Optical Engineering
Prof. Shanti Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Madras

Lecture – 44
Diffraction Grating

So, for the rest of the class I want to talk about the Diffraction Grating.

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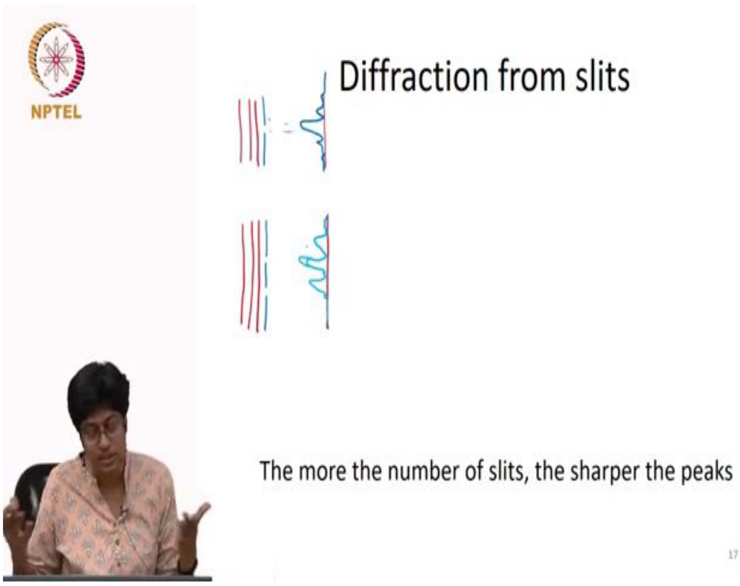

So, let us look at what happens when I have a slit now by now you should be able to yourself know or predict what is going to happen. If I have a single slit of light and I have a plane wave incident it is the slit is small enough on a plane over here. If I look at the intensity distribution I am not going to get one sharp region with shadow on either side, but instead I am going to get something like this, something like this.

If I now have two slits so let us say I have two slits and I have a plane wave incident each of these slits will create a pattern like this therefore, what I see on a screen now is going to be the combination of those patterns right. So, I might see something like this and something as I go on increasing, so I am sorry I have drawn this very similar. As I go on increasing the number of slits I get sharper and sharper peaks so the more the slits the sharper the peaks.

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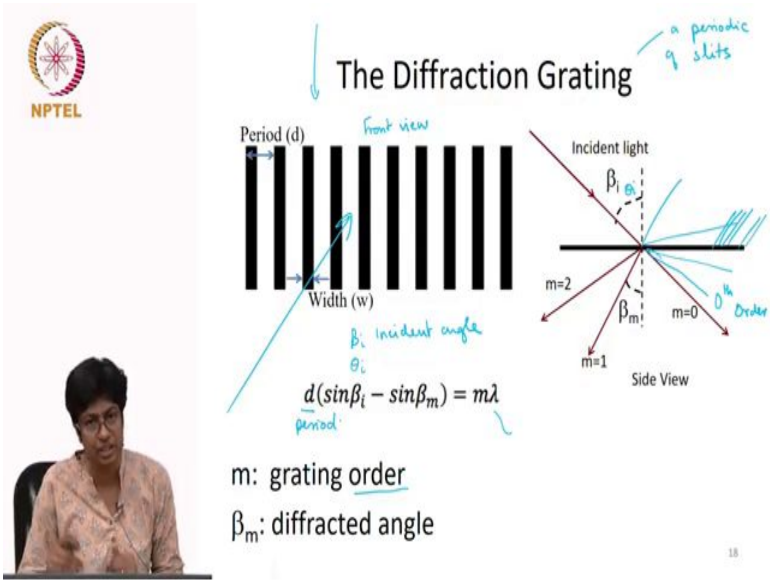

And so a diffraction grating is nothing, but a periodic collection of slits which plays a role in the output light is the width of each of these apertures and the period the spacing from one aperture to the next aperture. So, these two parameters w and d are very important when light is incident on the grating. So, this is a front view of the grating and now if I were to illuminate the light and I look at it from this point that is what you are seeing over here, so the grating exists like this right and you are illuminating the light.

One because I now have light constructively interfering at the select positions I have a number of what we call orders. One order continues as if there was no grating at all; this is the order that in a sense is following the geometric optics rules and that is why we call this the 0 th order. I may have some orders to this side I may have some orders to sorry I may have some orders to this side and if this is a reflective element I may have something's to this side.

So, instead of now you have one incident beam because of this periodic collection of slits you now generate a number of beams, but those beams do not take any random direction they take specific directions, according to the width the period the wavelength of the light the sorry.

The incident this and this period d, so this is what controls if the lambda the incident angle this is θ_i or β_i whatever you want. The incident angle, the wavelength and the period this controls the different directions the orders will take so if take very specific directions.

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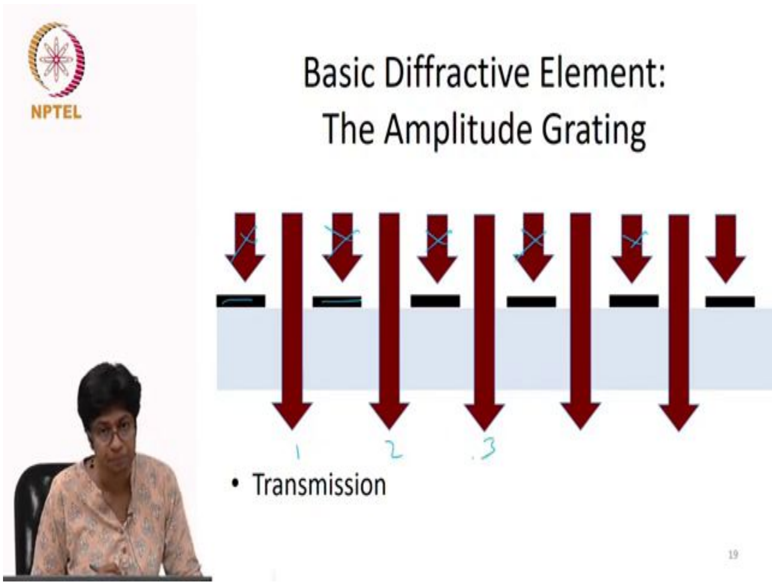

I can consider two kinds of elements I can conserve amplitude grating, so I have a periodic collection of slits; that means, in certain regions light is blocked, so it is blocked over here. And it is allowed to pass through in the other region, so it is a transmission element, but you can see straight away this is not a good element because these regions are blocks. So, I am losing half of the light right and you never want to have an element or an optical system where you throw away half of the light that is what you are doing with an amplitude grating.

