



Optical Engineering
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Lecture – 39
Application of interference

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Interferometry Basics


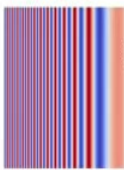
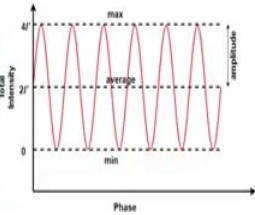


- Intensity equation:

$$I_i = I_0 [1 + \gamma \cos \delta]$$
- γ - the modulation depth = $\frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$
- I_0 - bias Intensity = $I_1 + I_2$

If $I_1 = I_2 = I'$, then $\gamma = 1$ and $I_0 = 2I'$



Good morning, in yesterday's class we looked at interference and we specifically took the case when you have 2 beams interfering. And we arrived at this expression and we looked at the various terms so we said I_0 is a term that relates to the individual intensities. So, you have 2 beams interfering and I_0 relates to the intensity of each of those beams. In fact, it is nothing, but the sum of those intensities and γ relates to what we call the visibility or the modulation depth of the fringes.

And we looked at how if you want your fringes to be very clear you need to have a large modulation depth. And surprisingly that meant sometimes throwing away intensity, because you need to make both the beams have the same intensity with the beam coming from the object you cannot always control the intensity you may lose a lot of light.

And therefore, you end up having to reduce the intensity of the reference to make them more equal and having more equal intensities will improve your modulation depth. In other words

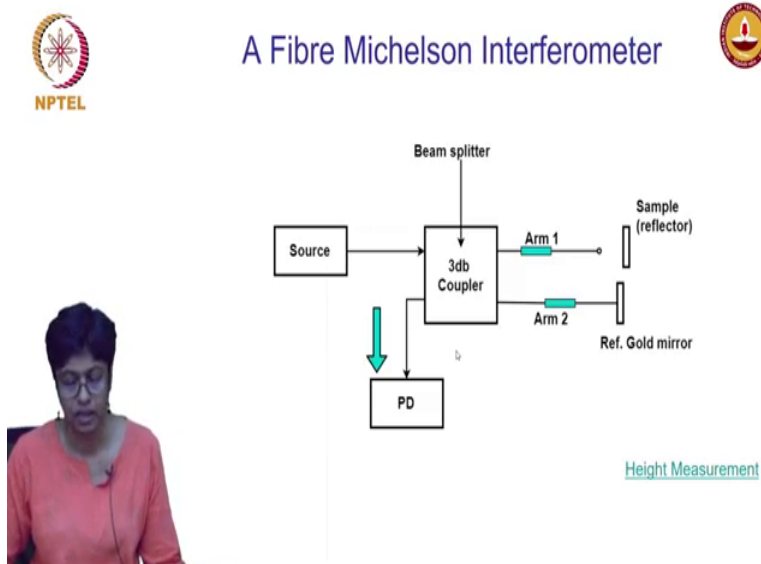
improve this difference here between the peak or the high intensity and the trough at the low intensity.

In the ideal case this should be the maximum intensity possible that you can achieve with 2 beams that are interfering. And this low point here should actually be a 0 where you have perfect cancellation. So you have perfect destructive interference and then your modulation depth is 1, because you are going from an intensity of 0 to the maximum intensity from these 2 beams.

We also looked at you when you look at two waves interfering how from the fringe pattern you actually get an idea of the shape of the wave right. So, we said for the patterns that are shown here you can imagine that this was created by a plane wave front and a tilted wavefront, whereas, this was created by maybe something like this a spherical wave front and a plane wave front.

And, the fringes are oblique and to be more specific let us say that wherever you have constructive interference they indicate the path where the optical path length difference has one value. And, clearly the way that optical path length difference is varying is what you can see in the fringe pattern ok.

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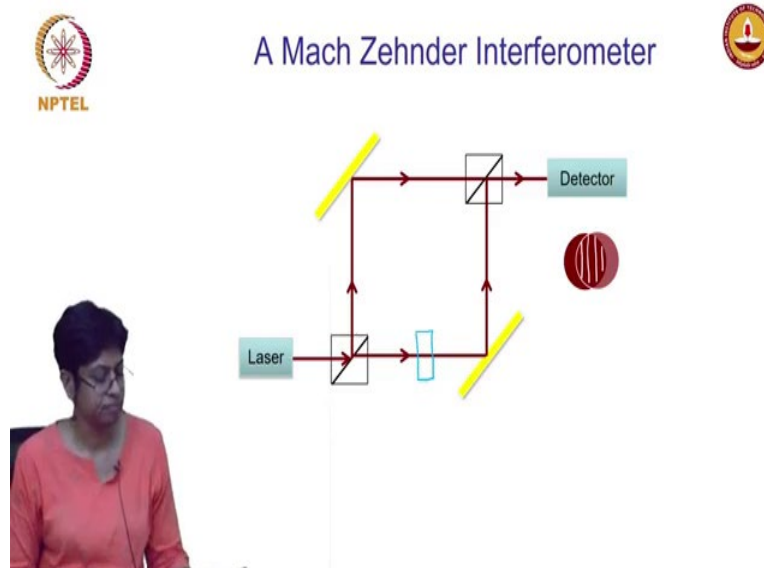
So, up till now what I have drawn is our free space interferometers, I just want to tell you that you could also have a fiber interferometer. So, you have light traveling within the fiber. The

principle is exactly the same. If however, you need it to measure some sample you would have that part in free space because; obviously, the light has to leave the fiber hit the sample reflect off the sample and then reenter the fiber.

So, this Michelson interferometer does not look like a standard Michelson where the mirrors are perpendicular to each other. Because here you are creating those two paths in fiber and you can align the fiber in any orientation it really does not matter. What is important is that this reflector is aligned such that most of the reflected light or a lot of the reflected light will couple back into this fiber and travel along this path.

