

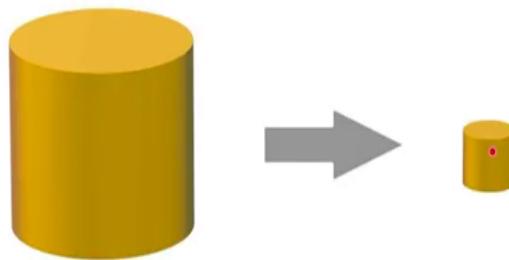
**A Brief Introduction to Micro Sensors**  
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**Lecture - 02**  
**Scaling effect**

Hello, welcome to lecture 2. In this lecture we are going to discuss Scaling effects.

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**Scaling effect**



Does the physics or science phenomenon be same if we scale up  
or scale down associated dimensions?



Now, what is scaling effects? What we are talking about the sensors or these devices they already exist, already exist means that they are present in macroscale currently, right. But, what we are actually doing we are making the same kind of devices or sensors for microscale uses or nanoscale uses.

Now, this is what we will call scaling like we are scaling down the device. Now, because of this scaling down what does it change does, the physics or science change or the phenomena change or what exactly, what are the things which exactly changes because of this scaling up or scaling down.

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## Why scaling effect is important?

- Scaling decides the performance of micro devices.
- Some micro devices do not function at larger scales.
- Scaling effects provide an excellent way to understand and teach the Micro/Nano systems technology.
- Simple scaling analysis based on simple modelling helps us assess the suitability of an application to miniaturization.



Now, before we go into that first thing is that why we should study the scaling and why this scaling is important. The first thing is that the scaling actually decides the performance of the device; it means, that let say whatever phenomena or whatever physics it is relying on, how that physics changes while we are going in lower dimension or in higher dimension. Then, some micro devices are there which will not at all work at like a macro scale or larger dimension, because that physics or that particular working principle will not be significant at larger scale.

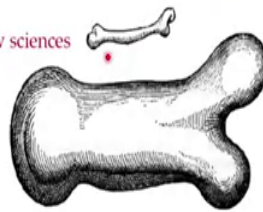
And scaling effect actually provides you an excellent way to understand the micro nanoscale devices how do they work and how they are different from our macro world. And simple scaling analysis based on simple modeling actually helps us assess that whether some device or whether one device is suitable for a miniaturization or not, right. Because, before making anything we need to understand that whether miniaturization is possible for that device or not and second thing is whether it will be suitable if the device is suitable for miniaturization or not.

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## Galileo's Bone

- To illustrate briefly, I have sketched a bone whose natural length has been increased three times and whose thickness has been multiplied until, for a correspondingly large animal, it would perform the same function which the small bone performs for small animal. From the figure shown here you can see how out of proportion the enlarged bones appears... Whereas, if the size of a body be diminished, the strength of that body is not diminished in the same proportion; indeed the smaller the body the greater its relative strength.

Dialogues concerning two new sciences  
by Galileo Galilei



Credit: Prof. Ananthasuresh, IISc, Bangalore

To explain the scaling effect, we will use this Galileo's Bone example. So, here what we are doing that, let us take this small bone and then enlarge it by 3 times in x y z dimension. Whereas, the thickness of the bone will be enlarge by the same amount what it is required for

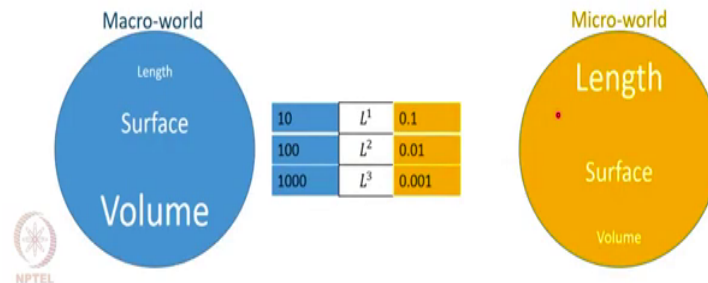
a 3 times larger animal, ok. Now, if you see then you will see that the largest bone will appear like this which looks like very much disproportionate, right.