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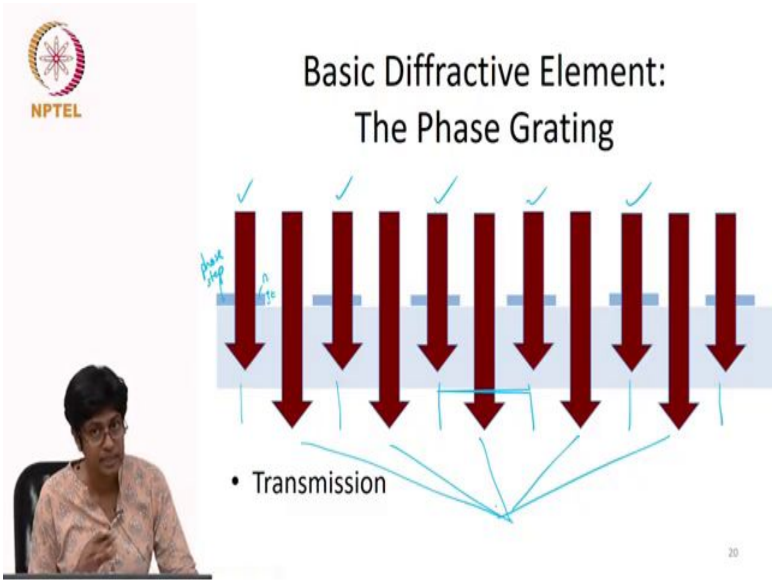

I could therefore, say let us have a face grating. I need some periodic structure. I need a structure that repeats, but causes a number of beams to form. I can do that by saying have one set of beams go through an optical or a phase step. So, they are now traveling this and this, and this, and this, and this are traveling a different optical path length because of this thickness and this refractive index. And that is what causes the multiple beams to form unlike the amplitude case where a blockage is what caused the multiple beams to form.

So, here you have this beam, you have this beam, you have this beam and these were blocked and here you have light coming through everywhere. But this has an optical path length difference; these all have the same optical path length whereas, these are all a different optical path length and that is what is causing the multiple beams.

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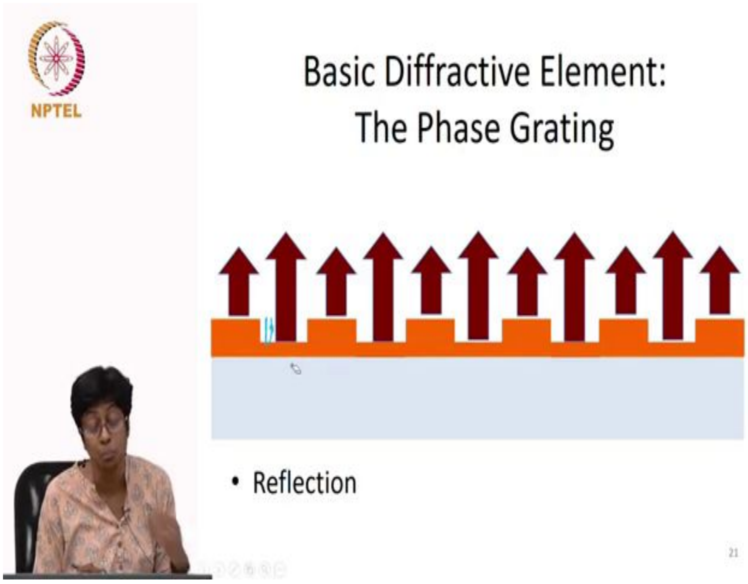

These are both examples for transmission elements I could have reflection elements, but the optical path length is now. Because this light though both sets are reflected this is reflected after going through twice this distance, so it goes down and it comes up so it has travelled twice a distance more than these beams.

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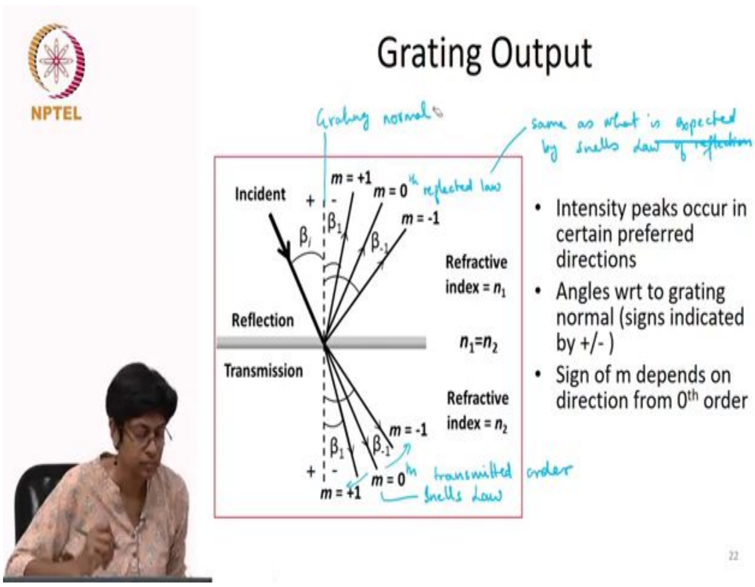

So, if I take a grating a grating splits up light into the number of orders those orders could be reflection or transmission depending on the type of grating you are using. And the convention is if you have a light incident on your grating like this all angles are measured with respect to the normal to the grating. So, it is positive if it goes in this direction the angles are negative if they go in this direction. How do I define the orders? Well if the beam was incident like this Snell's law of reflection says the reflected light should go in a certain direction that is the 0 th reflected order.

Same as what is expected by Snell's law of reflection if I look at the refracted? So, either I consider my grating to be very thin, so light travels just along a straight path or I take into account the refraction caused by this grating and I look at the beam that follows Snell's law of refraction that is the 0 th transmitted order.

The other orders are caused by where constructive interference happens between the multiple beams generated by this grating. And we call them the minus 1 orders or the plus 1 orders depending on which side of the 0 th order they are.

