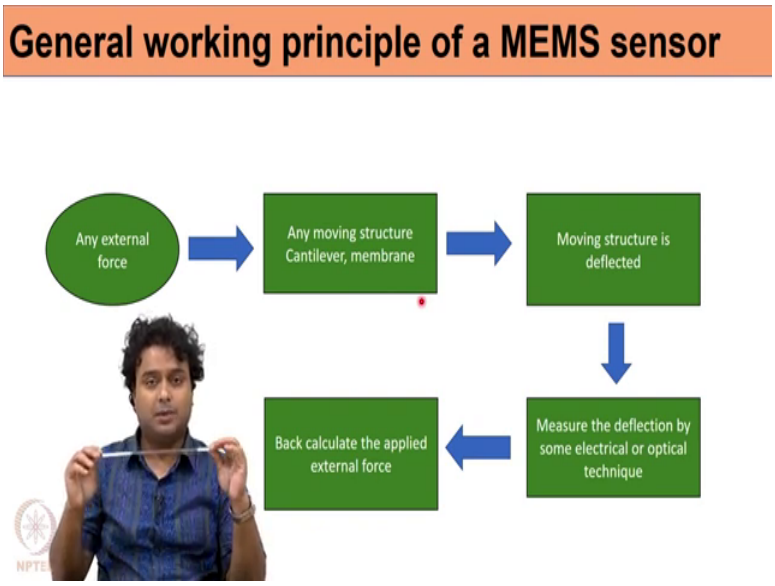

So, this slide I have presented earlier also, that the general working principle of MEMS sensor that you have some external force and that external force is ultimately moving some structure, this can be a cantilever.

Cantilever means like a simply supported beam at which is fixed at one end and other, other end can freely move right, like a cantilever or a membrane and, because of the external force this moving structure is deflected like this and we measure the deflection by some electrical or optical technique and then we can back calculate the applied force, if we know the force deflection relation.

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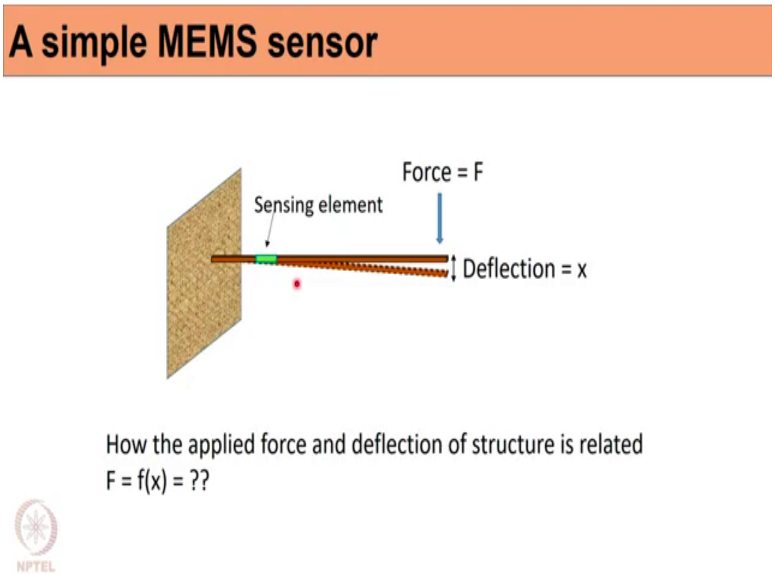

Let us take a simple MEM sensor, which can be let us say four sensor. So, this is a simple cantilever which is fixed at this ends. So, this end let us say it is put under inside some wall. So, this end cannot move or cannot even bend whereas, this tip can bend or move. Now, if we apply some, if we apply some force F, then what will happen? The cantilever we bent down right and it will be, it is tip will be deflected by some amount let us say x.

Now, the question is that how we can measure this deflection? This deflection we can measure by putting some sensing element like piezoresistive elements. So, piezoresistive elements are such that if we apply some amount of stress or strain on those element, then its resistance changes.

So, this piezoresistive element we can put here, where the stress is maximum and then we can measure the resistance. So, we can measure the resistance and while the force is not applied, the resistance will have some hello, let us say r 0 and while we apply some force, because of the stress is this r 0 will change and that change can be measured by some electronic circuit and then from that change in resistance we can calculate, how much is the deflection, but do we know that how this force and deflection is related?.

Now, how this force and deflections are related for that we need to understand like the mechanics like a how a beam deflects or how a beam move, what are the different constraints, etcetera and for that we are going to work on paper and then we will see exactly, how different force and deflections are related for different structures. Let us first take this simple example, which you have already seen probably at your 12 th standard where a mass is hold by a spring.