Now the point is the small bone really weak as it looks like compared to the larger bone. In fact, it is not because many a times we have seen that the smaller body even have greater relative strength. Now, what we are missing here is that just reducing the dimension by 3 times or 4 time does not mean that, all the properties also will reduce by 3 or 4 times. So, for that we need to understand that the how the scaling works.

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## Basic Scaling Law

- **Volume-related** phenomena decrease much more rapidly than **surface-related** phenomena, which in turn decrease rapidly than **length-related** phenomena.



This is the basic scaling law where, you can see that the let us say some phenomena which is related on the single dimension or like only the length scale, then it goes by 10 times means increased by 10 times or reduced also by 10 times, while we increase or decrease the size. Whereas, some phenomenon's which are like surface dependent they go by 10 times or point

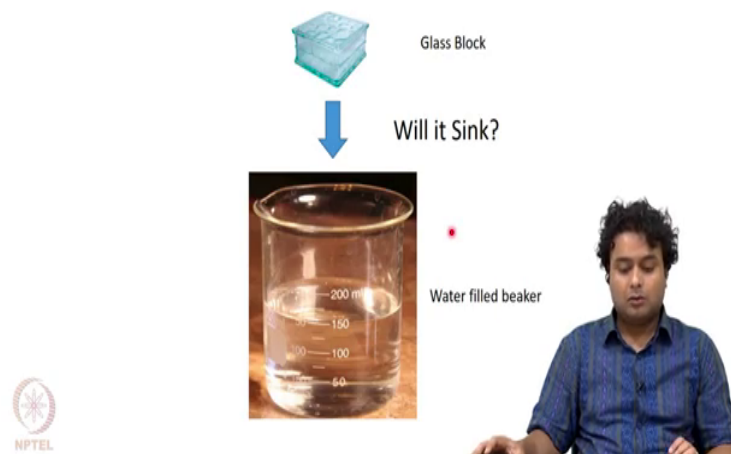
100 times higher or hundred times lesser, if we scale up or scale down. Similarly, the volume related phenomena's are like scaled up by 1000 times or reduced by thousand times while we increase the dimension by 10 times.

So, what does it mean, means that in macro world; in macro world the volume forces can be dominant right, because the volume forces are like it is  $L^3$  so, thousand times higher. Whereas, if we now let us say reduce the  $L$  by 10 times, then this force will reduce by 1000 times actually. Whereas, the surface or the length dependent forces will not reduced by that thousand times will not depend will not reduce by 1000 time, it will only reduce by 10 times.

So, some force which are volumes dependent or which have volumetric forces those will be dominant in the macro world whereas, the one dimensional or two dimensional forces like surface tension etcetera will be dominant in the micro world.

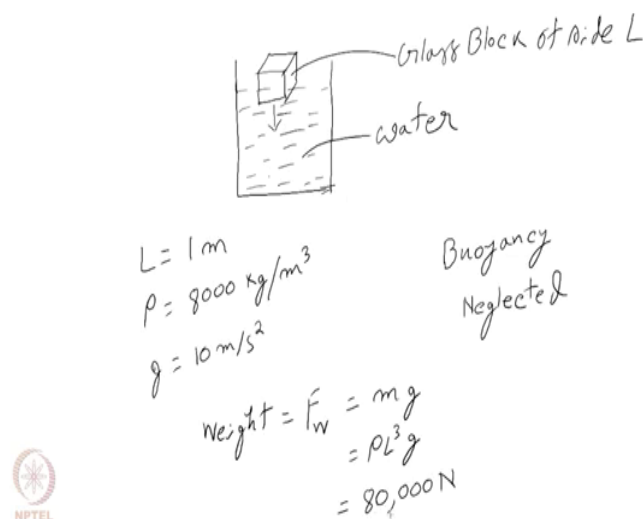
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### Example: Surface tension at small scale



Now, we will work out these example and we will see that how exactly this volumetric force and like area dependent force or length dependent forces are very much different as you scale up or scale down. And for that we will work out this example where we are taking simple a glass of water and we are taking glass blocks which have very high density than compared to the water. Now, the question is will the block sink or not and to answer this question we will work it out on paper.