So, you can see all the orders towards that have to be rotated anticlockwise to reach the grating. These are the minus 1 orders over here for transmission and these are the plus 1 orders over here ok. So, the order number or the order sign rather is determined by the location of the order with respect to the 0 th order and angles in general are defined or their sign is defined with respect to the grating Norman ok.

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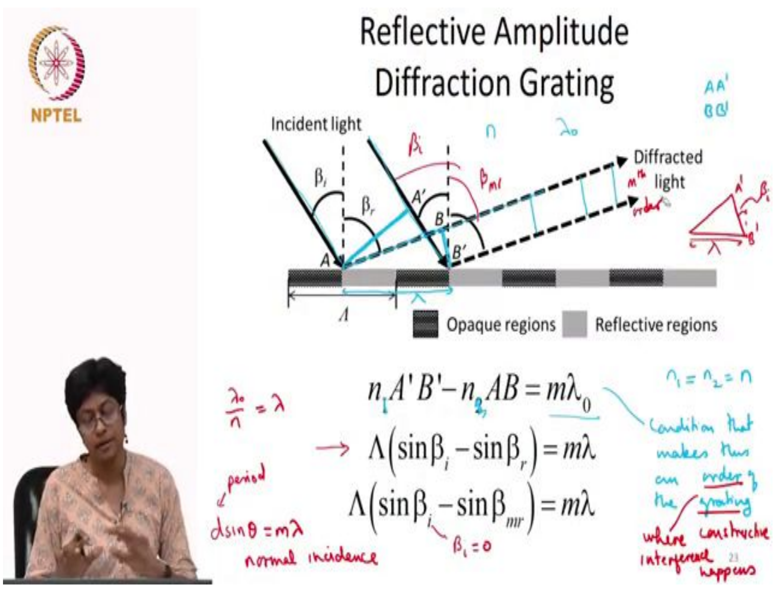

So, how do we arrive at the grating equation? I have a beam of light that is incident. And now let us just take two extremes of this beam: the one that hits this point and the one that hits a point corresponding to one period away.

So, this is one period we are using capital lambda to indicate a period now this is a plane wave incident on the grating. So, at this place this is the phase front of the grating sorry phase front of the plane wave; that means this is a constant phase.

What happens though is when the wave hits the grating at point A this reflects and starts to travel along this path, whereas this continues along this path. So, the size of this plane wave may change, but it is still a plane wave and since some other point I can look at, I can look at the phase front here or the phase front here or the phase front here.

Let us look at the phase front here that us when this part hits B dash and by the time this part hits B dash this is travelled from A to B. So, though I am interested at the wave front at its starting point AA' and then at BB'. I want this to be an order of the grating; that means this must correspond to where constructive interference happens.

So, if I look at the path length difference between these two rays they must be of some multiple of the wavelength. Now let us assume this medium is n of refractive index n; that means, n 1 minus sorry is equal to n2 = n.

So, the path length travelled from A' to B' I have put it as n 1 here, but it is all the same refractive index this one has travelled A to B in the same refractive index. So, these are both n and the difference in these path lengths if it is a multiple of lambda in this region. Then I will have constructive interference and this will correspond to the condition that makes this an order of the grating. When I say it is an order of the grating I am in effect saying this is where constructive interference happens.

So I can rewrite $\frac{\lambda_0}{n}$ as λ and if you do the geometry of this if just from trigonometry this A dash B' so you look at this triangle this corresponds to capital λ, this is the distance $A' B'$, this is the incident angle beta is right. So, this angle is β_i and so if I use this trigonometrical construct here, I will arrive at this expression and this is the standard diffraction grating expression you will often see it given as $d\sin\theta$ is equal to $m\lambda$.

In that case they are calling the period and this is for normal incidence. So, the sin has been absorbed into the index m and this is for normal incidence, so if I were to make beta i go to 0, then you would reduce to this. So, this may be something that you have seen before d sin theta is equal to $m\lambda$.

But the most general form of the diffraction grating equal to is this where you take into account not just the angles of the orders which is βm_r , so m corresponds to each order, but β_i which is the angle of incidence. So this is the βm_r if you consider this to be the nth order ok.

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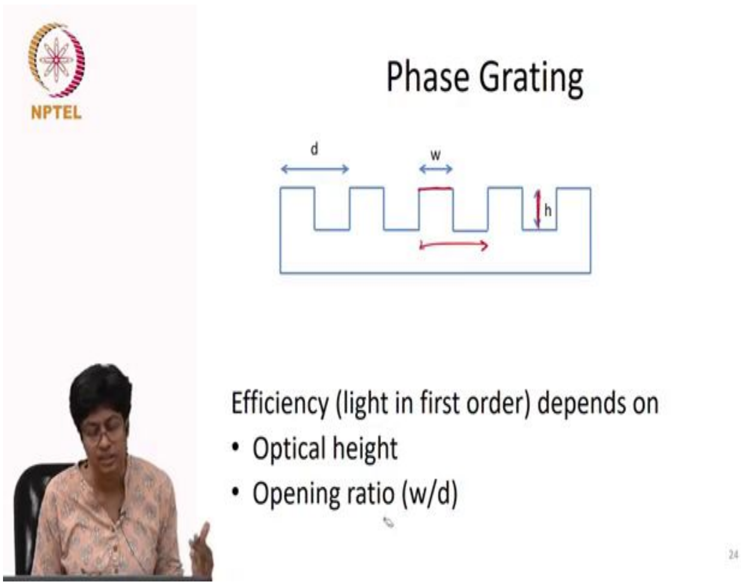

So, this construction tells us when light is incident on a grating and what is a grating a collection of periodically spaced slits they may be amplitude slits they may be phase slits. When light is incident on such a structure it breaks up the light and sends it into very specific directions called orders of the grating and those orders are determined by the period of the incident angle and the wavelength of the light.

What this equation does not tell us does not tell us how much of the light goes into the 0 th order, how much goes into the first order, how much goes into the minus first order or the tenth order the minus hundredth order do those orders even exist. It just says it is possible to get light into all these different directions with this kind of geometry.

So, how do we figure out the efficiency that is defined for a diffraction grating is how much light went into the first order compared to the light that was incident on the grid ok? And two things are important here: the height of your grating as well as the ratio of this part so this structure to the whole structure. So this is called the opening ratio w/d sometimes you see this also as fill factor w/d ok.

