

Optical Engineering
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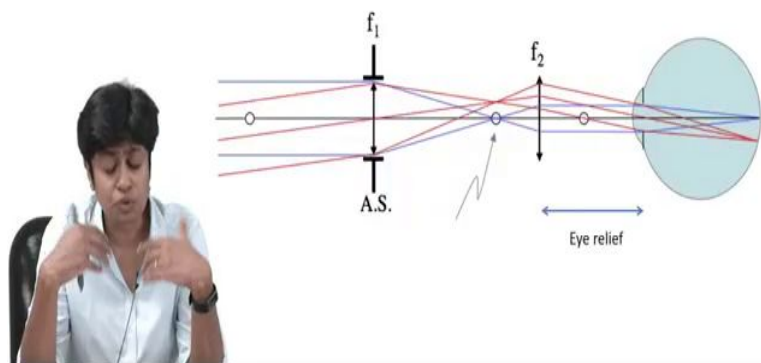
Lecture - 06
Aperture stop – Part 1

Good morning, we are going to continue with this course on Optical Engineering. Yesterday we looked at some important definitions, one was of Aperture stop. So, basically we looked at what are the optics that limit the cone of light traveling through the system and in extension of that we looked at some planes which were arrived at from the aperture stop namely the exit and the entrance pupils.

In addition to learning about these elements or these planes, we also needed to know which rays to trace through the system and we defined therefore, two important rays; one was the chief ray and one was the axial ray. And the definition of these rays are kind of interlink to the definition of the aperture stop because we say the chief ray goes through one part of the aperture stop of the axial ray will go through another part and so on.

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Position of exit pupil



So, continuing that or rather we ended the class yesterday looking at the exit pupil which is the image of the aperture stop seen through all the elements after the aperture stop. So, it is

very clear from that definition if you do not have any elements after the aperture stop, the aperture stop itself will be the exit pupil. But given that there are elements, then you will find the location of the image and that becomes an exit plane. And we said four applications where you as a user are going to look into your system the exit pupil must of course, lie outside the entire system.

Now, say what can you think would be different in an optical system that was being used in the daytime compared to an optical system that is going to be used at night. Let us say you had a night vision binoculars compared to a binoculars, you would use during daytime. What can you imagine you would have to do differently in your design of optics? Is there a difference?

Student: Yes (Refer Time: 02:30).

Yes, there is a difference, why is there a difference or what is the difference ok? So, larger aperture so that you get more light, but in terms of the exit plane what can I say? Same principle, you said larger aperture I must have a larger exit pupil right. What happens to our eye when we are viewing something in bright light as opposed to viewing something when it is much darker or dimmer?

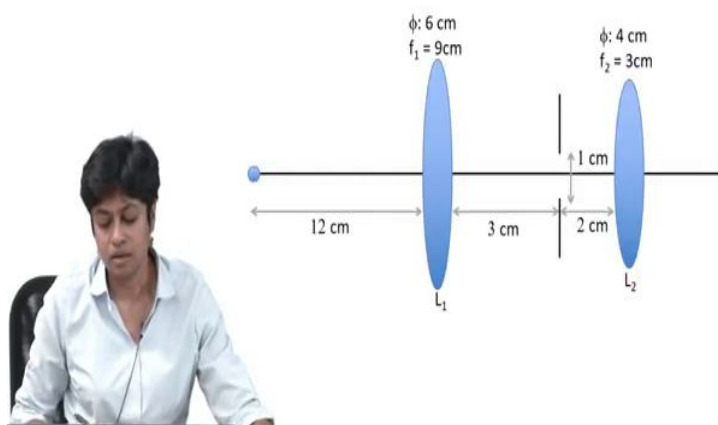
The pupil of our eye changes. Do you know the range of sizes our pupil can vary through ok? So, it can vary from about 2 millimeters or the very bright conditions the pupil of our eye will be to about 2 millimeters and it can go all the way up to a diameter of 8 millimeters if the lighting around goes down ok. That is the range for a human being. If you have seen a cat you can see the cat's pupil can be very much large as a remarkable change from a slit to a very large opening which is why they have so much better vision. Well one of the reasons why they have better vision at nighttime.

For a human being we are seeing the pupil varies between 2 to 8 millimeters. So, clearly a binoculars or any system that is going to be used during night time as opposed to the daytime, the design must be such that the exit pupil matches the pupil size of the human depending on the conditions and which they are using it.