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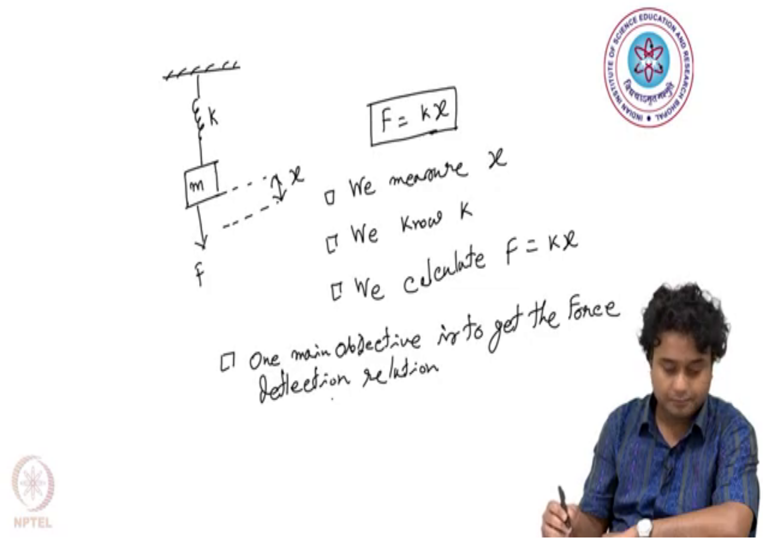

So, this is a mass m and this spring has a spring constant of k right. Now, if we apply a force F and, because of that let us say this mass is deflected by some amount x, then we know, from laws of mechanics that F is equal to k into x right. So, force is related to this deflection by a single spring, if the deflection is very small right otherwise, it will not be linear, it will go into non-linear domain.

Now, here the force is a linear function of deflection and this relation we can write from there. Now, what we are doing in MEMS devices that we measure x or the deflection by some optical or electronic technique, then we know k or the spring constant from its geometry. Then we calculate F, how F; because F equal to k into x ok.

So, one of our; one of our major objective, one main objective is to get the force deflection relation and if we can get a linear relation between the force and the deflection, then we can directly use the spring mass system like a model. So, it will make our analysis even more simpler. So, now we will take the simple case of one side fixed beam or cantilever.

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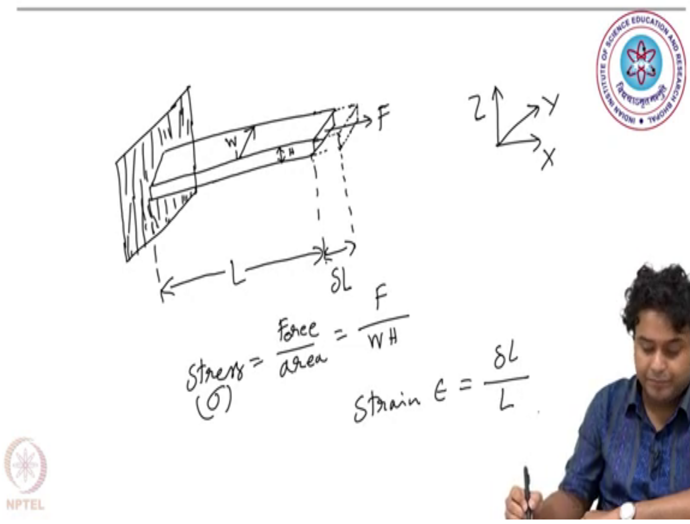

So, let us say this is a simply supported beam, one side is fixed to this wall and the beam has a length of L. So, its length is capital L with W and the thickness or the height of the beam is H ok. Now, we apply some amount of force F and, because of that so, let us assume that we have applied some force F and, because of that the beam has moved in the in this direction along its length by some amount delta L ok.

So, let us fix the coordinate first, this is X, this is Y and this is Z ok. So, this force F is applied in X direction and, because of that the tip has deflected by an amount delta L or elongated by an amount delta L. Now, by definition we know that stress equals to force by area. So, what is the force here F and what is the area? Area means this cross sectional area right the beam cross-sectional and that area is this is W into H

So, F by W H is that stress, let us call it sigma. Now, what is strain? Strain is the amount of deflection by its original length right. So, strain epsilon equals to; what is the deflection amount. As I told delta L divided by its the original length that is L right.