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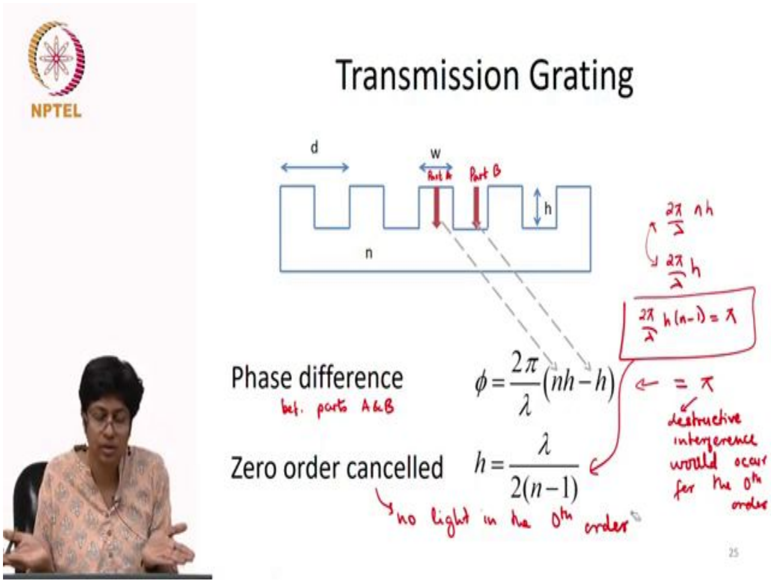

So, how do we do this to help us figure out efficiency? Let us consider a beam that is incident and we are only now looking at a part of that beam the path that travelled through this section of the gratings. So, it went through a height h of refractive index n, so the phase seen by this part of the beam is $2\pi/\lambda * nh$.

Then another part of the beam that is this part also would see a phase $2\pi/\lambda$, but there is no n here because it travelled through air. So, the phase difference then between let us say this is part A of the beam and this is part B of the beam.

So, the phase difference between parts A and B is going to be the difference between these two and that is what is given over here. Now I want light to go only into the four orders, now you can imagine what did I say the zeroth order was the zeroth order is the direction light would take if this was a refractive optical element.

So, I usually do not want to use that direction, if I wanted that direction I would not choose a diffractive element I will choose a refractive element. Where all the light goes into that order it is very efficient in a diffractive optical element, I want to send light into the order that is where it is different from the refractive elements.

In other words I want to minimize the efficiency in the zeroth order and how could I minimize that efficiency. If this phase difference was pi destructive, interference would occur for the 0 th order. And if I then use this expression that is $2\pi/\lambda * hn - 1 = \pi\, 2$ and I extract h from this is the height the grating has to have in order to ensure no light; that means, no light in the 0 th order.

If the light cannot go in the 0 th order it has to go into the higher orders and I have made it more efficient that way ok.

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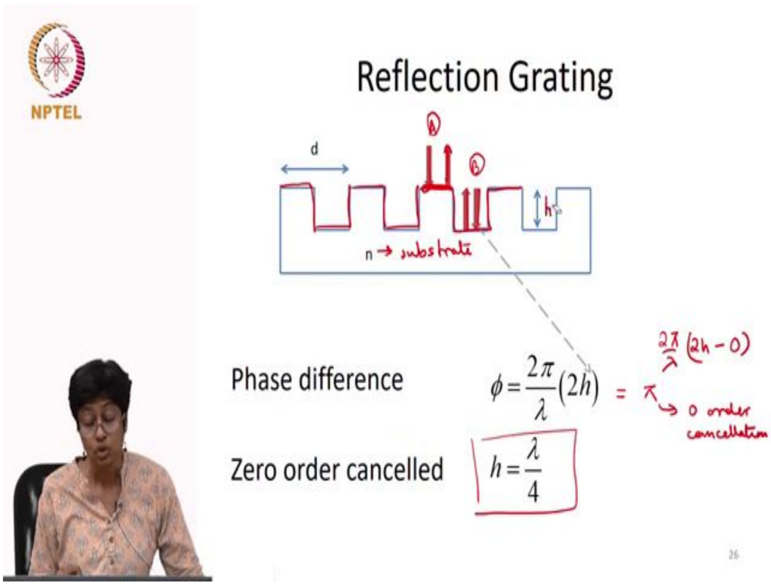

I can use a similar idea for a reflection grating here part A of the beam hits this surface and reflects here whereas part B travels this distance h and then another distance h and comes here. So, the phase difference now between these two is $\frac{2\pi}{\lambda} - 2h - 0$ because this has not if I consider this as my base my reference point ok.

Again if I set this condition to π this is for 0 order cancellation it reduces to this. So, in a reflection grating if I want to cancel the zeroth order I have to choose the height of the grating to be lambda by 4 whereas, in the case of the transmission grating I had to choose the grating height to be $\frac{\lambda}{2}$. As well as take into account the refractive index why do I have to take into account the refractive index? Think about it in the transmission grating as the name suggests the light is transmitting through the material.

It sees this n whereas, in the reflection grating we have some reflection coating here we are just using this material n as a substrate. The light never actually sees the material and therefore, the refractive index of that material does not matter at all what matters is only this height h. But the principle of how you cancel the zeroth order should be clear for both these cases and you can apply that even to any other diffractive element you might be designing ok.

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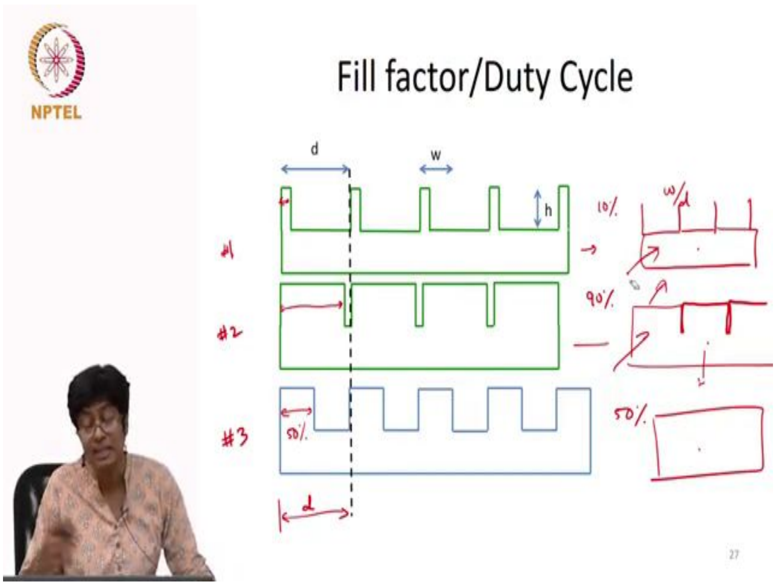

So, if you have designed the height of your grating correctly you have cancelled the zeroth order; that means, all the available light and all the incident light is only going into higher orders.

