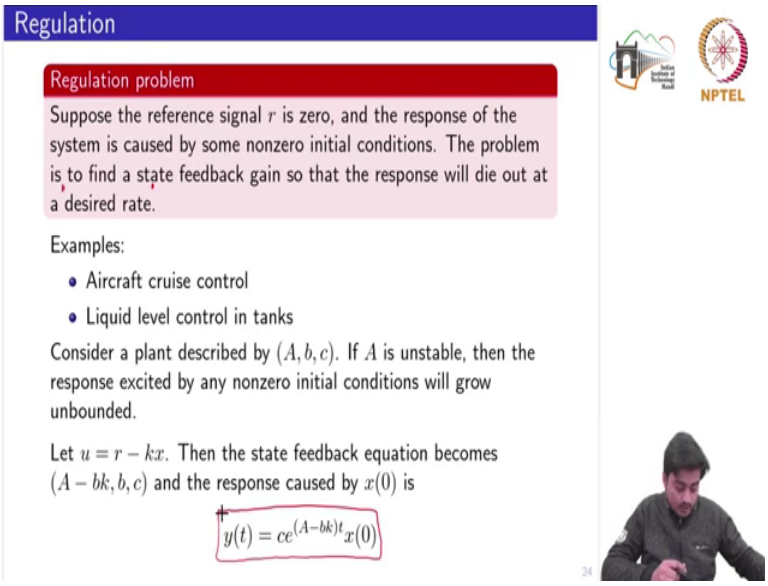
Linear Dynamical Systems
Prof. Tushar Jain
Department of Electrical Engineering
Indian Institute of Technology, Mandi

Week - 05
State Feedback Controller Design
Lecture – 32

(Refer Slide Time: 00:12)



So, so far we have discussed about the State Feedback Control Design problem for a generic case, where we have assigned the eigen values to some desired location. And the further classification of that generic state feedback control design problem are 2; first is the regulation problem and second is the tracking problem which we will discuss later.

So, the regulation problem deals with the effect; that suppose the reference signal r is zero and the response of the system is caused by some non-zero initial conditions. The problem is to find a state feedback gain, so that the response will die out at a desired rate

Now, this computation of the desired rate is basically consists of defining the objectives. The control objective which you can define in terms of the eigen values; if where you want your eigen values of the closer closed loop systems to be placed.

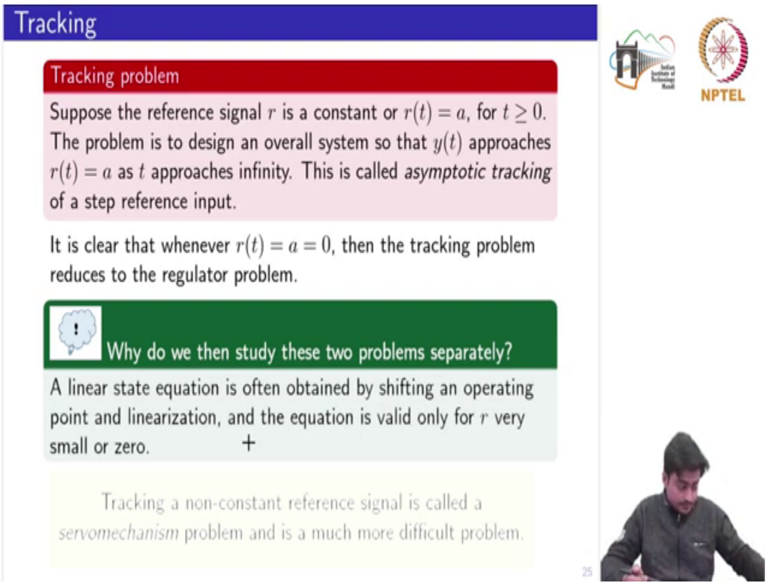
So, there are many examples. So, the first one the aircraft cruise control or the liquid level control in tanks. So, if the so consider the plant described by the pair A, b, c. If the system A is already a stable, then it is pretty much clear that for the zero reference signal and for non-zero initial conditions, the response of the plant would be a stable it would tend towards to zero.

But in that case you would not be able to achieve certain desired rate. So, we need to design a state feedback controller gain. So, that we could place the eigen values to some desired location. Now, if the if A is unstable then for any non-zero initial condition the response will go unbounded.

