# Optical Engineering

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## Lecture – 09
## Imaging equation for thick lens using ABCD matrix

Good morning. So, last week we looked at how we trace rays through a system using matrix optics and we in the last class looked specifically at how we can take the matrix, the ABCD matrix of any system, any optical system and convert it into a matrix similar to that of a thin lens. And doing so made analysis of systems easier; because the way we analyze a thin lens is simple and we could use that knowledge and analyze any system once we have done this conversion.

Doing so gave us some knowledge, it gave us the definition of some planes called the principal planes and what became important was, now you will make your measurements from those planes. So, if you are going to convert your system into a thin lens system, it is not a thin lens; but you are going to analyze it as if it were a thin lens.

In order to do so, you will now start measuring distances from these two important points on the optical axis, the principal points or in other words the principal planes that run perpendicular to the optical axis through those points. So, I want to just extend that idea a little bit, ok.

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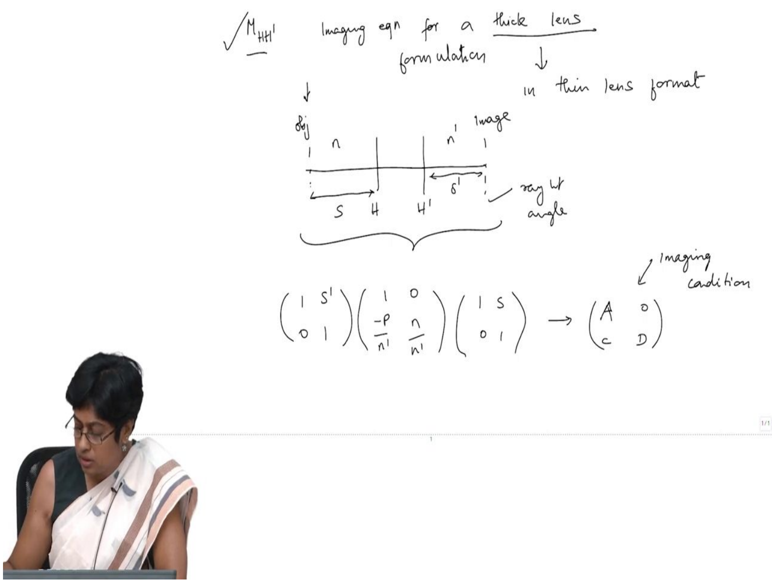


So, in the last class we had arrived at the system matrix in general and we said this is the matrix which we called H H’; because it now was converting the any system into that of a thin lens and the distances were being measured from the principals planes H H’. We took any optical system and said we can consider it to be in this form.

I can go a step further and say, well suppose I was doing imaging with this system. I took any optical system; all optical systems are not necessarily doing image, they may have lenses in them, but that does not mean they are all imaging systems. But suppose we want to look specifically at an imaging system; in that case I would say this matrix that we have developed, I need to now take into account an object distance in an image distance and I can develop another matrix which should represent or help me get to an imaging equation.

So, I want to get to an imaging equation for my thick lens, let us say for thick lens formulation; but this thick lens is now in thin lens format, ok. So, I will now say; that means, I have my system that is defined by these planes H H’, of course I have this matrix. I now have some distance and that is my object distance and we will call this S and I have an image, so my object is in this plane, my image is in this plane, I call this S dash.

So, just as we took the system matrix and added distances to find out where these principal planes were; so now, I am going to develop the matrix for this entire system. What does that mean, it means if I am given the ray height and ray angle at this point; that is the ray coming from a certain height of the object with the particular angle, I will be able with this new matrix that I develop, be able to say what is the ray height and angle at the image plane ok, that is what I am going to be able to do. $\begin{pmatrix} 1 & s' \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{p}{n'} & \frac{n}{n'} \end{pmatrix} \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}$

Yes, so that means, it is an imaging condition right, irrespective of the angle everything from one point on the object goes to one point in the image. So, this is nothing, but the imaging condition. And that is what we are trying to arrive at the imaging condition for our general matrix.

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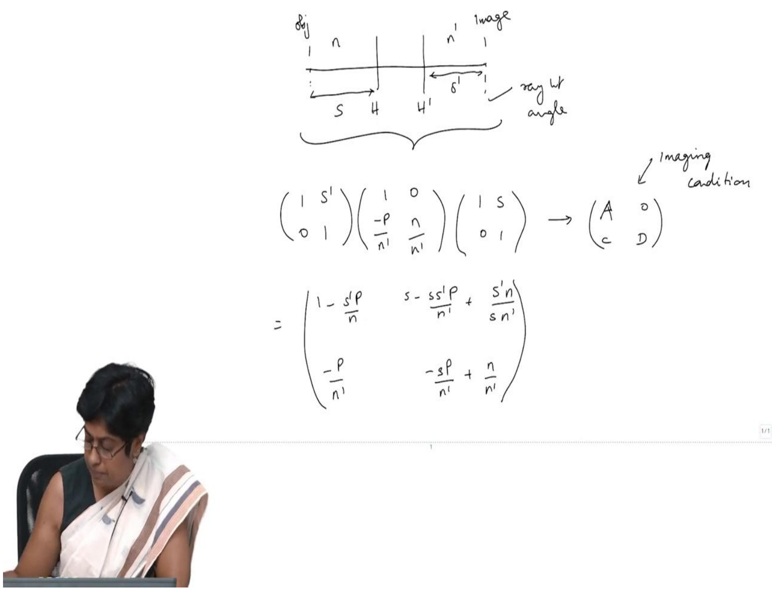