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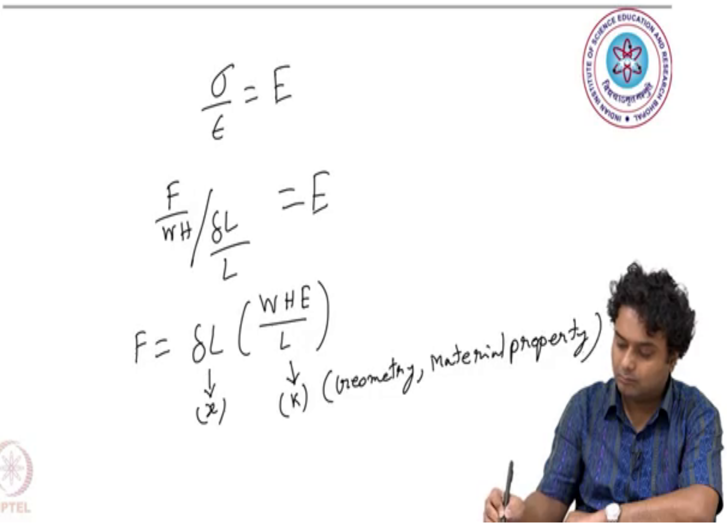

So, now we have stress and strain and from loss of elasticity we know that the stress varies linearly with the strain with a like how much is the deflection and so, the sigma by epsilon is a constant which is called Young's modulus and let us call it E. Now, this E, capital E is a constant and material property right, sigma and epsilon we have already got, sigma is F by W H and epsilon is delta L by L and this equals to E.

So, if we rearrange it, then we get delta L like F equals to; F equals to delta L into W H E by L right. Now, this F is the force and this delta L is the deflection or what we called earlier for the spring mass system as x right. So, this portion W H by L, this is stiffness constant or the spring constant of this particular cantilever for this particular direction force.

Now, this W H E L are all geometry and material dependent. So, once we know that then we can easily calculate the applied force from the measured deflection ok. So, now, for this case like for a one side fix axial stress MEM we know the, we know the force deflection relation right. Now, we will see that what is the force deflection relation; if we apply the force from top? That is this case.

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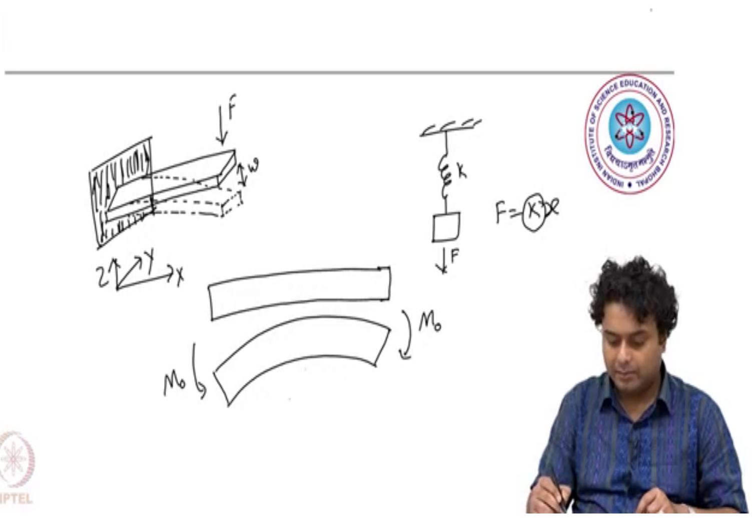

So, here the cantilever will bend by some amount right and this is what its deflection is and we need to understand that; we need to understand that how this deflection, let us call it small w, how this deflection is related to the force and for that we will be doing the same kind of analysis again. The just to remind you the whole point is that we want to ultimately model this system with a simple string spring mass system right and there you know that if we apply a force F and we have a spring of k then the F equal to k x and our goal here is to find that k ok.

Now, in the last case it was like a uniform deflection in just one dimension right and it was a linear deflection, but now it is bending, because of the, because of the vertical force the beam will bend towards like beam will bend downward. So, there will be a curvature introduced in the beam.

Now, before we go into this analysis, we need to understand if you properties of bend MEMS. So, for that we will consider pure bending. So, let us assume this was a simple cantilever beam like this and then, because of the moments it make, made this kind of curvature.