So, here you have light, the light incident on word light it this is a device that will splited into two parts. One arm allows the light to leave the arm and hit the object that we are studying, the object of interest the scatter or reflected light from this some of it will get captured by this fiber return. Here you can have a mirror and I can actually have a mirror pigtail directly onto the fiber so you never leave the fiber light comes back, and then these two combine and you can study this again with a camera or photo detector.

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Now, just to reiterate there are many different kinds of interferometers. The previous one was a Michelson, this is a Mach Zehnder. So, here again we are inter-creating 2 beams of light, but you are doing that with a beam splitter here and a beam splitter here. So, that in this interferometer no beam traverses the same path twice.

So, in the Michelson you know it goes up to the mirror and then it traverses that same path again. Here this beam gets split by the beam splitter it travels to this mirror travels onward like this and then travels to the detector, this one travels this way. Again I might introduce a slight tilt, so that if this is 1 beam falling on the camera the other arm is tilted slightly so that this is how it overlaps.

And then I am going to be seeing fringes in this region here ok, the fringes are going to be in the region of overlap. Why is this interferometer of interest? Well, I took for example, put a glass cell here and fill this glass cell with different liquids. And then depending like, so I will do a reference measurement where the cell is empty and I look at the fringe pattern. And then I fill that cell with the liquid of a certain refractive index and I look at the fringe pattern and from the difference I can extract information about the refractive index.

Or I could have a transparent block that is sensitive to temperature and as the temperature changes either that block expands or contracts. So, its length changes or its refractive index is a function of temperature and then again the fringes are going to change from one temperature to another and I could use that as a sensor.

So, there are many, many different ways that you could use this as a sensor right. Why do you want to make such measurements with an interferometer? If you think about it, what do you think is the sensitivity of your interferometer that limits the sensitivity, it is the smallest kind of measurement I can make.

Student: Resolution.

What is the resolution? Yeah in a sense.

Student: (Refer Time: 07:10).

Of my interferometer, what will limit?

Student: (Refer Time: 07:21).

It is only the he says it is the resolution of the detector. Is it only that I give you the world's best detector, I give you a detector that has not even been invented yet and it has super small pixels. Is it only limited by the resolution of the detector, what do you think limits when I ask

you these questions always if its optics you can almost always convert this question to a question about phase.

So, if I am asking you about how sensitive a measurement this is, how sensitive a measurement of phase is what I am asking. And, what is the problem with phase with these phase measurements, remember your intensity equation and your interference equation. What is your interference equation? $I_1 + I_2$ plus the visibility a modulation depth into a cosine what limit, so you are really interested in that third term.

Student: (Refer Time: 08:43).


What limits anything which is a cosine, say I have a phase difference I think we had this discussion yesterday. If I have a phase difference that relates to a length difference of 2λ and a phase difference that relates to a length difference of 3λ , 4λ , can I really make up I cannot know.

So, finally, it's λ that matters and actually it's $\lambda/2$ that matters, so in a sense of course, that assumes a detector can detect that, but that is usually not the limiting case. The limiting case is that 2π , 4π , 6π , 10 million π to an interference pattern look all the ways the same. So, that is my limit with interference that I am limited to measurements in terms of λ or really $\lambda/2$.

So, if I have a change in length say I am using an interferometer to measure change in length. And the change in length is λ or the change in length is 2λ , I cannot tell I can tell that there is a change, but I cannot tell whether it was $\lambda/2$, $\lambda/3$, $\lambda/4$ λ right.

So, that is my limitation there are ways to get around it right, I could use two wavelengths. Then something happens for λ_1 something happens for λ_2 and if I can sort of create an effective λ which is a function of λ_1 and λ_2 . So, there are many ways to get around that, but if at the most basic level that is my constraint ok.

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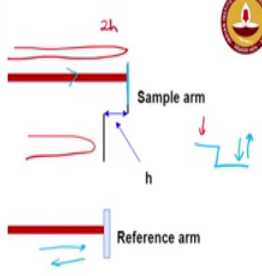
Height Measurement

Intensity due to interference

$$I_1 = I_0 [1 + \gamma \cos(\delta_1)]$$

$I_1(x, y)$ desired

$$I_2 = I_0 [1 + \gamma \cos(\delta_2)]$$



$$\delta_i = \frac{2\pi}{\lambda} OPD_i$$

where OPD is the optical path difference
between sample and reference arms

$$h = \frac{OPD_1 - OPD_2}{2}$$

Step height retrieved

[Back](#)

So, let us say I wanted to measure height and how would I do it. I have available to me the information that I have available to me is the intensity that I am measuring. And, again I am saying this I am always writing it out like this please note it is always that I am measuring this ok.

So, I have a camera image of an interference pattern that gives me the fringes, so the points of constructive interference and the places of destructive interference I have that information. Now, say I wanted to measure a height so I am not drawing the whole interferometer here I am just showing you the last part of the sample arm and the last part of the reference arm. The reference arm the beam comes hits the reference mirror and reflects back, in the sample arm what we initially do is have the sample beam or the investigating beam fall on one region of the sample.

So, my sample has a step in it and initially we align the systems, so the beam falls here. So, it hits this part and it reflects off this part and that travels back and I can capture that interference pattern and that is what I have written here as I_1 . Now I move the sample so that the beam falls here.

So, I can either move the sample or I could move the beam. It does not matter the point is my second reading is to have the beam fall at a different position of the sample. If I do that I will get a different interference pattern why, because the path length of this beam is different from

the one the earlier case. So now I have a second one, now θ does not change why does θ not change θ and γ do not change why not.

Student: (Refer Time: 12:49).

I am using the same 2 beams and θ and γ functions of those same 2 beams right what has changed is the δ the phase has changed. So now, I have these two equations or I have these two intensity distributions, what I am interested in is finding not δ_1 or δ_2 I want to find out the difference in optical path length.