So, you have to take that into account when you are doing your design right. It is not enough to say the exit pupil must lie outside of course, that is true, but its size also depends on what are the lighting conditions you are working under because the human eye will respond to that and change according to that ok. So, you need to take all these things into account when you are doing your design.

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Example: Calculating Aperture Stop of a system



So, I have not actually told you how to find out the aperture stop. We did a nice definition and maybe the words were clear, maybe the words even were not clear, but basically I gave you a definition of aperture stop. And in all the examples we looked at yesterday we had very nicely one lens and one aperture very simple systems. But of course, your systems are not always going to be so simple. So, how do you decide which of the elements in a system is actually the aperture stop?

So, I am going a step further. I am not going to go to 100 elements or 50 elements. Let us just take 2 elements with power. So, in this example, here I have 2 lenses with power of course, their lenses and in between that I have an aperture of opening 1 centimeter ok. So, I have 3 elements in this system and now I want to calculate which is the aperture stop.

Now, I can take a guess given that the diameter of the lenses are much larger than the diameter of this aperture. It is quite likely that even in this case the aperture will turn out to be

the aperture stop right, but that need not be the case always, keep that in mind. It depends upon the focal lengths of the positions because it depends how the light is going to be bent and traveling through this system ok.

So, we will use this example to calculate and find out which of these elements actually is the aperture stop of the system. So, if you can draw this quickly because I will go to a new sheet where we will actually do the calculations. I want you to have these numbers. It is not drawn to scale and our goal is to calculate the aperture stop of this system comprising two lenses and one aperture. This ϕ here denotes the diameter of the lens ok.

So, it is that that tells you that this distance is 6 centimeters for the first lens and its 4 centimeters for the second lens. So, that is to do with the physical size. So, if you remember we kept talking about the aperture stop as being the element that limits the cone of light that reaches the image plane. So, I am going to calculate aperture stop by calculating angles. I am going to calculate the angle extended by each of these elements and find out which one has the smallest cone; which one has a smallest angle. So, my method of calculating aperture stop is calculating angles ok.

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Image of L_2 as seen through the first lens L_1

$$u = -5 \text{ cm} \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$f = -9 \quad v = \frac{uf}{u-f} = \frac{(-5)(-9)}{-5+9} = \frac{45}{4} = 11.25 \text{ cm}$$

Size Magnification $M = \frac{-v}{u} = \frac{-11.25}{-5} = 2.25$

Lens subtended by this image

$$y_i = 2.25 \times 2 = 4.5$$

$$\theta_1 = \tan^{-1} \frac{4.5}{12+11.25} = 10.95^\circ$$

2) Angle subtended by L_1

$$\theta_2 = \tan^{-1} \frac{3}{12} = 14^\circ$$

3) Angle subtended by L_2

$$\theta_3 = \tan^{-1} \frac{4.5}{12+4.5} = 2.6^\circ$$

So, in the first calculation I am going to calculate the image of the second lens as seen through the first lens ok. So, the object distance and now this is where sign conventions are

going to be very important. The object distance, can you tell me what is the object distance from your figure? Your second lens is the object now, do not get confused that it is a lens, it is just now an object, you are treating it as an object. What is you, the lens you are imaging with is the lens L1 right. So, what is the object distance?

Student - 5 cm.

-5 cms right, because the object is to the?

Student: Right side.

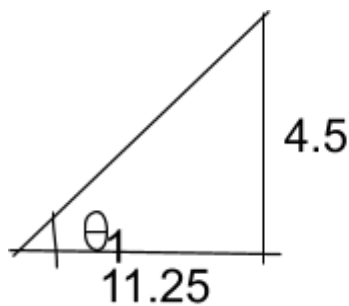
Right of the lens. Now this is another point which is slightly tricky. The focal length, yes it is a convex lens, but I am working in image space. So, I will not put down +9 cm. You are given the focal length as 9 cm, I will put down -9. I am not going to write down the units that are cm everywhere ok. So, although it is a convex lens, we are working in image space and by this sign convention I must take that fact into account by indicating a minus sign over here.

Now, we are using a very simple Gaussian lens formula which is this: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$, $v = \frac{uf}{u-f} = \frac{(-5)(-9)}{-5+9} = \frac{45}{4} = 11.25$ cm. Where does the image occur? It should not be surprising to us, it occurs on the same side as the object because the focal length is 9 cms, but the object distance was 5 cms the object is less than focal distance away. We have a virtual erect magnified image on the same side as the object. What is the object? Lens L2 right ok.