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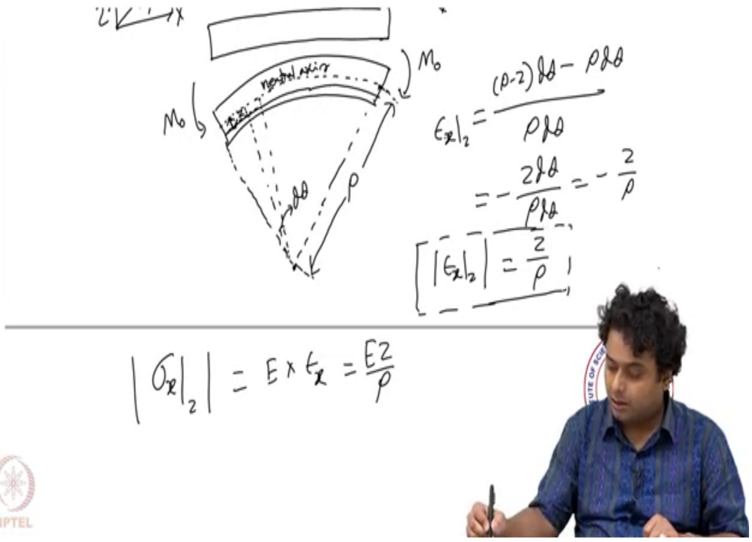

Now, there will be so, if for this beam you see, the top portion will be elongated, but the bottom portion will compressed. So, there will be at some region in the middle or some line in the middle, which will be at the same length and that we can call as neutral axis. Above neutral axis all the fibers, if you consider it like fibers then all the fibers has been elongated and below the neutral axis, this is complexed. What is the stress on a fiber just at a distance, let us say at a distance Z from the neutral axis.

So, at this line which is at a distance Z from the neutral axis at a it is below Z distance from the neutral axis, what is the stress or strain on this fiber or on this layer? So, for that what do you know that strain epsilon x right, because this is x axis you can see this is the x direction. So, this elongation is in x axis right.

So, sigma x at a distance epsilon x at a distance Z equals to this deflection by the original length. Now, what is the deflection? Deflection is it is current length dip minus its past length. So, what is the current length? Current length if I take a if I take an element like this d theta here, let us take this angle as d theta then what is the length? What is current length? Current length is radius of curvature or the radius at this point is rho minus Z right, because the neutral axis is.

So, we can consider this radius rho from the center to the neutral axis on the center to the neutral axis, it is rho and this layer is just Z distance below the neutral axis. So, the radius at this layer is rho minus Z into d theta minus its original length. So, what was its original length original length of the all the layers was seem equal to the neutral axis. So, what is the length of the neutral axis layer.

So, that is rho d theta right. So, current length is rho minus Z d theta minus and the old length was towards rho d theta divided by what? Divided by rho d theta. So, now, you will see, it will come as minus of Z d theta divided by rho d theta equals to minus of Z by rho. So, let us this minus is coming, because this layer is below the neutral axis.

So, it is in under compression. So, the modulus of sigma x at Z equals to Z by rho. Now, this is strain. So, what is stress? Stress is; stress sigma x at Z mod of that equals to Young's modulus that is E into epsilon x and that is E Z by rho ok. So, this is one important relation, we will need later also.

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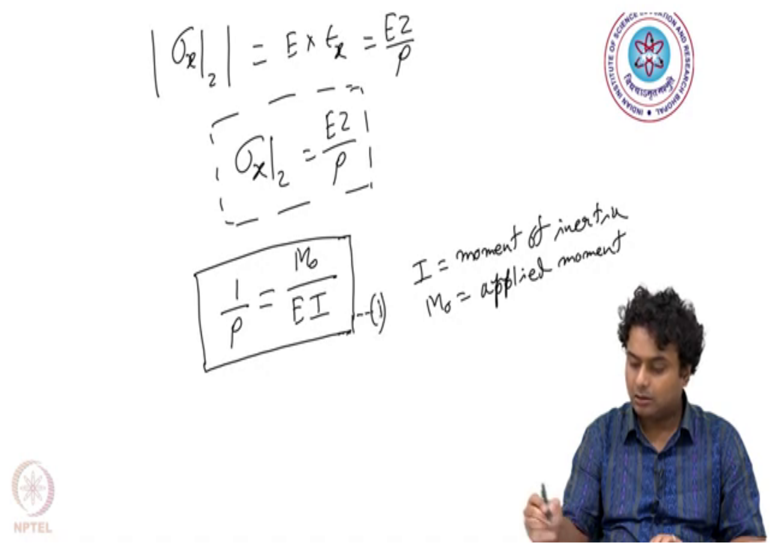

And sigma x at Z equals to Z by rho, this is also you might need later ok. Now, we have that stress and strain relation and we will use one relation between the acting moment and the radius of curvature for pure beam from mechanics, which we will use that relation directly, we will not derive it here, because as it is not in the scope of this course and according to this formula the radius of curvature, if the radius of curvature 1 by rho is equals to applied moment divided by E into I.