So, δ_1 relates to the optical path length seen by the first by the beam in the first case δ_2 is the optical path length seen by the beam in the second case. And if I could somehow get this information and take the difference between these optical path length, I should be able to extract height from that agreement.

In fact, this is not right here actually it's OPD 1, OPD 1 is this distance and this is the first case. So, the difference here gives me twice $2h$ actually right, so I actually should have a 2 here I think right, but you get the point, it is the difference that I need.

Now the problem is I have this in this intensity I_1 the distribution I have I_2 I want the δ s, but I do not know θ and I do know γ . Can you think of some wave we could solve? I have this is unknown. I want this to be desired right, but I do not know this, I do not know this, I do not know is there any way to solve this sorry.

Student: Take a ratio.


I cannot take a ratio here because it is there buried in a cosine and so on right, it is not just you know $a \times n$ something $a \times y$ or something like that right it is it is not a simple expression. So, the problem here is I have more unknowns than knowns right now if I had as many unknowns as if I had as many knowns as unknowns I could solve this problem if I have more unknowns.

So, I am going to do some more experiments and get some more data, so that I get more knowns how I can do that. Well I do not know θ and I do not know γ and you might ask I am surprised you are not asking well it can take just measure them I cannot I measure I_1 and I_2 .

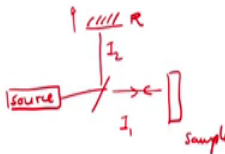
The thing is in a practical system you can measure the incident the intensity you are sending into the system. Once it goes inside the system splits, reflection to get an accurate measurement of that is really very hard, may not be easy for you to put some device inside and measure what is that actual intensity falling on the detector.


So, it is not practical. You are not going to get accurate results if you try to go and measure I_1 and I_2 right. So, you do not want to go by measurement of I_1 and I_2 you do not want to calculate I_0 and γ and in that sense. But what I can do is this if somehow I can introduce a known phase change ok, then I can make another measurement. I have not changed the intensity of the beams involved in the interference.

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Phase Shifting Interferometry





- Intensity equation:

$$I_i = I_0 [1 + \gamma \cos(\delta)]$$
- Three Unknowns
 γ - the modulation depth
 I_0 - bias Intensity
 δ - required phase
- δ obtained by adding a known phase

$$I_i = I_0 [1 + \gamma \cos(\delta + \epsilon_i)]$$
- solving a set of equations with different values of ϵ_i - known phase shifts, $i = 1, 2, 3 \dots$

But I am introducing a phase known phase change, so how do I do that? This is called Phase Shifting Interferometry, so my standard interference between 2 beams gives me this expression. Now let us say and I am as we know the unknowns are I_0 , γ and the desired phase δ .

Let us say I introduced a phase change that is going to change this equation by this phase term appearing here. Yes I have not changed I_1 and I_2 , but I have somehow introduced a known phase change how could I do that how could I introduce a known phase change and an additional phase change.

Student: (Refer Time: 17:40).

So, using not a known sample I mean even if you say known sample I want to keep my setup with my sample, I do not remove the sample and put another sample that will change things right. Like the beam coming from the object has to be the same, I am just adding an additional phase. I am not taking away the object beam. Remember I_1 is the beam coming from the object. If I take away the object it is not I_1 anymore it is a different value expression.

So, let us say we had the Michelson, so I_1 that is my source beam splitter this is my reference mirror and here is let us say is my sample, sample which is reflective. What is reflecting is and coming is what I am calling I_1 and this is I_2 . Now, if I remove this and put a known sample that is different I know because it will be in having the properties of that reflector not of this sample.

So, I do not want to change that I am not changing I_1 and I_2 if I do that then I naught is not constant anymore the equations I make my life even more complicated, I want to introduce an extra face how do I introduce an extra face.

Student: (Refer Time: 19:16).

Sorry.

Student: Shifting a reference (Refer Time: 19:21).


So, shift the reference arm. In other words, how can I introduce phase? I can introduce phase by introducing path length. So, all I do is move this mirror. If I move the mirror I have introduced an extra path length.

The path length difference will change by I have moved it a certain amount that is twice that length. And let us say it is in air so I do not have to worry about refractive index, then I have 2π by λ into twice the distance I have displaced the mirror that becomes an extra phase which is in one arm not the other arm.

So, I have an extra phase difference now so if I make a measurement now. So, I have I_1 which is this measurement and I do this repeatedly I can now get three equations I have three unknowns, but I have three equations three knowns and then I can solve it right. I need to have at least that many equations as there are unknowns right, so if there is no limit I do not have to stop at three I can do more than three, but I need at least 3 such measurements.

And that is exactly the principle of phase shifting interferometry you take a reading introduce a known phase shift take another reading introduce another known phase shift and you go on doing this so what does that allow you to do my initial equation is this.


(Refer Slide Time: 20:51)




Four Step Method

$$I(x, y) = I_0 + I' \cos[\varphi(x, y) + \varphi(t)]$$


| | | |
|---------------|--|-----------------------|
| $I_1(x, y) =$ | | $\varphi(t) = 0$ |
| $I_2(x, y) =$ | | $\varphi(t) = \pi/2$ |
| $I_3(x, y) =$ | | $\varphi(t) = \pi$ |
| $I_4(x, y) =$ | | $\varphi(t) = 3\pi/2$ |






Now, let us say we pick those known phase shifts not to be some random value, but some specific value and let us say we use the values that I have listed over here. So, my initial expression is $I_0 + I' \cos \psi$ of this right, but now I am adding an extra ψ and I am adding initially its 0. So, I have the original expression then I add $\pi/2$ I add π and I add $3\pi/2$ how will this expression change what is \cos of some angle plus $\pi/2$.