But even there how do I know how much goes into the first, the second, the third, the minus first, the minus second, the minus third. So, we need to look at the fill factor or the duty cycle, so let us think of this before we try to do any maths for it. Let us say I took a substrate and I fabricated a grating on top or I took three different such different substrates of the same material and in each case I fabricated a grating they all had the same period. So, you can see this grating 1 and number 2 and number 3 in all cases the period is the same this is the period in all cases.

But the fill factor is different because the width of this in this case is so small here it is really very large and here it is 50 percent. So, if I take the ratio of w by d maybe this is something like 10 percent maybe this is something like 90% and here it is 50%

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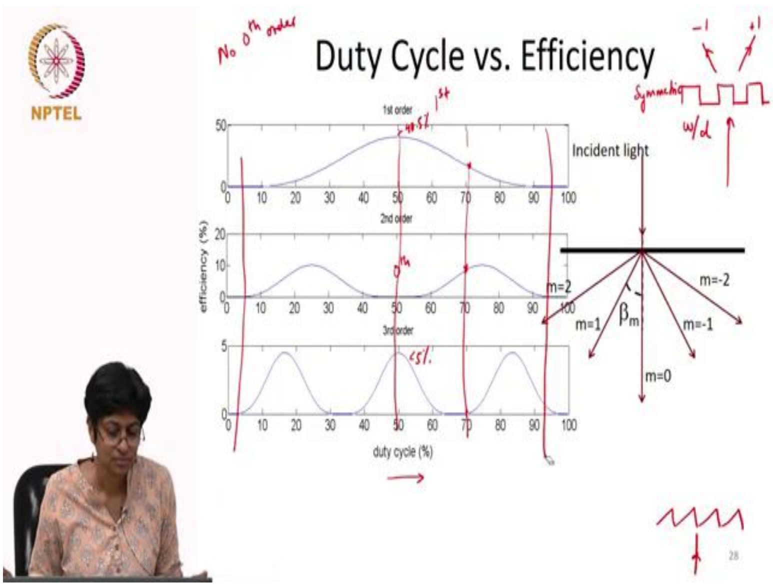

If you measure or calculate the efficiencies in all of these orders you will find four gratings with what we call a binary grating. If you study the efficiency in their existing orders as you change the duty cycle that is as you change w by d right.

So, what these three graphs show is for the 1st order remember you have chosen the height correctly so there is no 0 th order ok. So, it is only the higher orders and this structure is symmetric. So, if there is a light incident here some amount of light goes into the 1st order and an equal amount goes into the minus 1 and plus 1 orders, because this is a completely symmetrical structure whatever is satisfied. So, constructive interference on this side is satisfied for this equal and opposite angle on this side.

So, if I have 10 percent light going into this direction I will have 10 percent light going into equal and opposite direction, because this is a symmetric structure right what do I mean by symmetric structure. If you remember the Fresnel prism of the diffractive prism this is not symmetric the light comes here it will preferentially send light into one direction compared to another direction, because the structure is not symmetric with respect to the incident light whereas, this is symmetric ok.

So, in this structure in these graphs we are varying the duty cycle and looking at the efficiency in the 1st order, the 2nd order and the 3 rd order. And you can see at the 50 percent the 1 st order has about 40 it is actually 40.5 percent is the efficiency.

There is no 2 nd order, the 2 nd order has 0 percent and the 3 rd order has less than 5 percent; this is for a duty cycle of 50 percent. If I go to some other duty cycle say 70 percent you can see the 1 st order has some efficiency there is some efficiency in the 2 nd order there is almost nothing in the 3 rd order.

Now, think about this if I look at this structure or this structure that is grating 1 and 2 if I were to go on reducing the width of grating 1 or increasing the width of grating 2. So, to say I took this structure and I made it even narrower so it was something like this my substrate. It is almost as if I have no grating now take this structure and say I make the period like this it is almost as if I have no grating this is almost becoming this.

And if I look at this or this or this if I have no grating I do not have orders the orders come because I create zones and those zones create beams and those beams interfere and give me certain preferential directions.

But if I remove those beams as I almost do when the fill factor becomes very small or very large then I get very little light in order. And that is what you will see if you go to duty cycles which are very large above 90 percent or less than 10 percent or even less than 20 percent you see that the amount of light that is going into the orders is very small. So, definitely 10 percent or 90 percent is almost giving me a structure that is not a grating and that is why there is no efficiency there is no light here.

In that case all of the light is just following geometric optics it has no 0 order cancellation because for that to happen I needed to have a number of beams that could cancel here I am just saying there is 1 beam it just goes through that a grating as if it sees a slab of glass.

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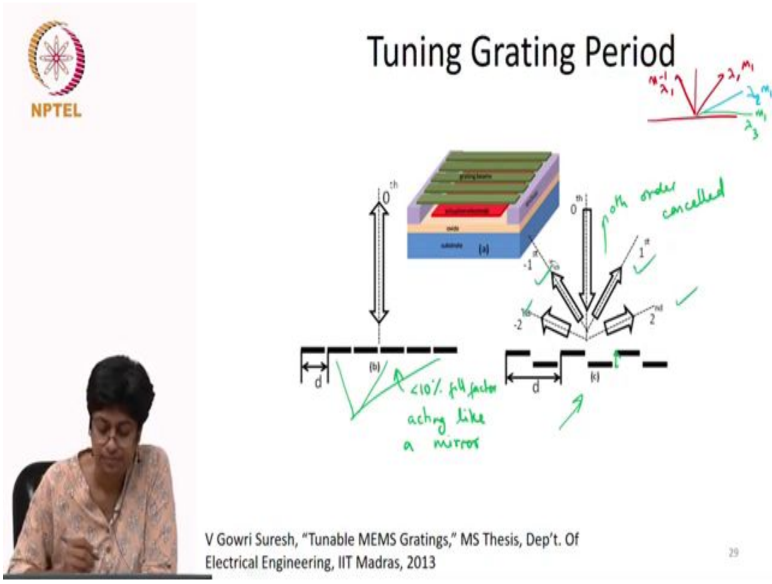

So, you can do a lot of funky things with gratings, gratings are used in spectrometers because recollect every expression that we have arrived at has arrived at 4 1 wavelength. And that means, I will get a number the 1 st order minus 1 st order this is for lambda 1, but if I have now a second wavelength incident maybe that lambda would go here this is the first order for lambda 2.