So, this is very important and we are not going to derive this, this mechanical engineering students, already knows it and if anyone is interested then you can find it from the laws of mechanics or the beam theory book.

So, for a pure bending case, where the applied moment is M and the curvature radius of curvature is rho and moment Young's modulus is E and moment of inertia is I, we can write this relation between the radius of curvature and the applied moment. Here, I is moment of inertia and rho E and M 0 has been already M 0 is the applied moment. Now, another important assumption or a formula we are going to use from geometry that is let us say if we have a one side fixed beam like this and then, because of deflection, it has made this kind of curvature.

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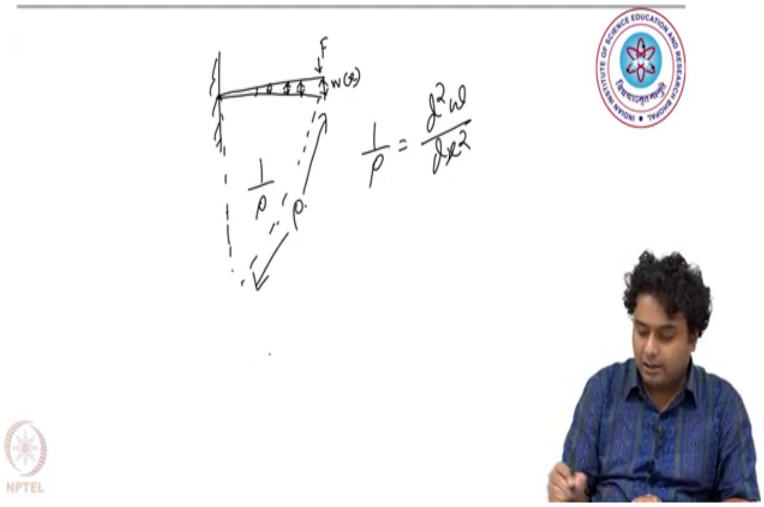

Then if this angle of deflection, if it is very small theta then we can write that 1 by rho, where this rho is the radius of curvature of this geometry ok, of this geometry, this is rho is equivalent is equal to let us say this is the force and this is W, W and deflection W at a distance x.

So, at this point the deflection is this much, at this point the deflection is this much and the tip the deflection is even more. So, this is deflection W at any point x is related to the radius of curvature by this rule, which you can find from geometry ok, but what do we need to remember here, that this theta is very small then only we can write this expression otherwise, the this equation will not hold, the another other non-linear terms will also come into the picture ok.

So, now we know that a pure bending case and small deflection, we can have this two expression that 1 by rho is equal to M 0 by E I and 1 by rho is equal to d 2 w by d x square where w a is the deflection at any point x.

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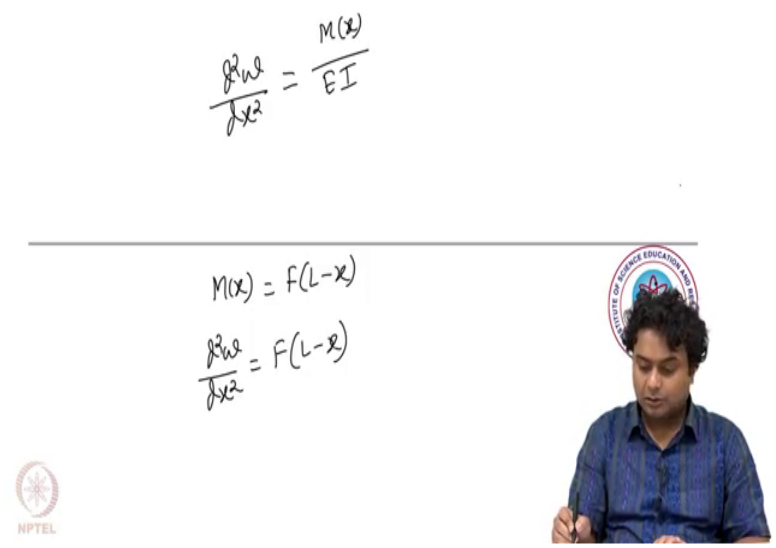

So, we can write then on this two equation we can write that d 2 w d x square equals to M x, because this moment at different at different distance from the fixed point, the moment also will be different. So, this is also a function of x Mx by E I right. So, now we have a relation between the radius of curvature and the moment, but what is the moment at every point for our case, if we see this cantilever beam then at different point the moment is different.