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Four Step Method



$$I(x, y) = I_0 + I' \cos[\varphi(x, y) + \varphi(t)]$$

I_1 

I_2 

| | |
|--------------------------------------------|-----------------------|
| $I_1(x, y) = I_0 + I' \cos[\varphi(x, y)]$ | $\varphi(t) = 0$ |
| $I_2(x, y) = I_0 - I' \sin[\varphi(x, y)]$ | $\varphi(t) = \pi/2$ |
| $I_3(x, y) = I_0 - I' \cos[\varphi(x, y)]$ | $\varphi(t) = \pi$ |
| $I_4(x, y) = I_0 + I' \sin[\varphi(x, y)]$ | $\varphi(t) = 3\pi/2$ |


$$\frac{I_4(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)}$$

What is it by plus pi minus what is it by 3 by pi 2 all I am doing is changing my expressions like this. So, this means if I have not when I say I have I_1 of x or y ; that means, I have a camera image and I have the pixel information everywhere right, this is my I_1 .

And again for I_2 I will have the same information, but of course, the intensity now on each of those pixels is different ok. So, if I carry out the operation shown here in other words all I am doing is simple pixel traction and then dividing it by another pixel subtraction, what is this give me what is I_4 minus I_2 by I_1 minus I_3 give me do it quickly and tell.

Student: (Refer Time: 22:52).

(Refer Slide Time: 22:56)




Four Step Method

$$I(x, y) = I_0 + I' \cos[\varphi(x, y) + \varphi(t)]$$

| | |
|--------------------------------------------|-----------------------|
| $I_1(x, y) = I_0 + I' \cos[\varphi(x, y)]$ | $\varphi(t) = 0$ |
| $I_2(x, y) = I_0 - I' \sin[\varphi(x, y)]$ | $\varphi(t) = \pi/2$ |
| $I_3(x, y) = I_0 - I' \cos[\varphi(x, y)]$ | $\varphi(t) = \pi$ |
| $I_4(x, y) = I_0 + I' \sin[\varphi(x, y)]$ | $\varphi(t) = 3\pi/2$ |

$$\tan[\varphi(x, y)] = \frac{I_4(x, y) - I_2(x, y)}{I_1(x, y) - I_3(x, y)}$$




You will get nothing, but the tangent, so now, you see how do you remember why we start this whole discussion yesterday we said there is so much information in phase no detector measures phase directly therefore, we're going to do an interference and use the intensity of that interference pattern to extract phase.

And this whole exercise from then to now has been well how do I get out phase a single intensity pattern a single intensity of an interference does not give me phase. In order to extract phase I need to make at least 3 measurements mathematically, but you see if you make 4 measurements and you pick this as your phase shift you then arrive at a very simple method by which to get the phase. Now I just have to measure four different patterns with my camera, do this pixel subtraction, put it into this formula and then take the argument of those measurements and I will get the phase at every point ok, you see any problems with this.


Student: (Refer Time: 24:05).


Yes, no well come back to that.

(Refer Slide Time: 24:17)



Phase-Measurement Algorithms





Three measurements $\varphi = \tan^{-1} \left[\frac{I_3 - I_2}{I_1 - I_2} \right]$

Four measurements $\varphi = \tan^{-1} \left[\frac{I_4 - I_2}{I_1 - I_2} \right]$


Hariharan Five Measurements $\varphi = \tan^{-1} \left[\frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1} \right]$

Carré Equation


$$\varphi = \tan^{-1} \left[\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)} \right]$$


So, people use you. We know we need at least 3 measurements. There are a variety of algorithms out there starting from the minimum 3 to almost an infinite number of measurements ok.

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Phase-Measurement Algorithm for N Intensity Measurements








N Measurements $\varphi = -\tan^{-1} \left[\frac{\sum_{i=1}^N I_i \sin \alpha_i}{\sum_{i=1}^N I_i \cos \alpha_i} \right]$

$\alpha_i = \frac{2\pi i}{N}$ for $i = 1, \dots, N$

Technique is also known as synchronous detection

Why do you need to have so many options? You might think well 3 is what I need. I have just convinced you 4 I hope I have convinced you that 4 is good. It makes things easy. Why do I need to do 5 and six 6 10 and 100 and so on.

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Which algorithm should you choose?

- Different algorithms differ in their mathematical properties, especially in their sensitivity to different factors:
 - errors of the phase shifter
 - noise
 - external vibration
- Incorrect phase shifts between steps
 - Non-linear errors due to vibration
 - Incorrect phase shifter calibration


It turns out that every algorithm will be sensitive to some errors more than others ok. Now, it always sounds nice mathematically we did that little derivation and we arrived at $\tan \phi$ in terms of those 4 intensity measurements you have made. But, all of this rests on the fact that you are introducing a known phase shift and how accurately you are doing that.

The moment you introduce a phase shift and if you introduce an error along with that you say I am introducing $\pi/2$, but it is actually $\pi/2$ plus some δ that error is going to cascade into all your readings. You may not be in some cases if you are if you have a known fixed term even an unknown fixed error, maybe it will subtract out, but it may be that the error has a range typically the error will be anywhere between this and this value.


So, every time you are introducing a slightly different error you can not subtract it out. So, these algorithms will be more sensitive to some errors that are more sensitive to some error errors than other errors. And since you can introduce that known phase shift in different ways some methods will be more prone to creating some kinds of errors than other errors.

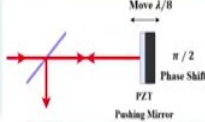
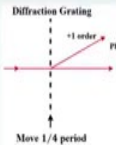

So, people will do a quite an extensive study to see what is the kind of method we are going to use what are errors typical by that are introduced by this method, what is the range of values of the errors introduced by this method which algorithm will be least sensitive to these kinds of errors and then you would take that algorithm ok.