So, this may be m 1 this is m 1 this is m minus 1 for lambda 1 I might have m 1 for lambda 3 and so on. So, a grating splits colours and that is why they are used in spectrometers, but I could also think of doing tunable gratings I could play around with the period. So, I could also play around with the period and this schematic just shows you if I had a grating where in this case it almost is not like a grating because these gaps are very small most of the light that is incident is going to reflect off.

So, it is like it or its acting like a mirror here this grating has less than 10 percent fill factor. So, its acting like a mirror and if I could somehow pull down every alternate period and that is what you see here. Suddenly it starts behaving like the diffraction grating and I have a number of orders and if I pull it down by the right height the 0 th order would be cancelled and only these orders would exist.

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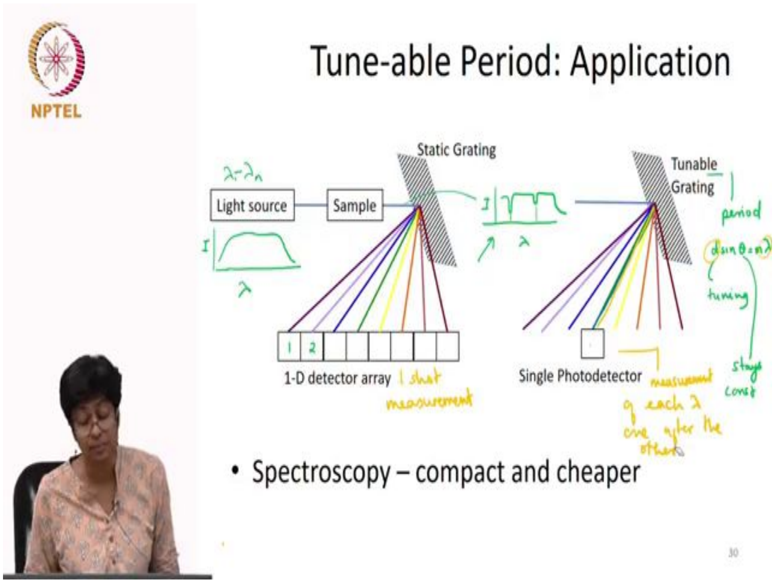

So, I could create a tunable grating. Why would I need to do that? Look at an application that let's go back to spectroscopy. I have a light source; that means I have a range of wavelengths those different wavelengths are going through a sample and from the way the wavelengths change through the sample.

So, let us say if I looked at intensity versus lambda here it might have something like this. So, all wavelengths exist in almost the same intensity, but after the sample if I looked at it so I looked at it here, maybe some wavelengths have been absorbed.

From this I can tell what that sample is or what the concentration of some chemical in that sample is because say this wavelength was absorbed this wavelength was absorbed. But in order to do this in order to generate this intensity versus wavelength graph, I need to be able to measure the value of the intensity at each wavelength. How do I do that? I use a grating.

So, in this simple schematic you can see the light comes through the sample it hits this grating it is just static grating and it splits out the different colours.

So, I need a detector where from the location, so from this pixel if this pixel has light on it let's say pixel 1 has light on it, then I know this wavelength is present and I know the intensity of that wavelength. Pixel 2 another wavelength is present. I know the intensity of that wavelength and from this ray I can then draw a graph like this or arrive at a graph like this.

This means I must have a 1 dimensional detector which is a ray of pixels and that is quite expensive to have instead could I not use a single photo detector, the problem is how do I know which wavelength is falling on it.

Well if I change migrating from a static rating to a tunable grating what happens is when I change when I say tuneable what is being tuned the period is being tuneable. So, remember d sin theta is equal to m lambda if I am tuning d such that theta stays constant four different wavelengths; that means, as I can tune this so that initially the green wavelength is incident on the photo detector.

But I tuned the grating so that in the next instance maybe the blue wavelength will be incident or I tuned the period so that in the next instance the orange wavelength is incident. So, the photodetector I need only a single photodetector it stays in one place, but because I am changing this parameter each wavelength in turn is getting imaged onto this photo detector.

So, here I get a one the difference then is I get a 1 shot measurement and in this case I need to take measurement of each wavelength one after the other ok. But it makes it more compact and it could because for certain wavelength ranges the 1 dimensional arrays is very expensive so it could even make it cheaper ok.

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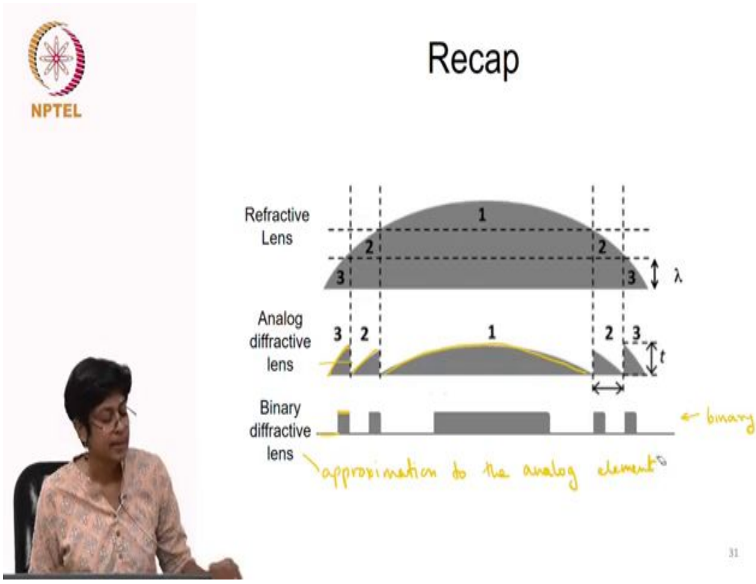

So, coming to the end almost and let us just do a quick recap we started out by saying since a change of phase of 2 pi does not really optically make a difference can we convert a refractive element to a diffractive element by removing all the unnecessary 2 pis. And we saw that yes we can do that and we will end up with an element like this and we do call this diffractive if these dimensions are this height in the order of wavelength. This structure is still quite hard to fabricate with current technology because it has these curved structures.