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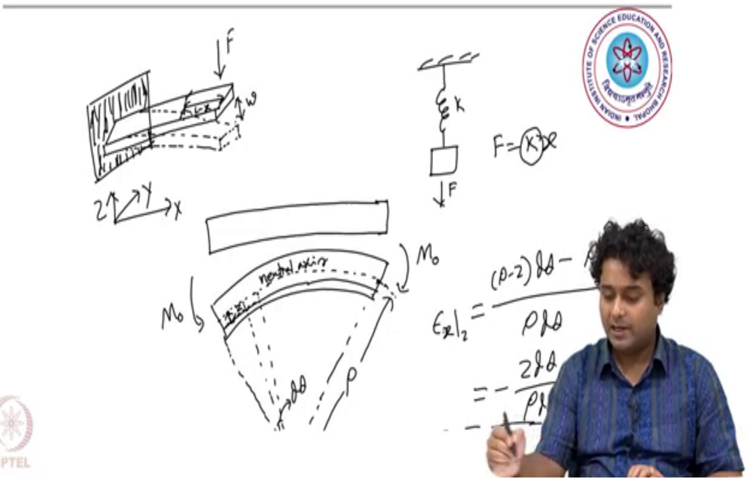

And let us say if I take this phase, which is at a distance x from the fixed base then this is at a distance L minus x from the point where the force is applied. So, the moment at this point, because of these forces is F into L minus x. You already know that that moment is force and the distance of like a at any point the moment is the force into the distance from the from that particular force right, from the line of force you know that the moment is the force into the distance from that line of force.

So, here the moment will be F into L minus x. Now, we can write that M x is equal to F into L minus x right. So, we get that d 2 w d x square is equal to F into L minus x ok. Let us call this equation, let us call this equation as equation 1 and this equation 2.

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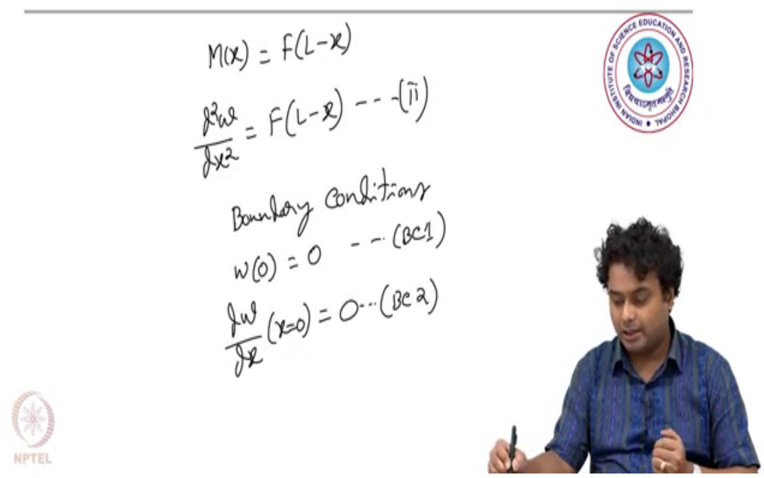

Now, we have the differential equation we need to solve it, but before solving we need the boundary conditions also and what are the boundary conditions? So, the boundary conditions; first boundary condition is w, my need this is a small w which is the deflection and the capital W, I have earlier wrote it for the width of the beam, this w or the deflection at x equal to 0 x equal to 0 means the point which where the beam is fixed to the wall.

At that point the deflection is 0 right at this point, at this point the deflection will be 0 definitely right and also another thing is as this is a very small deflection, at this point the slope also will be 0 means d w d x at x equal to 0, also will be 0. So, this is our so, this is our second boundary condition, this dw dx at x is equal to 0 is also 0. So, this is boundary condition 1 and this is and this is boundary condition 2.

Now, if we solve the differential equation then you see that one side it is d 2 w d x square and another side there we have x. So, it will come as x square and then x cube right.