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Phase Stepping in Free Space

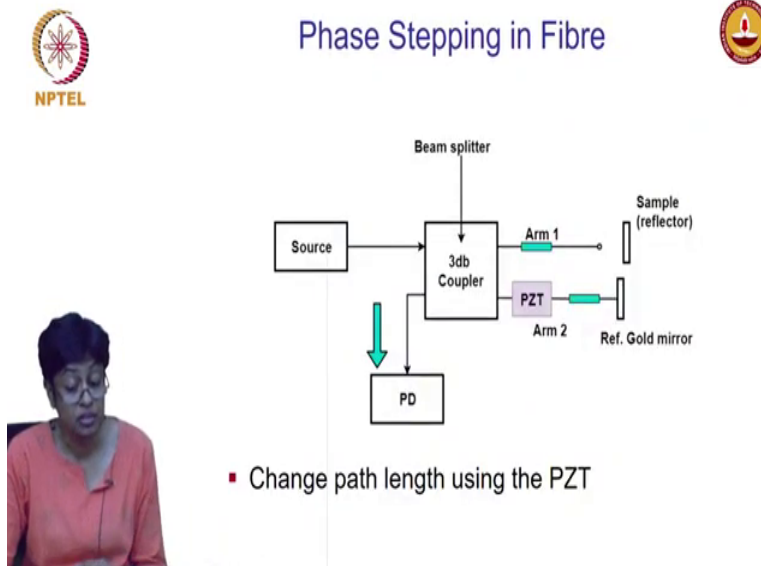


| Technique | Experimental Setup |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Moving Mirror |  <p>Move $\lambda/8$ $\pi/2$ Phase Shift</p> |
| Grating Translation |  <p>Diffraction Grating +1 order $\pi/2$ Phase Shift Move $1/4$ period</p> |
| Tilting Mirror |  <p>$\pi/2$ Phase Shift</p> |

Here are some examples of how you could introduce the phase shift as the first thing that comes to mind always is just shift the mirror π by 2 I mean, if I am talking about 633 nanometer wavelength; that means, half of that length. So, you are talking about 100 nanometers is record the movement required to do a π by 2 phase shift.

So, it has to be very accurate right, so usually you will have the mirror mounted on a piezo device and you are applying a voltage and its moving through that distance you could also have a grating. And, do not worry about it too much which can have a grating when you move the grating the phase in the order will change slightly or you can also tilt your mirror to create a phase there are other techniques too these are some of the common techniques right.

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In the fiber interferometer it is quite simple to introduce a phase shift because you get a ph, so the cylinder with fiber wound around it. And you just splice this fiber into your reference arm and then when you apply a voltage that cylinder will contract and the fibers wound very tightly around it, so the length of the fiber also changes, maybe the refractive index also a little bit. And you can calibrate this to say for this voltage I get this phase change right.

So, it does not matter how you do it there are a variety of ways of doing it, but to crack the problem of getting phase out of that interference pattern you need multiple measurements. And known phase shifts introduced between each of those measurements are ok. There are in fact ways of also extracting phase without doing multiple measurements. Bagath is working on that for his PhD and maybe if you are interested you can talk to him about that.

So, you can also just beat the whole system and say I will take one intensity reading and somehow extract phase from that ok. But for this class this is good to know and this is actually a technique that many people still use, it is a standard phase shifting is a standard thing ok. So, with that we will close this part and let me move to the next so are there any questions on this part ok.

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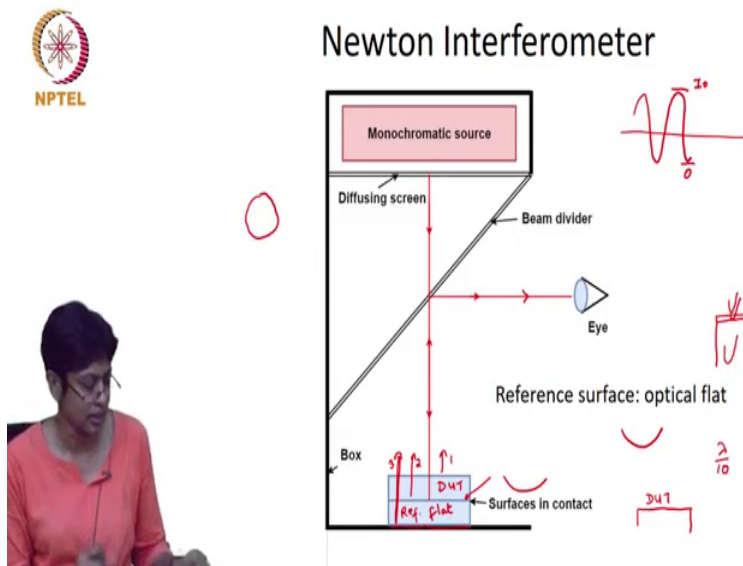
Interferometry for Testing



So, with this part you should have got a basic introduction into what is interference, what are the conditions for interference, why it is important and how we actually extract phase ok. So, for the next few classes we are going to look at applications. briefly I have mentioned some of the applications I want to go into a little more detail.

So, it should be fairly clear that I can use interference to tell if there are 2 waves if there is a difference between 2 wave fronts. So, I can extend this idea to say if I am interested in testing an optical element the shape of that element. If light were to travel through or reflect that element the wave front is going to change according to that element's shape.

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So, by studying the interference pattern I should be able to say something about the elements' shape itself ok. So, a Newton interferometer is used commonly for testing optical elements. How does the Newton interferometer work? I always need at least 2 beams. So, here is the source, light travels through the source from the source through this beam splitter. So, it is not a it its partly sending some light through that light comes and hits this element now this actually is 2 elements one is a reference and on this you put your device under test ok.

So, the device under test is resting on this reference surface. It has to be an optical reference; it is called an optical flat; that means, its surface has been polished to a very high flatness because we are worried about the variations in λ . So, it's polished to a very high flatness its fairly easy to get optical elements with a roughness of less than λ by 10 and you can if you need to you can even do λ by 20 ok. But so this is your reference flat this lower and this is the device under test where do I get the 2 beams from I need 2 beams.

Student: (Refer Time: 32:05).

It is a reflection measurement, so I might have the 2 beams of interest coming here where the 2 beams coming from.

Student: Reflection from (Refer Time: 32:19).

Reflection from this surface and interface.