So, what if we did an approximation and instead of this curved structure we approximated and said wherever the height is above a certain value I will have this as having 1 height, wherever it is below a certain value I will have the height as 0 and if I do that everywhere right. So, I am putting a cut off and saying height above something makes the height 1 height below something make the height 0 I end up with what is a binary diffractive.

Now, this serves the purpose except we should expect differences in behaviour because this now is an approximation to the analog element.

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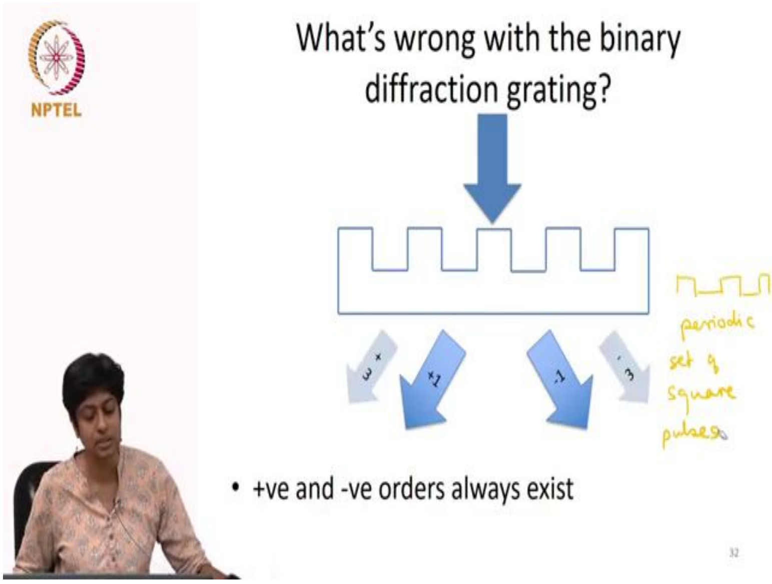

And although we looked at this already, let us look at it now with a more critical eye and say, what is the problem with a binary diffraction grating. We always want light elements or devices or systems to be as efficient as possible. So, when light is incident if my goal was to redirect light with the diffraction grating I always send light into a plus 1 and a minus 1 order or plus 2 and a minus 2 plus 3 and a minus 3.

I cannot get away from that and as I pointed out earlier it is because of the symmetric nature of this particular structure ok.

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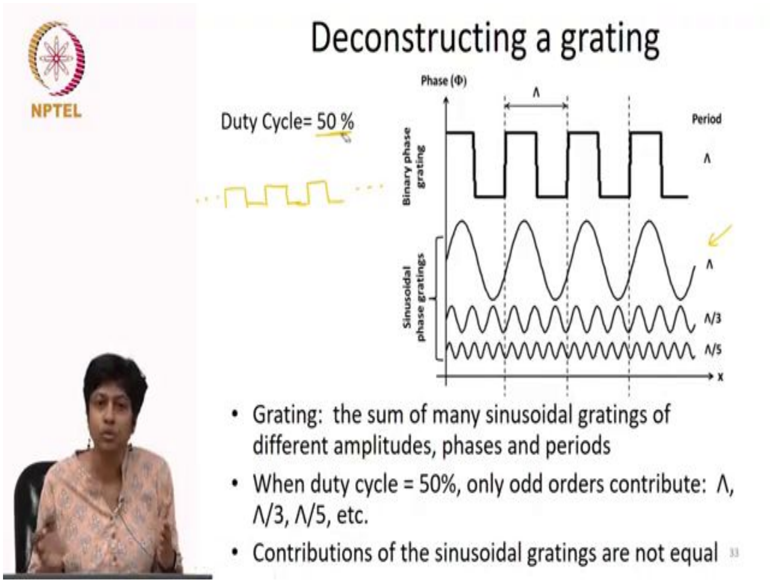

How do I get rid of this or why exactly does that happen? You could think of this in terms of a Fourier analysis of the grating. So, if I gave you a signal like this and I asked you to Fourier analyze this signal, this is a periodic set of square pulses. It is a nice analogy to help us understand what is going on with the grating, if I thought of this is a periodic set of square pulses, what would the harmonics that are required to create this periodic set of harmonics of square pulses be?

And you will see if you have a 50 percent duty cycle you have a number of harmonics that are present in fact all the odd harmonics are present. So, you can think of your periodic set of pulses as being a large number of them as being the sum of a different set of harmonics of different amplitudes of different frequencies. When I add them up together they give me this and if it is a 50 percent duty cycle I will only have the odd numbers.

So, I will have plus 1 minus 1 order plus 3 minus 3 order plus 5 minus 5 order but not the even numbers. If the duty cycle changes of course, you will have the other orders and you remember you can get an idea of that if we go back to this graph when we did the 50 percent duty cycle you can see the 2 nd order went to 0 percent. Whereas, if you had a different duty cycle then all the orders the 1 st, the 2 nd, the 3 rd they all existed ok.

So, a nice way to understand the orders of a grating is just to think of this as light seeing this structure which is made up of a number of harmonics and light takes the path corresponding to each harmonic ok.