Now to first of all, we need to make the closed loop stable which we have done so far by using u the control signal s r minus k times x where k is the feedback controller which we need to design such that the response caused by the non-zero initial condition is given by this.

So, we have seen this equation a number of times and this is only the response due to the initial conditions, but the response due to the external signal reference r would be zero because the reference signal is zero. So, this problem could be pretty much dealt in a similar way what we have done so far for designing the state feedback controller design.

(Refer Slide Time: 02:48)



The more involved problem is the tracking problem where, so, suppose the reference signal are is a constant or r of t is some or let us say the magnitude is a for t greater than equal to zero.

The problem is to design an overall system that is the closed loop system, so that the response y of t approaches the reference signal at a as t approaches infinity. So, this is called the asymptotic tracking of a step reference input. Now, within these 2 classification of the state feedback control design problem; now if you suppose if you put r of t as or a is equal to 0 then the tracking problem would reduces to the regulatory problem.

Meaning to say that for so, if the first an implication of this one that for any given a if we design the controller as a regulator as a regulator then would it be possible that the tracking would always happen for any non-zero constant reference signal.

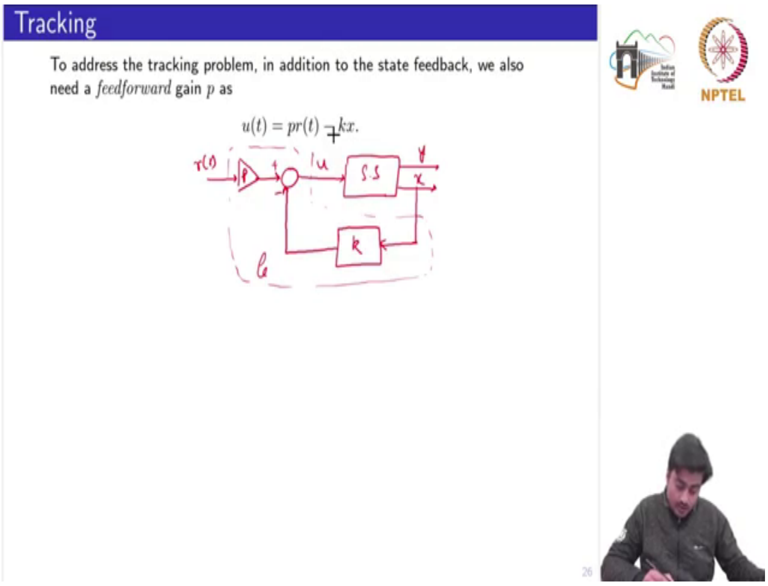
So, the answer for this would be certainly not; because, it would depend on the DC gain of the closed loop system also. So, the first question raises then why do we then study these 2 problems separately? So, the first; so the straightforward answer to this one that most of the linear systems what we are discussing are derived basically by linearization of a non-linear system around some operating point.

Now, these operating points may change from time to time depending on the operating conditions. So, whenever the operating condition changes the reference signals would also change. So, we need to ensure that for the same system when we do the linearization around different operating points, resulting in 2 different reference signals that all those signals should be trackable.

Now, there is so, suppose now if the reference signal is no longer a constant signal or it is a time varying signal let us say a sinusoidal signal then this that problem is called a servomechanism problem and that problem is a much more difficult problem.

So, we would not be addressing the servomechanism problem, but along the solution of the tracking problem those problems would also be dealt with. So, now, we will see that how you can achieve the tracking for any constant reference signal.

(Refer Slide Time: 05:37)



So, to address the tracking problem in addition to the state feedback, we also need a feedforward gain p defined by this. So, now, if you see a structure of the state feedback system this is the state space system where we have two outputs let us say y.