Student: (Refer Time: 32:31).

He says reflection from here and here.

Student: (Refer Time: 32:34).

Now, he says here because he is psychoanalyzing my voice and saying it does not sound right ok, I will tell you it is this reflection and this reflection ok. So, the surface that we want to study is usually placed here so let us say I wanted to study a convex surface I would place it like this ok. But, I will turn back [laughter] and ask you a question why I am not worried about this reflection. Your first instinct was to say that it is from there so I can assume I have 3 beams coming back. Why am I not worried about 3 it does not matter let it be there.

Student: (Refer Time: 33:42).

I am worried about the third beam. Is it going to bother me when I try to analyze the surface, no it is not going to bother me that it does. It is a problem and I do not want it I do not want that beam there.

Student: (Refer Time: 34:11).

No, you have not even answered this question. I have many more questions for you, why do not I want that beam there. So, first of all this picture is not really drawn to scale my reference flat and the surface or the device under test then they will not be of the same thickness ok. So, if I were to draw this to scale it to be much more like this is the device under test and this is the reference why does that matter, why does that make a difference.

Student: (Refer Time: 34:53).

Remember.

Student: (Refer Time: 34:56).

I said do not make the assumption if I give you 2 beams from the same source you have interference do I always have interference.

Student: (Refer Time: 35:06) it needs.

It needs half coherence and I have put a monochromatic source up there. By the way this is not a laser this will be something like a sodium vapour lamp or something right. So, it is more monochromatic than the light you get from a tube light, but it is not a laser source ok. So, it has a shot much shorter coherence length in a typical laser source right.

But by having this so far I have a reflection from here I have a reflection here these 2 beams must interfere for this system to be useful to me. That means, a path length difference between the beam coming of the upper surface and the interface must be less than the coherence length and the beam coming from here. This is way more than the coherence length; the pathing difference is larger than the coherence length.

So, it does not play a role in the interference, but can it still cause problems. Remember if I have a good interference pattern my visibility is going to be high right; that means, if the peak intensity here is some value I naught this could even be 0 if I have got it perfect right. But, if I have another reflection coming in some other light coming in that light is not playing a role in the interference it is going to create some it is going to have some level here.

It is like a dc light that exists everywhere it does not play a role in the interference, but it will affect the contrast, so I don't really want to have a reflection. So, one is I ensure that the flat reference flat is large enough so that if there is a reflection it does not play a role in the interference. And secondly, I can make this region dark or black absorbing so that I minimize the amount of reflection ok.

Now there have these two beams coming back. What am I going to see? Remember we are drawing it as lines, but actually that is how you have a diffusing surface that is making scattering and making the beam larger. So, you are looking at a beam of some larger area ok, because I need an area over which to see fringes for it to be useful to me there is no point if its lying within a beam of infinitesimal size I must be able to see ok.

(Refer Slide Time: 37:36)

Newton Fringes: Optical Flat

- Fringes formed between two optically flat surfaces with a wedge between them

- Air gap = αx
 Dark fringes $2\alpha x = n\lambda$
 Bright fringes $2\alpha x + \lambda/2 = n\lambda$
 Distance (d) between two consecutive bright (dark) fringes: $\lambda/2\alpha$


What do the fringes look like?



So, let us say this is the reference flat and this is another flat. I am testing a flat I want to make a flat, so I place it one on top of the other. Now to our eyes it may look like these are placed flush against each other there is no air gap, but it is never that is never the case there is always a gap.

And, it can if you have not designed your flat accurately, if it is not flat it could even have a wedge in it, so when you place it one over the other it may be that you have a wedge like this ok. What kind of fringes are we going to see or how do the fringes look like you are looking from the top. So, this is a side view and now I am asking you look from the top what are the fringes that you will see, straight line fringes right.

(Refer Slide Time: 38:41)



Newton Fringes: Optical Flat

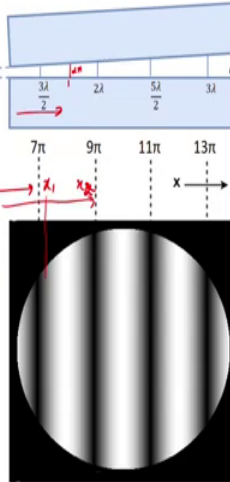
- Fringes formed between two optically flat surfaces with a wedge between them
- Air gap = αx

Dark fringes $2\alpha x = n\lambda$

Bright fringes $2\alpha x + \lambda/2 = n\lambda$

Distance (d) between two consecutive bright (dark) fringes: $\lambda/2\alpha$

Handwritten notes:
 $2\alpha x_1 = n\lambda$
 $2\alpha x_2 = (n+1)\lambda$
 $x_2 - x_1 = d = \frac{\lambda}{2\alpha}$
 Fresnel eqn \rightarrow always a π phase shift on reflection in air $n > 1$



Why straight line fringes because you have the reference wave the way of coming of the reference flat is a plain wave and the other wave is tilted and you have these two waves that I interfere right. So, you will have straight line fringes there is a linear change in the optical path length. Now, you could if it were as simple as this you will say if the angle the wedge is alpha then the air gap is going to be alpha x where x is this distance let us say this is x equal to 0 right.

So, the alpha x at any height this is alpha x where if this is x measured from this origin as defined here right. So, the optical path length then is 2 alpha x because 1 wave travels to the lower surface and reflects back up. So, that is 2 alpha x and I am telling you the dark fringes will occur here what does that seem right. Why are the dark fringes occurring at n lambda or does not that sound more like the constructive you know should not you expect constructive interference whenever you have a multiple of lambda.

I am now asking you to go back and think about your electromagnetics light after all is an electromagnetic wave hitting a dielectric surface. So, you have air and you have a dielectric surface that happens on reflection.

Student: (Refer Time: 40:29) there is a pi.