But what we wanted to find out was an angle subtended by this image. So, I have not got there yet; I have only the location of the image I. So, this just tells me location, I still need to calculate the size of the image in order to get the angle subtended by the image. So, to get the size I will use magnification and magnification is $m = \frac{-v}{u} = 11.25$, the object distance was -5 and so, this becomes $m = \frac{-v}{u} = \frac{-11.25}{-5} = 2.25$ We know it's magnified; now we know exactly how much it's magnified by 2.25 times. This is equal to the image height by the object height. Again you have been given the size of the object that is the radius of that first lens sorry second lens. So, I can find the image height y_i , it is $2.25 \times 2 = 4.5$ cm ok. So, its radius is 2 cm right, in other words it's 4.5. Now I have all the information; I need to calculate the cone of light I want to calculate the cone subtended by this image and I know I must for a new

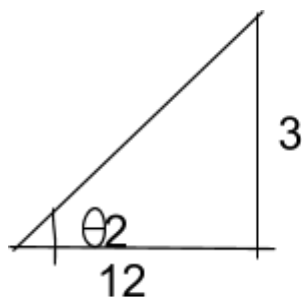
aperture stop definition says as seen by a point in the object space. So, to make this since I have to make it consistent I would do every calculation from the same point in object space and I will make it easy and say we will choose the point from which I am making all these observations.

Let me just choose the object location ok. So, in our example in our overall system the object was located 12 cms away from the first lens. So, the angle I need to find out is I have an object here. I now say I have 12 cms to some point and then I have another 11.25 cms to this image of the first element and I have 4.5cms. This is the angle subtended by the image right



So, I am interested in theta which is $\theta_1 = \tan^{-1} \frac{4.5}{12+11.25} = 10.95^\circ$ So, the cone of rays that the aperture the size of that second lens would allow through the system taking into account bending is 10.95° ok. So, I have one piece of information ok, but that is only this element. I now need to do the second element and I used up all my space here ok.

So, I now need to calculate the second one right. For the second one, let us just do the angle subtended by the lens L1 directly. Now for L1, there are no other imaging elements before it. So, I do not have to calculate any image now I can directly calculate the angle and what will I get this dimension is the nothing, but the size or the radius of the lens.



$$\theta_2 = \tan^{-1} \frac{3}{12} = 14^\circ$$

if I did not have any other element in this system, I would say now very clearly the second lens is aperture stop of the system. It subtends a smaller angle and that itself is interesting because the diameter of the first lens is larger, but because the rays bend it's diameter directly is not playing a role in the maximum cone of light that makes it through the system.

So, if I did not have the third aperture, I could at this point say the second lens is an aperture stop of the system ok. But there is another element. Now that element has no power, but again when I am calculating aperture stop I do not have to worry about that. I will now use that element as the object image through lens L1 and carry out exactly the same procedure that I carried out in this case here ok.

So, I am not going to carry out that entire procedure, it is going to be exactly the same as what we have done in the first step. We can just maybe write out our parameters u in this case is minus 3 because the aperture is 3 centimeters away from lens L1. We are using lens L1 so, the focal length is minus 10 where in image space that is why the negative sign comes although it is a convex lens right. And again I will calculate the location of m image, the size of the image and I will find out the angle subtended by the image and that will turn out to be 0.75 is this dimension; this is 12 plus 4.5, this is theta 3.

So, $\tan^{-1} \frac{0.75}{16.5} = 2.6^\circ$. So, when I look at these simple system three elements, the element that limits the cone of light that reaches the image plane is the aperture stop. In this case, we kind of guessed that at the beginning because its diameter was so much smaller than the other elements.

But we now have mathematical proof that the aperture that lies between these two lenses is actually the aperture stop of this system ok. So, this is how you calculate aperture stop. So, this is the logic and the point that you should be getting now is it is not a diameter alone that controls the angle of rays that go through because this is an optical system.

It has elements with power; that means, it has elements that bend rays. It is quite possible that rays get bent and are able to make it through the system around or are not able to make it

through the system depending on how they are bent through the system and you have to take that bending into account.

And that doing this calculation where you find the image because the image is a result of how light has bent and travelled through the system right. And therefore, that gives us a way of calculating the location of the aperture stop or which of the elements is actually the aperture stop.