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So, we have a glass beaker and inside the beaker we have water and then we have a glass block, now the question is whether the block will sink or not, right and this block this is glass block of side L. So, this is like a cube where each side is L like length is L and this is water.

Now, let us first start with the our known world, that is macro world and let us assume that L is equal to or each side of the glass block is 1 meter, then we take the density of the glass as

8000 kg per meter cube. So, this is much higher than density of water and  $g$  that is gravitational acceleration is equals to 10 meter per Second Square. Actually it is 9.81, but ah for our simplicity of calculation let us take it as 10 meter per second square.

Now another important thing is that you know that whenever some block or object is merged into the submerged or like a is dipped into the water or any kind of liquid then there is a buoyancy force, which tries to push it up, but in this case we will neglect buoyancy. So, in this case we will neglect buoyancy.

So, what I am considering here that the weight of the block that will try to like make the block sink and buoyancy will try to push it up, right. So, we are neglecting buoyancy, but there will be another force that is surface tension force and we will see that how the weight and surface tension force actually compares in this scale. So, let us first calculate the weight of the block.

So, what is weight? Weight let say gravitational force or weight is equals to  $F_w$  and equals to  $m$  into  $g$  where  $m$  is mass of the block and that mass is  $\rho$  into  $L$  cube into  $g$ , because mass is  $\rho$  into volume where  $\rho$  is the density, right. And  $L$  cube is the volume, so mass density into volume into  $g$  and that if you put all the values, then you get about 80,000 Newton. So, the weight is 80,000 Newton, ok.

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Surface tension coefficient  $= \gamma = 0.073 \text{ N/m}$

Upward surface tension force  $F_{ST} = \gamma S$

$$S = 4L$$

$$\uparrow F_{ST} = 0.073 \times 4 \times 1$$

$$= 0.292 \text{ N}$$

$$\downarrow F_W = 80,000 \text{ N}$$

$$F_W \gg F_{ST}$$

Block will sink



Now, I assume that the surface tension coefficient is equals to gamma is equals to 0.073 Newton per meter, ok. Now, what is surface tension force so, this will at upward. So, upward surface tension force, let us call it  $F_{ST}$  is equals to gamma into  $S$ . What is this  $S$ , this  $S$  is actually that like the length of the interface or the perimeter of the glass block. So, what is this  $S$ , in this case as the block is like a cube. So,  $S$  equals to it has 4 sides it is in contact with the water right. So,  $S$  equals to 4 into  $L$ . It is like this glass block and this has 4 sides 1 2 3 4, right and, this 4 side are 4 sides are in contact with the water. So, the perimeter is 4  $L$ .

So, what is the surface tension force?  $F_{ST}$  is equals to 0.073 into 4 into 1 and that is equals to 0.292 Newton and earlier we have seen that  $F_W$  was equals to 80,000 Newton, we are neglecting buoyancy, ok. So,  $F_W$  is much greater than surface tension,  $F_W$  is downward



force it is acting downward whereas,  $F_{ST}$  is acting upward and downward force is much higher than the upward force. So, block will sink.

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$$\begin{aligned}
 L &= 1 \text{ Mm} = 10^{-6} \text{ m} \\
 F_W &= \rho L^3 g \\
 &= 8000 \times (10^{-6})^3 \times 10 \\
 &= 8 \times 10^{-14} \text{ N} \\
 F_{ST} &= \gamma S = 0.073 \times 4 \times 10^{-6} \\
 &= 2.9 \times 10^{-7} \text{ N} \\
 F_{ST} &\gg F_W \\
 \text{Block will float}
 \end{aligned}$$

Now, let us try the same problem keeping everything same just in lower dimension. Means, now I take a block of 1 micrometer, earlier we took a block of 1 meter, now we are taking a block of 1 micrometer and 1 micrometer is equals to 10 to the power 6 minus 6 meter, right ok. So, now, let us again calculate the weight as well as surface tension force. So,  $F_W$  is equal to  $\rho L^3 g$  is equals to 8000 kg per meter cube into 1 micrometer cube, means 10 to the power minus 6 whole cube into  $g$  is 10 and that gives us 8 into 10 to the power minus 14 Newton.