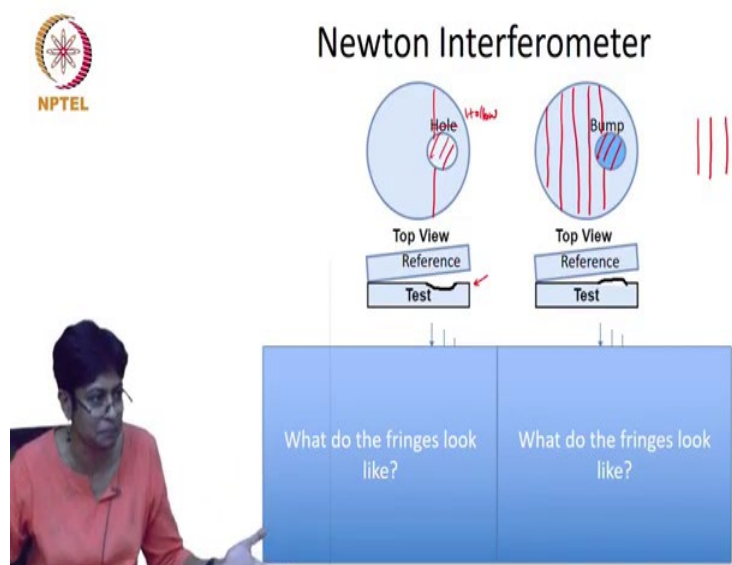
There is a pi phase shift so if you remember when you were when you were studying boundary conditions and Fresnel equations. Your Fresnel equations would have told you there is always

a π phase shift on reflection when you have air and some n which is greater than 1 then you have a π phase shift.

So, there is an additional π phase shift that is arising when you count if you look at this $2\alpha x$ that distance that is only taking a phase change that has occurred due to the path length, you need to take into account all of the phase changes to get the interference condition. So, because of that extra π phase shift the dark fringes occur here and therefore, the bright fringes occur here. But it does not matter you take distance from one dark fringe to the next or from one bright fringe to the next if you just take $2\alpha x$ is equal to n plus 1 lambda and $2\alpha x$ is equal to n minus we can do that $2\alpha x_1$ is n lambda and $2\alpha x_2$.

So, let us say this is the location this is x_1 and this is x_2 right this will be n plus 1 lambda it is the next dark fringe. And if I take the difference between these two so x_2 minus x_1 or the distance between them is going to be nothing, but lambda by 2α . And that is true whether I take the distance between two bright fringes or two dark fringes ok.

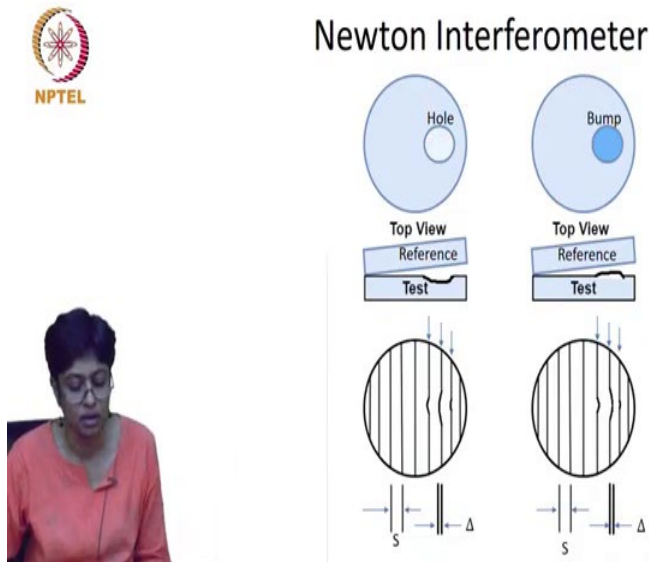
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Let us say I had a flat, but the flat has a small depression in it like here ok, so not a hole actually it is a hollow. I would rather than say a hole and say a hollow or the flat has a bump in it like this. What will happen to the fringes now if these were two flats on each other and you know there is a slight angle always between them? The fringe pattern is a set of straight lines in the region of overlap.

But now, in one area there is a depression or a bump what will happen to the fringes now any idea, everywhere other than this region the fringes are going to look the same right. Now somehow these fringes have to connect the fringe above this hollow and the fringe below this hollow, it will connect along the path where the optical path length difference is the same.

(Refer Slide Time: 43:45)




So, if you look at the fringes they will look something like this. So, you can actually use this to tell whether you have a depression or a bump in your optical flat, why is it you might know the way they make these flats is by grinding. So, you need to find out if in some region it is not been ground enough you need to know where and you have to grind that region or you have to grind the region around. So, if it is a depression you have to grind the region around to make it all flat if it is a bump you need to grind in that region to flatten that region.

So, you need to know that and this is a very Newton interferometer is a very easy instrument to use there is no alignment. Because the path line difference is so small it is like you are not aligning anything. You are putting your test element there and the reflection from the top and the bottom of this element is what is interfering right.

And, you just visually are looking at the pattern and deciding what to do so it is a tool that was used a lot earlier when a lot of the grinding was done manually. People would be doing the grinding that is put in there, look at the patterns and then go back and know where to grind it. Now of course, a lot is automated, but you would still use such interferometers and then we analyze the fringe pattern and tell the machine what to do or where to do what ok.

(Refer Slide Time: 45:15)



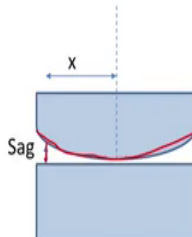
Newton Fringes: Curved surface

$$\text{Sag} = \frac{x^2}{2R}$$

$$\text{OPD} = \frac{x^2}{R} + \frac{\lambda}{2}$$

$$\text{Dark fringe } \frac{x^2}{R} = n\lambda$$

Distance of nth dark fringe from centre



Ratio of spacing between fringes

$$\frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}} \approx 1 + \frac{1}{2n}$$

Study curvature at low 'n'

$$x = \sqrt{n\lambda R}$$

So, what happens if by device under test the element under test is not a flat, but a curved element say you are doing making a convex lens what the fringes would look like over here. So, again it's always controlled by this path length difference and I can calculate if this is part of a sphere if this is part of a sphere. Then the sag is going to be determined by the radius of curvature of this sphere and the location from the center by this expression and again is an air glass reflection.