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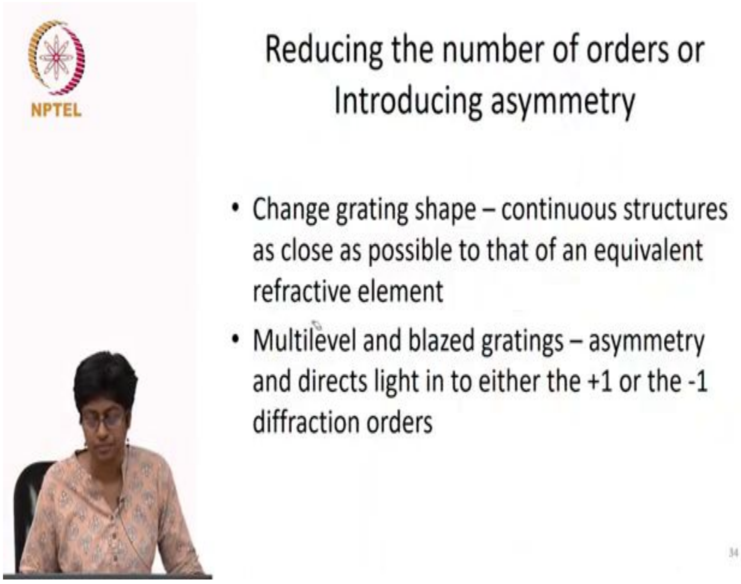

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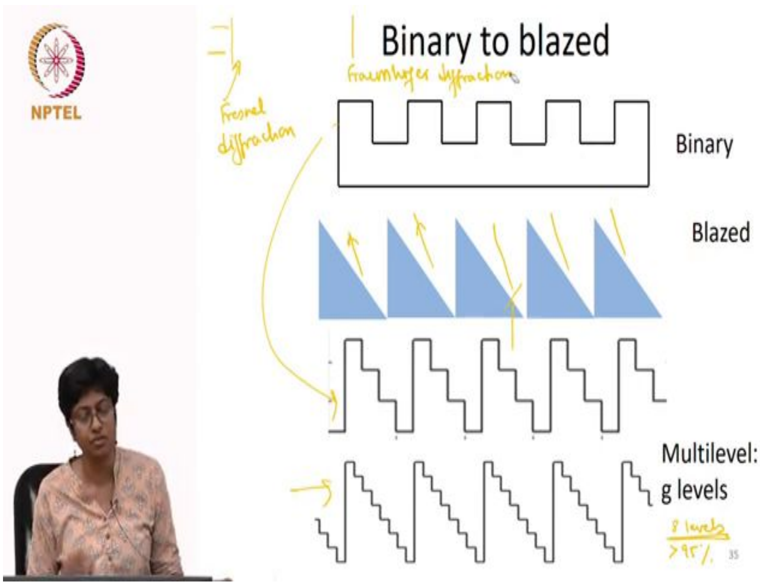

If I want to get rid of the problem of always having a plus 1 and a minus 1 order then I have to break the symmetry somehow and this blazed prism structure would blaze the symmetry.

So, light incident on this would all bend in one direction, one preferential direction, so you could have like almost 90 percent of the light going only into one order.

I could all otherwise convert this to a structure with multi levels like this and it turns out if you have 8 levels and above you can have much greater than 95 percent efficiency right. So, these are some ways of making a binary element more efficient, but of course, there are limitations you might think well I should always do this, but it becomes difficult in terms of fabrication.

So, with that I will wind up the summary of diffractive optics. We have not gone into a mathematical analysis of it because you can actually spend an entire semester to an entire course on diffractive optics. What you should have learnt in this discussion on diffractive optics is that diffraction is inherent in optical systems. Because you have the edges of the optical systems where you cannot neglect the size or you cannot assume lambda tends to 0 or you can and in those cases you considered diffraction a problem and you have to solve it somehow.

You can look at it from a different approach and say since diffraction affects intensity and phase of light I can create optical elements using diffraction and we have looked at the grating as one example of how you do that. I should mention two things that I think I forgot to mention: one is when you are looking at a diffractive element.

So, you have light incident and you are looking in the region near the element we call this Fresnel diffraction and if you are looking in a region far away its Fraunhofer diffraction these are the scientists I mentioned right at the beginning Fresnel and Fraunhofer.

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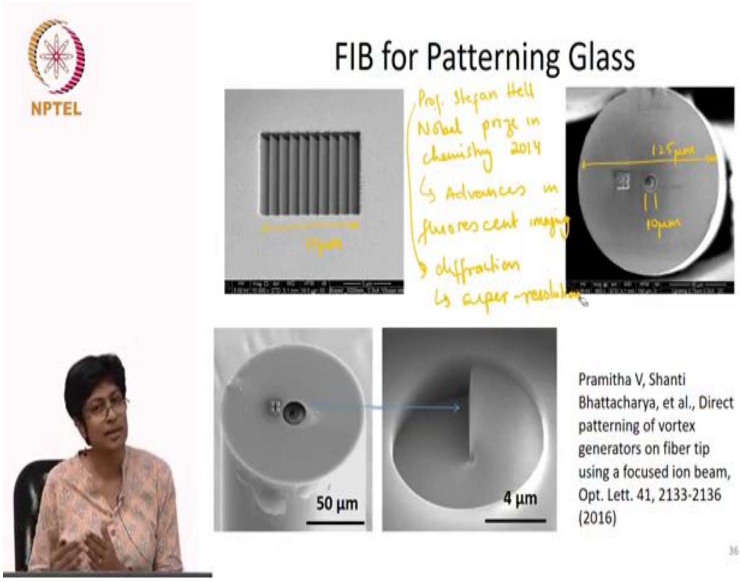

And I will end with just some pictures showing you some structures that we have fabricated here. So, this is a blazed grating. You can see this scale represents 5 microns so this is about 10 microns. So, that is the size right this is actually a grating a circular grating fabricated on a fibre tip so this is the diameter of this it is a fibre. So, the diameter is 125 microns again this is roughly 10 micron diameter and these are some other kinds of elements also fabricated on fibre.

So, just to show you or give you an overview of what all is possible with the diffractive optics. I want to end with one example and that is when we do imaging we talk about diffraction being a problem, but please do look up professor Stefan Hell.

So, maybe some of you will remember his name. He won the Nobel Prize in chemistry in 2014 four advances in fluorescent imaging. And you can imagine that an imaging resolution of your imaging system is always a major issue and the best systems are diffraction limited and Stefan Hell used surprisingly diffraction.

So, he used a diffractive element to create a super resolution system. So, I am not going to talk about the system today, but I will urge you to go and read up on that because it is interesting that he used the idea of diffraction which normally an imaging person will think is a problem for imaging. He used that to achieve better than a diffraction limited system and so that is very interesting ok.

So, I hope that you have a good overview of diffraction and will help you also with the programming exercises that you have been given.

Thank you.