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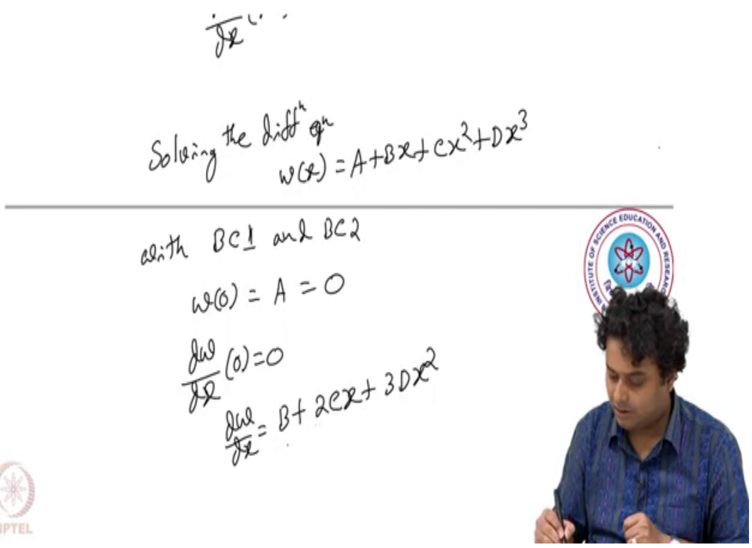

So, solving this equation we get that w x equal to A plus B x plus C x squared up to x cubed right. Now, A B C D all this coefficients we can calculate from the boundary conditions. Now, with BC 1 and BC 2, we can write that say BC 1 is w at 0 equal to 0. So, equals to A only equals to 0 and B C 2 is d w d x 0 equals to 0. So, d w d x is B plus 2 C x plus 3 D x square right equals to and a if we d w d x equal to 0 means then we can write that at x equal to 0 at x equal to 0 equals to B only equals to 0 right.

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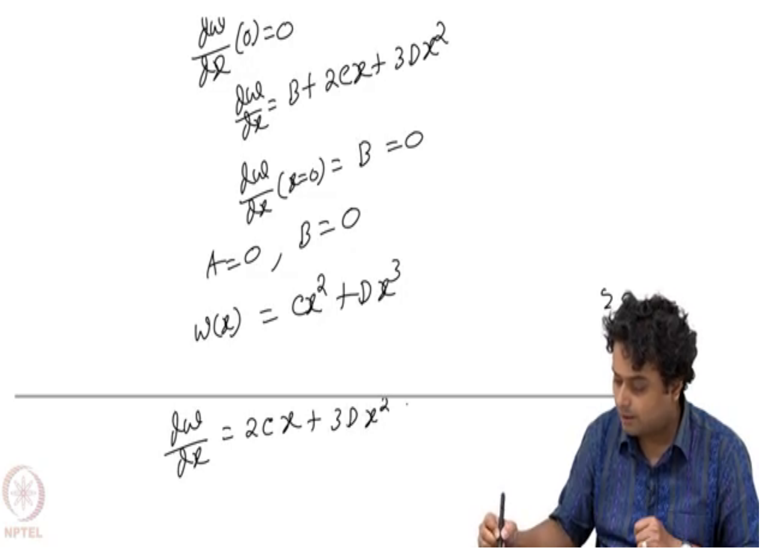

So, A equal to 0 comma B equal to 0 from boundary condition 1 and 2 ok. Now, from equation 2 so, let us first write then what is my w x? Then w x is means says A and B are 0. So, C x squared plus D x cube right and d w d x equals to 2 C x plus 3 D x square right and d 2 w d x square equals to 2 C plus 6 D x right.

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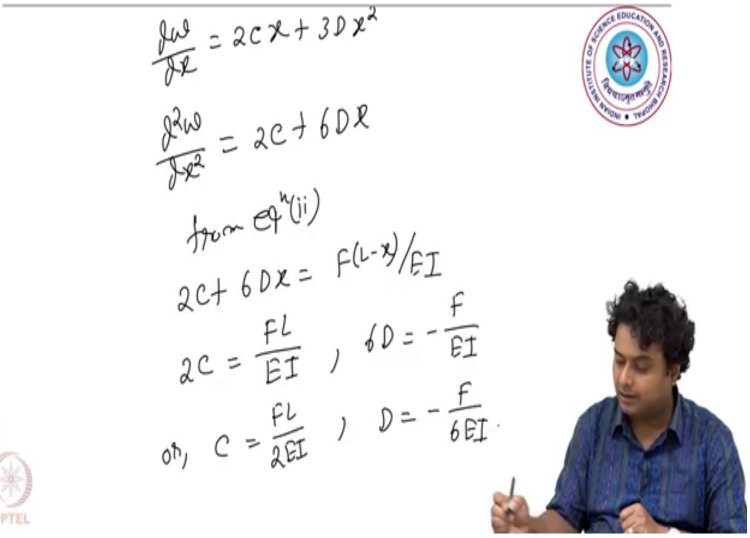

Now, from equation 2 we can tell, we also know that d 2 w d x square from equation 2 we can see that d 2 w dx square equals to F into L minus x right. So, and this equation this expression is valid for all the axis. So, we can write from equation 2, 2 C plus 6 D x equals to sorry, here will be d 2 w d x here, it will be my E I, because d 2 w d x square equals 2 M x by E I M x equal to into L minus x.