So, there is an extra lambda by 2 path lengths to think about or that pi phase and so the dark fringe is going to occur in this region what the fringes look like now.

(Refer Slide Time: 46:06)



Newton Fringes: Curved surface

- Fringes formed between an optical flat and a surface with short radius of curvature
- Surface deviates by 2λ (ie 4 fringes)



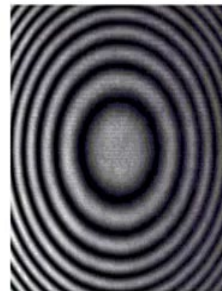
Student: (Refer Time: 46:11).

(Refer Slide Time: 46:15)



Newton Fringes: Curved surface

- Fringes formed between an optical flat and a surface with short radius of curvature
- Surface deviates by 2λ (ie 4 fringes)




If they were to fly it is going to be a set of concentric circles right. And not only does it tell you the fringes just by having a look at them you get an idea of the surface, but when the number of fringes is also telling you something. Because you know each fringe is happening at λ by 2 right so 4 fringes means you have a 2λ deviation.

So, you are even getting an idea of the actual path length difference, so it is not just the shape you are getting you are actually quantifying to some level. And usually people measure from dark fringe to dark fringe rather than from bright fringe to bright fringe and that is because it is hard for us to accurately find the center of the bright fringe.

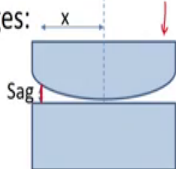
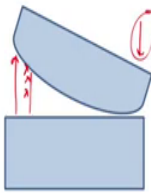
Where you might your detector might even be saturating over that region and you are not able to locate the center of the bright fringe, but the dark fringe is null is an more accurate measurement can be made of ok. So, although bright fringe to bright fringe or dark fringe to dark fringe distance is the same most measurements are made using the dark fringe ok.


(Refer Slide Time: 47:36)



Newton Fringes: Curved surface

- Observation of full circular fringes:
 - Max sag is $\lambda/2 \Rightarrow$ One fringe
- For errors less than this, centre of symmetry displaced
- Surface deviates by $\lambda/4$

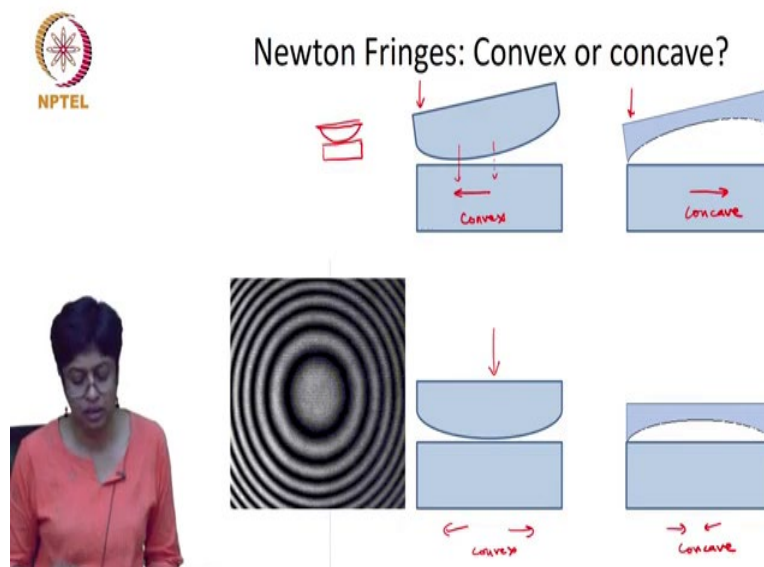


You can go a step further. You can say that well if it is a, if I have a bag of lambda I can see a fringe, but see the side is less than lambda. What do I do right to say this optical path length is less than lambda? How do I make up then I am not going to get a full fringe I need a full fringe. In order to so I need an optical path length to be at least a full fringe in order to see the fringes right.

So, you can move your test object and you can move it, so that you introduce extra path length. And that is what they show over here if you were if this distance were less than lambda by 2; I am not going to see a fringe. But, I can tilt it so I am basically pressing down on this side here and this is an exaggerated figure you are not going to make it of course so large it is going to move up a little bit, but that is going to add those extra lambdas in the path length.

So, you will have extra lambdas here right, and if you are doing this in a known way you then can say and if applied this much pressure then you see these fringes and you will get some idea about the type of right. So, you may either just use it not quantify, but to qualify is it a spherical surface is it a flat or so on from the nature of the fringes, you get if you get straight line fringes then you know that it is a flat for example ok. So, you can use it to qualify yeah not quantified things.

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How do I know whether it is convex or concave, whether it is convex or concave? I am going to get concentric fringes. How do I know whether it is convex or concave so again I will use movement. So, if initially I have placed it like this what you can do is press down here now when you press down here the center that was originally here has moved here.

So, your fringe pattern will also move so the center of your fringe pattern moves. So, if you are applying pressure on the left and the fringe pattern moves to the left then you would say you have a convex surface. On the other hand, if you press on the left and the fringe pattern moves in the opposite direction then you know you have a concave surface or you could say I will press in the center.

And, if you press in the center what happens to this fringe pattern you are slightly squeezing it down so the fringe pattern will expand. So, if the fringe pattern expands then you say it is convex and if the fringe pattern contracts then you will see its concave.

So, just an observation of fringes or how they behave when you tilt this element or move this element can give you an idea about the nature of the element ok. So, we started looking at applications and we have seen how an interference pattern can allow us to very quickly tell something about the shape of an optical element and we will look at more applications in the next class.