So, if we carry out this product, right. And I will just do that here, ok. So, I have just carried out that multiplication; a particular term and equated to 0.

$$\begin{pmatrix} 1 - \frac{s'p}{n} & s - \frac{ss'p}{n'} + \frac{s'n}{sn'} \\ -\frac{P}{n'} & \frac{-sp}{n'} + \frac{n}{n'} \end{pmatrix}$$

$s - \frac{ss'p}{n'} + \frac{s'n}{sn'} = 0,\ \frac{n}{s} + \frac{n'}{s^{i}} = p \quad \frac{1}{s} + \frac{1}{s'} = \frac{p}{n}$

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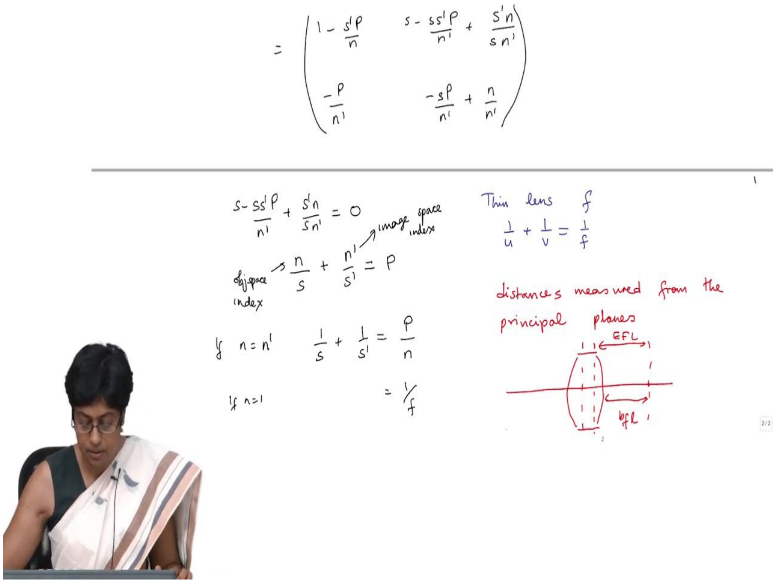


And the moment I do that, this will reduce to n by s plus n dash by s dash is equal to p, just rewriting and this should look familiar to you. Now, when we had a genuine thin lens with focal length f, we arrived at an imaging equation which was $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. In this case we had assumed that the object refractive index, this index of the object space and the refractive index of the image space were equal and we arrived at this equation.

And you can now see that the equation we have here is very similar; n is the index of the object space, this is of the image space. If I assume they were equal; so let us say if $n=n'$, then I would have $\frac{1}{s}+\frac{1}{s'}=\frac{p}{n}$. Or if they were equal to 1, I would just have. So, let us say if n = 1, then this would just be 1/f.

And you have exactly what we expected to get, you have an imaging equation; but remember how what we start with, we have not used the thin lens anywhere in this, yet we have an imaging equation that looks identical to the equation of imaging for a thin lens, ok. This is a really powerful result; because you are saying that I can take an optical system which has 100 lenses in it with all the spaces in between and I can now deal with it as if it were a thin lens.

The important point to take is that, any measurements that I make will now be made from the principal planes. Why is this important? It is important or rather maybe I should say when is it important to have this ability? When I want to deal with the rays in object space and the rays in image space and I don't really want to know or maybe it is not possible for me to know what is actually happening within the system.

But I have the information of the input and I can maybe measure the information of the output rays, I can use this knowledge and of course, you will also need the principal plane locations. So, the important point is distances are measured from the principal planes. And just to reiterate, if you are talking about a system; in this case I am just doing a simple thick lens, right. But of course, this applies to a system as I said which could have many lenses and spaces in between.

Once you have located the principal planes of your system, the focal distance is then defined as the distance; so the effective focal length is the distance from the principal plane. And there is some confusion in some literature talking about back focal length, front focal length also being mentioned measured from the principal plane, some literature says from the vertex of the lens; we will always define it as from the vertex of the lens.

So, in this case this is the back focal length, ok. So, we will move on from matrix optics here; of course, you will get an exercise assignment on this. But with these few examples that you have taken, you should be able to see now how you can use the idea of matrix optics to convert a system to a simple system that can be analyzed simply, ok. You should also have the idea now it is quite simple to do ray tracing with this, you do not have to worry about the sign.

Once you have got your sign convention and your inputs are correct, you do not have to worry about where the image is forming for the next surface or the third surface or the fourth; just multiply a matrix in the correct order. And then if you have got your input information; that is ray angle and ray height of the input ray, you will get that information for the output ray. And all of this is happening in a two dimensional plane, so we said this is happening for the meridional rays, as they traverse through the system, ok. So, I hope that is clear yes, ok. So, we will move now to the next part.