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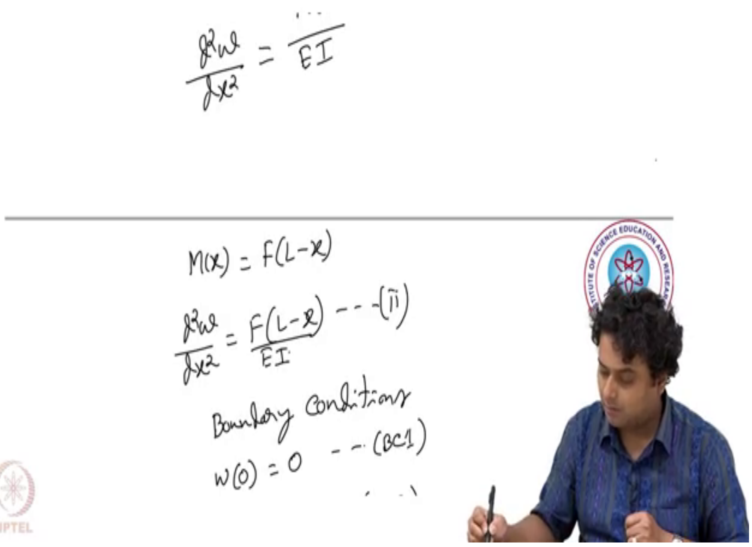

So, there will be E I ok. So, F into L minus x divided by E I right. So, and this expression is valid for all the axis. So, we can write, we can tell that 2 C equals to FL by E I and 6 D equals to minus F by E I or C equals to F L by 2 E I comma D equals to minus of F by 6 E I right ok.

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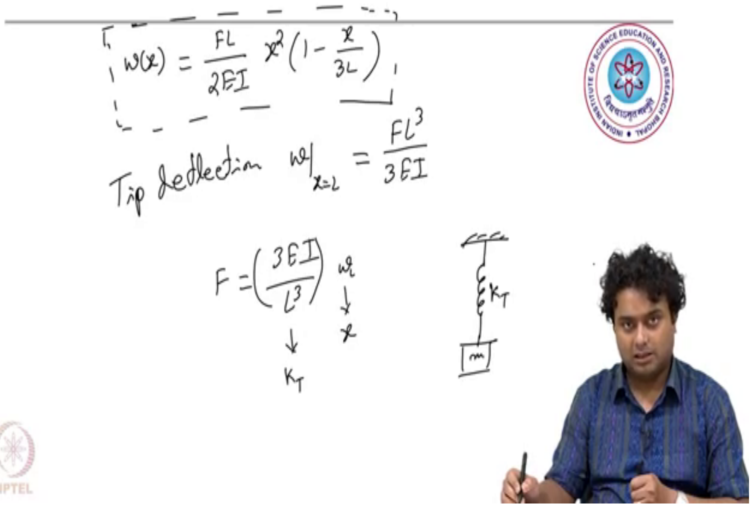

So, w x or the deflection equals to F L by 2 E I into take x square out to 1 minus x by 3 L right. So, this is our deflection versus force relation for any x right at any point at any point of the beam, we can find that how. What is the deflection with respect to the force? Now, what is the tip deflection? Tip deflection w at x equal to L equals to, if we put x equal to L, we will get F L cube divided by 3 E I.

So, if we rearrange it, we can write it as F equals to 3 E I divided by L cube into w at x equal to L. So, this is the spring constant or transfers spring constant, because now, the force is applied in transfer direction earlier, the force was applied in axial direction and this is the deflection. So, in now we can model this beam considering just a spring connected to the tip of the beam. Now, we can model this beam with the help of the spring mass system, where this is the spring constant K T and accordingly, we can do the further analysis.

So, just note one thing that the earlier for the axial stress case, we got the; we have got this spring constant which K, which was the axial beam for the axial case right, but the force was in the direction of the beam length or in the axial direction whereas, in this case the force is at the transfers direction. So, with the same beam while changing the force direction, the spring constants are different and the analysis are also different.