So, this is the weight force while the block or this is the gravitational force or gravitational force while the block is of size 1 micrometer, each side is 1 micrometer. Now, what is a

surface tension force  $F_{ST}$  is equal to  $\gamma$  into  $S$  and that  $\gamma$  is  $0.073 \times 10^{-4}$  to the power minus 6, because now this is 1 micrometer. So,  $4 \times L$  and that  $L$  is 1 micrometer that is  $10^{-6}$  and that gives us  $2.9 \times 10^{-7}$  Newton;  $2.9 \times 10^{-7}$  Newton.

Now, if we compare the surface tension force with the gravitational force or with the weight, then it is almost  $10^7$  times or  $10^6$  times higher than the weight, right. So, if surface tension is much much higher than if weight, right. So, the block will float; why it will float because now the upward force is much higher than the downward force, ok.

Now, as we see here that we did not change any physics right, we just change the scale. And, because of that scale change earlier the weight force was 80,000 Newton that became  $8 \times 10^{-14}$  Newton; whereas surface tension force became  $2.9 \times 10^{-7}$  Newton.

Now, point to be noted here is you see surface tension force earlier was 0.292 Newton and now it has become  $2.9 \times 10^{-7}$ . So, the surface tension force has also decreased and by several order, right. Surface tension force has also decreased by several orders, but the weight force or the gravitational force was 80,000 Newton and from that it came down to  $8 \times 10^{-14}$  Newton.

So, the gravitational force decreased by much larger amount right by much larger amount the volumetric force has actually decreased, because gravitational force is volumetric force and it has a cube like in it is power it has 3. So, it became  $10^{-18}$ , whereas for surface tension force it has only  $L$ . So, it has reduced by  $10^{-6}$  right.

Now, where is that critical length, where the gravitational force or the weight and surface tension force are exactly equal at that point the block will just float, right.

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What is the critical length where  $F_w < F_{ST}$ ?

$$F_{ST} > F_w$$

$$4\gamma L > \rho L^3 g$$

$$L^2 < \frac{4\gamma}{\rho g} = \frac{4 \times 0.073}{8000 \times 10} = 3.65 \times 10^{-6}$$

$$L < \sqrt{3.65 \times 10^{-6}} = 1.91 \text{ mm}$$

$$L_c = 1.91 \text{ mm}$$



So, the question is what is the critical length where  $F_w$  is equals to  $F_{ST}$  right; so, let us let us compare it, right. So, at which point  $F_{ST}$  will be just crossing the  $F_w$ . So, let us call it not equal to rather just crossing. So, that it will just float where the surface tension force will be just higher than the block of weight of the block.

So, that what is the surface tension force it is  $\gamma$  into  $S$ , that means,  $\gamma$  into  $4L$  will be greater than  $\rho L^3 g$  and then we get that  $L^2$  less than  $4\gamma$  by  $\rho$  into  $g$  and if we put all these values, then this is  $4$  into  $0.073$  and below we have  $8000$  into  $10$ , and that becomes  $3.65$  into  $10$  to the power minus  $6$ , ok.

So,  $L$  becomes square root of  $3.65$  into  $10$  to the power minus  $3$  and that is equals to  $1.91$  into  $10$  to the power minus  $3$  meter. So, let us make it millimeter. So, the  $L_c$  or the critical length is equals to  $1.91$  millimeter and what does it mean, means that as we are sinking down the

block from 1 meter to 1 micrometer at 1.91 millimeter it will just cross over the weight, right. So, at we need minimum size of like minimum size of 1.9 millimeter to cross it over. If we make it even lesser the block size that each side of the block then it will definitely float and above that it will not float.

And, I remember that here we are always neglecting buoyancy, if we add buoyancy then we will have one additional upward force. In that case we will even require lesser amount of surface tension force. So, even larger block can float there right.

So, you see here we got the critical length as 1.91 millimeter right and as you have see in many times that some insects actually can walk on the water. And, if you see then their legs this usually this insect legs diameter can be much lesser than this 1.91 millimeter almost it is about 2 millimeter right and insect legs diameter can be even lesser than 2 millimeter and much less than 2 millimeter and their weight is also very small. So, those insects can easily walk on water using the surface tension forces.

Thank you.