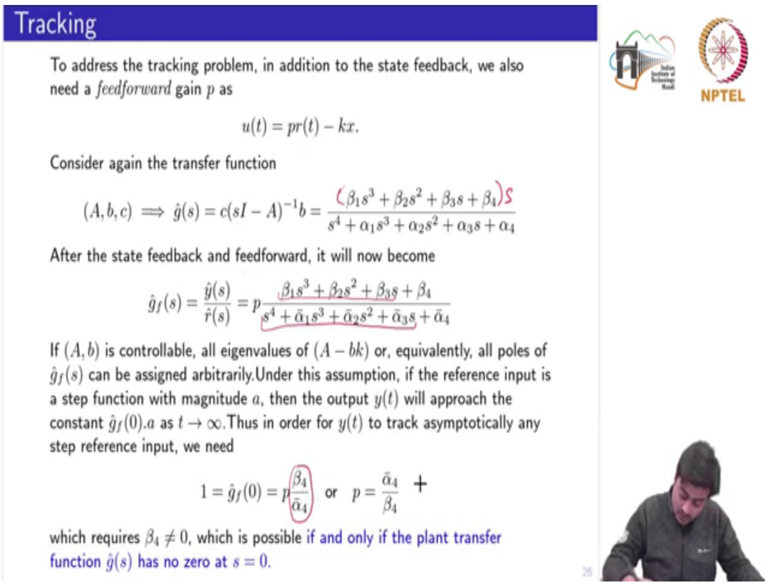
And say suppose, we also have the measurement of the state and this is u. So, so far we have considered the design of the state feedback controller by considering k, taking the feedback from x and putting it back to the state space system.

Let us say this is 0 or say for example, lets put it r of t. So, this would be plus and minus. So, here u would become r of t minus k times x, but here we have also introduced another degree of freedom by p which would turn into another gain p by reference signal r, ok.

So, now we have two degree of freedom for the designing of the controller one is the state feedback gain k and another is this p. So, this is the controlled structure. So, this complete part we will call the controller and this is nothing but this equation u t is equal to p r minus k x.

Now here, we need to design 2 gains; one is p and k such that the y becomes equal to the reference signal or to a constant value a.

(Refer Slide Time: 07:33)



Consider once again the transfer functions what we had seen earlier for the pair A, b, c denoted by g hat s.

So this transfer function is written for n is equal to four, ok. So, we have seen earlier all these beta i's and alpha i's of the plant. Now, after applying the state feedback and the feed forward, the overall transfer function of the closed loop system would become from y hat over r hat, ok.

So, the transfer so the denominator would contains all the eigen values, all the desired eigen values denoted by n to the power written in terms of a characteristic polynomial and their co-efficients with this alpha i bars and the numerator would not change, as we have seen earlier.

Now, in addition to this overall transfer function, we have additional parameter p associated to the overall transfer function. So, if A, b is controllable if the original A-b pair is controllable, then all the eigen values of the state feedback of the new state feedback state matrix A including the state feedback gain or equivalently all poles of this g f hat can be assigned arbitrary hm.

This is one of the results we had studied earlier. Now, under this assumption, if the reference input is a step function with magnitude a then the output y of t will approach the constant g hat f at 0 dot a as t tends to infinity.

So, we basically we are interested in computing what would happen to the y in time domain as t tends to infinity; which is equivalent to computing the DC gain of this transfer function also. And the DC gain can be computed by putting s is equal to 0 in this overall transfer function.

So, if I put s equal to 0 all this part would go away and similarly, all this part would go away and we would have y hat of 0 is equal to g hat f 0 times a because a is the DC value; DC value of the reference signal.

Now, as all these numerator and the denominator part will go to 0, the remaining part is beta 4 by alpha 4 bar and we want this g hat f of 0 is equal to 1, in that case only we would have y hat is equal to r hat in the steady state.

So, on the right-hand side we would have p times the ratio of beta 4 by alpha 4 bar. So, if we substitute p as the inverse of this factor which requires that beta 4 should not be equal to 0, the tracking would be possible if and only if the plant transfer function g hat s has no 0 at s is equal to 0. And because if there is an s, if there is a 0 available at the origin then we know that the numerator cannot be changed.

The i the roots of the numerator polynomial cannot be changed. So this s would stay as it is. If this is s stays as it is then the DC value would definitely be 0. Now, in that case we cannot track any reference signal because the response would always be 0. Since, the denominator is always stable Harwood's polynomial.

So, we compute p by this then the; so there are 2 design parameters the state feedback gain k where first we have computed. So, that we could place the eigen values at the desired location. The second design parameter has been computed such that the tracking to a constant signal is possible; which is computed in this way, ok.